Plurals, Cardinalities, and Structures of Determination

Christopher U. Habel

Universität Hamburg, Fachbereich Informatik
Schlüterstr. 70, D-1000 Hamburg 13

Abstract

This paper presents an approach for processing incomplete and inconsistent knowledge. Basis for attacking these problems are 'structures of determination', which are extensions of Scott's approximation lattices taking into consideration some requirements from natural language processing and representation of knowledge. The theory developed is exemplified with processing plural noun phrases referring to objects which have to be understood as classes or sets. Referential processes are handled by processes on 'Referential Nets', which are a specific knowledge structure developed for the representation of object-oriented knowledge. Problems of determination with respect to cardinality assumptions are emphasized.

1. Introductory remarks

Most approaches to 'processing reference' are concerned with the case of singular NPs and deal with the complications of plurals only peripherically by remarks of the kind "The plural case can be considered analogously." But such hopes are only partially justified: the plural case is worse and therefore more interesting.

In the present paper I will discuss some specific problems of (in)definiteness with respect to plurals from an AI point of view. The heart of any knowledge-based system (KBS) - man or machine - is his/her/its knowledge base (KB), containing different types of knowledge (cp. sect. 2). The KB reflects the KBS' view of the world; in other (e.g. Jackendoff's, 1983) words: a projected world. (Giving emphasis to projected worlds and thus to mental models leads to a psychological foundation of semantics.)

The case easiest to manage is that of a complete and consistent KB. But in normal life - of man as well as machine - this almost never occurs; the knowledge is incomplete or inconsistent (or both). There are some reasons (cp. sect 3, 4) to see both types of problem as closely connected, as twin problems, abbreviated by I&I. It is important to extend the KBS' faculties with regard to the maintenance of I&I. This includes:

- Recognition and detection of I&I
- Correction of I&I, i.e. forcing completeness and consistency
- Dealing, i.e. arguing or 'thinking', with incomplete or inconsistent knowledge.

These tasks for maintaining I&I is of specific importance in processing reference.

2. The frame of representation

In representing the knowledge about the world (not linguistic knowledge) of a KBS I distinguish three types, *knowledge of facts*, *knowledge of rules*, and *knowledge of objects*, which is represented by 'Referential Nets' (RefN). The formal objects, which can be understood as internal (or mental) proxies for entities of the real (or other possible) world(s) are called 'Referential Objects' (RefO). RefOs can be seen as underdetermined formal objects (UFOs) in case of incompleteness, or as overdetermined (OFOs) in case of inconsistency.

For representing the knowledge of a KBS and the meaning of utterances I use a propositional 'semantic representation language' SRL. For processing, e.g. storing and retrieving, referential relations SRL contains specific 'description operators', which are from a formal point of view variable-binding, term-making operators. Here I will neglect the details of SRL and exemplify only those SRL-concepts which are involved in knowledge about objects (cp. Habel, 1986). The totality of RefOs and their properties (see below) form a net-like knowledge structure: the *Referential Net* (RefN). RefNs are based on three types of formal entities: *referential objects* (RefOs) as system-internal proxies of the objects of the world, *designations* of RefOs, i.e. terms (as opposed to formulas) of SRL and *attributes* to RefOs and designations. From a formal point of view (Habel, 1985, 1986) these (double-attributed) RefNs form a relation with

$$AARefN \subset (R\text{-}ATT \times REFO) \times D\text{-}TER \times D\text{-}ATT$$

Remarks:

1. REFO is the set of all referential objects at a specific point of time (I neglect time-indeces in the present paper); D-TER, R-ATT, D-ATT are the set of SRL-expressions of the types 'designating term', 'attribute to RefOs', 'attributes to pairs of RefOs and D-terms'.
2. Bracketing RefOs and their attributes reflects that in AARefNs the 1st component is functional dependent of the 2nd.

A first example will illustrate the concepts of the RefN:

(1) John's children will travel abroad during their summer vacation.

leads to the following entries in a RefN (only the most relevant parts are formulated; attributes are omitted in the present sect.):

(1') r.1 —— 'John'
r.2 —— ALL x : child_of (r.1, x)
     \ SOME x : travel (x, r.3, r.4)
r.3 —— "abroad"
r.4 —— "during r.2's summer vacation"

Remarks:

1. There are proxies for objects in a narrow sense as well as for some in a wider sense, e.g. w.r.t. locations (r.3) or time (r.4). Their SRL-designations will not be formalized here.
2. "ALL" is the intensional class-building operator, which differs from the formula-making universal quantifier. "SOME" is the indefinite plural term-making analogy to the definite "ALL". (On "SOME", the definite descriptor "IOTA" and the indefinite "ETA", which are used in (5'), cp. Habel 1982, 1986).

3. RefNs: Under- and overdetermination

In the following I will mainly deal with proxies for concrete objects, especially persons. A first analysis of the situation in question shows that a hearer of (1) possesses a RefO representing "John's children" without the obligation to know more details about them. e.g., though s/he does not have to know how many they are it is possible to refer to them definitely. With the introduction of the additional concept 'attribute of a RefO' it is possible to deal with the I&I problem, i.e. the problems of under- and overdetermination of formal objects. (Furthermore, the use of attributes leads to knowledge representations which allow easy and quick access to the objects in question, e.g. in anaphora resolution and generation). A more adequate analysis of (1) should lead to a representation, which represents the plural explicitly (and not only implicitly via "ALL"):

(1") card $\geq$ 2 —— r.2 —— ALL x : child_of (r.1, x)
human /

using a *cardinality attribute* to the RefO r.2 which represents the essential property that r.2's real-world counterpart is assumed to consist of more than one human being; the sortal attribute "human", which will be used here only, exemplifies another type of attribute, namely *sortal attributes*.

Remark:

By this attribute mechanism I represent the meaning of numerals, e.g. "John's two cars" leads to

card = 2- r.9 ——— ALL x : car (x) & own(r.1, x)

In text generation the communicative goals determine which designation(s) and R-ATTs are used to form the content of the message.

What counts as determinate depends on the type of attribute in question. Each type of attribute possesses its own set of completeness and consistency conditions. In the case of cardinality, the determinacy condition is given by

(2) **Cardinality Condition**:

Each set has exactly one cardinality.

This condition defines the ideal-state of the cardinality attribute which a system aspires to. The actual knowledge with respect to cardinality concerns a 'range of possible cardinalities'. From this follows what under- and overdetermination (I&I) are:

- in the case of underdetermination some cardinalities are possible, eg. the cardinality is greater or equal 2, but the exact value is unknown,
- in the determinate case only one cardinality is possible, i.e. the exact cardinality is known,
- in the case of overdetermination more than one cardinality is assumed, which violates the cardinality condition.

I will go on with John's children:

(3) The boys will visit France. Mary and Sue will go to Italy.

Analogously to (1) the RefN has to be extended to:

(3') card $\geq$ 4 —— r.2 —— ALL x : child_of (r.1, x)
card $\geq$ 2 —— r.5 —— ALL x : child_of (r.1, x) & boy(x)
SOME x : visit (x, 'France')
card $\geq$ 2 —— r.6 —— ALL x : child_of (r.1, x) & girl (x)
CONTAINS (r.7)
card = 2 —— r.7 —— CLASS ('Mary', 'Sue')
SOME x : visit (x, 'Italy')

Remarks:

1. "card(r.6) $\geq$ 2" because it is possible that there are further daughters of John. Note, that all boys - "card(r.5) $\geq$ 2" - visit France but only some girls, namely those represented by r.7, visit Italy.
2. I assume that the competence of calculating attributes is used in the maintenance of RefNs. By this "card(r.2) $\geq$ 4" is calculated from card(r.5) and card(r.6).
3. There exists an operator "IS_CONTAINED" dual to "CONTAINS", which I neglect in this paper (cp. Habel, 1986).

r.7 can be seen as determined with respect to cardinality since an exact value is assumed, whereas r.2, r.5 and r.6 are underdetermined. As a last example for cardinality computations, let us take the input

(4) John has four or five children. Three of them are girls.

That leads to the following changes in the RefN:

(4') card = 5 —— r.2 —— ALL x : child_of (r.1, x)
CONTAINS (r.5)
CONTAINS (r.6)
card = 2 —— r.5 —— ALL x : child_of (r.1, x) & boy(x)
SOME x : visit (x, 'France')
card = 3 —— r.6 —— ALL x : child_of (r.1, x) & girl (x)
CONTAINS (r.7)

Remark:

In a first step (corresponding to the first part of the input) card(r.2) is set to 4 or 5. In a second (inferential) step card(r.2) is computed to 5 based on the cardinalities of r.5 ($\geq$2) and r.6 (=3). In a third step card(r.5) can be computed to exactly 2.

Now we turn to overdetermination, i.e. inconsistencies. Suppose someone tells the KBS (or you):

(5) The oldest, Peter, travels to Spain.

What is there to do now? Where are the problems, how are they noticed, and how can they be solved? Before rejecting (5) with "That is impossible!" let us discuss the changes in the RefN:

(5') card = 5 —— r.2 —— ALL x : child_of (r.1, x)
card > 5
CONTAINS (r.5)
CONTAINS (r.6)
card = 2 —— r.5 —— ALL x : child_of (r.1, x) & boy(x)
card $\geq$ 3 ——
SOME x : visit (x, 'France')
CONTAINS (r.8)
card = 3 —— r.6 —— ALL x : child_of (r.1, x) & girl (x)
CONTAINS (r.7)
card = 2 —— r.7 —— CLASS ('Mary', 'Sue')
card = 1 —— r.8 —— 'Peter'
IOTA x : oldest(x, r.2)
ETA x: visit(x, 'Spain')

Remark:

The newly created RefO r.8 is integrated in the RefN by two links: on the one hand via CONTAINS from r.5 "the boys"; this link is inferred by use of knowledge about Christian names in English. On the other hand via the oldest-connection to r.2. Thus the cardinalities of r.2 and r.5 (in 4') have to be changed, which is realized by assigning a second cardinality attribute. (This reading of the sentence and interpretation of the net assumes a third son, "Peter", which visits Spain only. Note, that the inheritance about visiting France can be blocked via the 3rd designation of r.8.

The points of inconsistency or overdetermination can be located at the cardinality of r.2 ("card=5" vs. "card>5") and of r.5 ("card=2" vs. "card$\geq$3"). What is reasonable to do now? There are several possibilities:

- Reject the newest input. But why should "card=5" be preferable to "card>5" (or "card=2" to "card$\geq$3")?
- Try to solve the inconsistencies. Ask other people or undo inferences.
- Try to live with inconsistencies. Be aware that reasoning can be dangerous.

Why is it convenient and possible to follow the third strategy? On the one hand, though there are inconsistencies with respect to the cardinality of r.2 and r.5, these inconsistencies are localized and do not infect the whole KB. (This strategy of marking inconsistencies and thus avoiding infections of the KB, i.e. putting inconsistencies in quarantine, follows Belnap (1976)). Therefore the system is justified in answering questions with regard to other parts of the RefN.

On the other hand, locating points/areas of inconsistency and waiting for future information can lead - by means of inferences - to the solution of the inconsistency in question. One possible correction of the inconsistencies in (5') could be achieved by detecting that the informants used different concepts of 'daughter', e.g. 'daughter', 'adopted daughter', 'stepdaughter'. In the present example the "updating of the boys", i.e. the new "card(r.5) $\geq$ 3", was not given explicitly but was inferred from the male Christian names 'Peter'. It is possible that the inference in question, which uses common knowledge about Christian names, was misleading, because John's oldest daughter is nicknamed, she is "a girl named Peter" (as Russell's wife, who was known as *Peter* Spence).

Remarks:

1. Another way of analysis, namely concerning designations but not cardinalities, leads to a different solution with respect to r.8. Peter can be seen as a person visiting both France and Spain. Note, that this reading would also be based on a careful analysis of card(r.5).
2. The parallel example in German would lead either not to an inconsistency at all or to another type of inconsistency since gender-information of the article would distinguish between two cases: "Der älteste, Peter..." ('der' - 'masc.') leads also to (5'), but the possibilities for the solution of the overdetermination mentioned above are not usable in this case. "Die älteste, Peter..." ('die' - 'fem.') leads to linkage of r.8 to r.6, "the girls", and no inconsistency of cardinality would appear. But, most hearers would be suprised with the strange Christian name of the girl.

The similarities and differences of under- and overdetermination, i.e. the justification of the twin-concept I&I, can be seen best by discussing the appropriate response to questions like "How many children does John have?". On the one hand with respect to an underdetermined case, e.g.

(6) card $\geq$ 5 r.2' ALL x : child_of (r.1, x)

induced by "John has five or more children".

In the case of underdetermination (6) the KBS knows that it has incomplete knowledge and therefore it is justified in answering "Five or more, but I don't know exactly". In the case of overdetermination (5)the KBS knows that it has inconsistent knowledge. Therefore it should warn the questioner, e.g. by responding with "Presumably five or more, but I have contradictory information". Note, that it would be reasonable for you to use the concept of "John's children" in a similar way if you only have the information in question.

4. Structures of determination

From a formal point of view the cardinality attributes are examples of approximation structures similar to the information lattices introduced by Scott (1970); cp. Belnap (1976). The lower part of the structure of determination (see Fig. 1), "UD-CARD", represents the underdetermined and the upper one, "OD-CARD", the overdetermined cardinalities. The determined cases are represented by the "D-CARD" level, which is the symmetry axis of the structure. D-Card is the set of singletons over the set **N** of natural numbers (including zero); UD-CARD consists of the not-singleton elements of the power-set of **N** with the partial ordering induced by the set inclusion. OD-CARD is built up by introducing a 'dual to each UD-CARD' element, which is symbolized by square brackets "[_]".

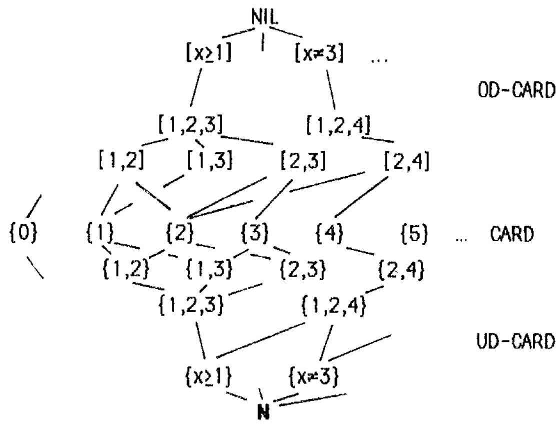

Fig. 1: Approximation structure CARD of cardinality attributes

The D-CARD elements stand for "the cardinality is exactly the n which forms the singleton in question". UD-CARD represents a set of possible cardinalities. The minimal entity in the approximation structure, namely **N**, holds no relevant information, since "Card=**N**" stands for "the RefO has a cardinality", and this is true for all RefOs. ('Card' is a set of cardinalities 'card'.) Getting input from communication or inferential processes, leads to climbing up the structure, which reflects the enrichment of information with respect to cardinality, or to no change in knowledge about the attribute. The ideal-level is reached at the D-CARD-level: an exact cardinality is assigned. Further input causes (in the good case) no change and in the bad case of inconsistency climbing up into the OD-CARD-region.

The structure of determination does not possess lattice properties; only the UD-CARD and the OD-Card parts are lattice-like. The sudden change at passing from UD-CARD or D-CARD to inconsistent OD-CARDs destroys the lattice properties (see below).

The approach of structures of determination, which is exemplified here with the case of cardinality attributes, can be used analogously with respect to other types of attributes. The base of all such structures are lattices, e.g. those of sortal attributes, which can be interpreted as approximation lattices. This means that climbing up the lattice can be understood as increasing information. (Note that the ALL-element in this interpretation is the bottom-element). In a (half) formal way, a structure of determination is built up from a Scottian approximation lattice (AL) by the following method:

1. Delete NIL from the approximation lattice AL.
2. Devide the rest in the level of determination (LoD) which is formed by the direct neighbors of the (now deleted) NIL and the underdetermined part of the lattice (UD-AL) which is given by those elements of AL which are neither NIL nor in LoD.
3. With respect to UD-AL construct a dual counterpart of overdetermined elements. This is called OD-AL.
4. Glue OD-AL with UD-AL via the level of determination LoD.
5. The ordering relations can be defined in the canonical way.

As mentioned for the case of cardinality attributes such structures of determination do not possess lattice properties. This is proven in Habel (1986). The same phenomenon is observed by Belnap (1976) with respect to his set of *epistemic states*, E. The lattice properties are violated at the passage to inconsistency (overdetermination). Nevertheless, the most relevant properties of Scott's approximation lattices also hold for structures of determination, especially the *ampliativity by input* (using Belnap's terminology). One very important difference between Scott's approach and determination structures concerns the NIL, which is the (!) failure element of ALs. In contrast, structures of determination contain many different failure elements, namely all beyond the level of determination. Thus a condensed history of informing and disinforming is abbreviated by the OD-attribute. (A chararcterization of Scott's approach could be: "All failures are equal, namely disastrous.") Repair processes, which e.g. can be triggered by input from an especially competent or believable informant, e.g. with respect to my example by John himself, lead to climbing downward in the structure. Note, that repairing is informing of a specific type. In contrast to normal informing it leads downwards; this changing of the direction demands a specific prior decision based on the experience that something was going wrong.

I conclude this section with a remark on overdetermination: Overdetermined objects are a specific type of *impossible objects* (cp. Rapaport 1985), which constitute a test case for every semantic theory. 'Impossibility' or 'non-existence' (as used in some approaches to this topic) refer to the real world and not to projected worlds, which are in the mind.

5. Conclusion

In this paper I have only dealt with I&I problems concerning the subtype of referential knowledge. Obviously, a similar approach is appropriate for the other subtypes of knowledge, i.e. for other formal objects. (Notice that essential properties of RefOs, such as cardinality, can also be seen as part of factual knowledge.) In the case of factual knowledge underdetermination or overdetermination concerns truth values. Belnap's (1976) four-valued logic with a lattice-theoretic semantics has influenced the concepts of the present paper from a logical point of view.

Some types of RefNs and of structures of determination are implemented as parts of prototypical text-understanding systems by the KIT-projects at the Technical University Berlin.

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