

USE OF A MINIMUM-ELLIPSE CRITERION IN THE STUDY OF ION-BEAM EXTRACTION SYSTEMS*

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Summary

Ion-beam extraction systems may be optimized by ray-tracing codes. As a general criterion for comparing the geometry-dependent phase-space distributions, we first calculate the minimum-area ellipse that encloses all particles of any given two-dimensional phase-space distribution. Then, the relation between ellipse area and contained beam fraction is established by systematically finding and eliminating those particles that contribute most heavily to the emittance. Prescriptions for finding the minimum ellipse and beam fractions will be presented. The minimum and rms ellipses are compared for two code-calculated distributions that represent ion-beam extraction geometries.

Introduction

Particle-beam kinematic properties often are described using the concept of average (rms) values.¹ An alternative method that we are proposing is to fit the two-dimensional phase-space distribution with the minimum-area ellipse that just encloses all particles. Ray-tracing codes like SNOW² or AXCEL³ show that the predicted transverse phase-plane distributions at the extraction region are quite nonuniform. Therefore, the calculated 4rms emittance,

$$\epsilon_{4rms} = 4 \left[\frac{\overline{x^2} \overline{x'^2} - (\overline{xx'})^2}{\overline{xx'}} \right]^{1/2}, \quad (1)$$

does not encompass all the particles. Furthermore, the rms ellipse is different in shape and orientation from an encompassing, minimum-area ellipse.

Many particle distributions exhibit extended tails that substantially increase the emittance; but the tails are occupied by only a small particle fraction. Eliminating particles in the tails would lead to much lower emittance beams without serious current reduction. Our minimum-ellipse method has been extended to a calculation of the minimum emittance versus beam fraction.

The proposed algorithm is a formal approach, not involving any physics of particle dynamics. The algorithm was developed and tested using a variety of artificially constructed distributions and is applied here to optimize the design of an ion-beam extraction system using the ray-tracing code SNOW. The algorithm is also well suited for the design of low-energy beam-transport sections; the procedure also can be applied to measured beam data.

Mathematical Background

Transverse phase-plane distributions are represented by (x,x') coordinate pairs for all trajectories. The minimization algorithm is valid for distributions projected into one phase plane or radial sections of the total distribution for cylindrically symmetric cases. For the latter, particles have currents proportional to their radial position.

Two different ways of representing ellipses are used: the "analytical parameters," leading to the ellipse equation

$$x' = \pm (B/A)(A^2 - x^2)^{1/2} + Cx, \quad (2)$$

with its emittance resulting from

$$\epsilon = A \cdot B, \quad (3)$$

and the Twiss parameters⁴ with the ellipse equation

$$\gamma x^2 + 2\alpha x x' + \beta (x')^2 = \epsilon, \quad (4)$$

with

$$\beta\gamma - \alpha^2 = 1; \quad \gamma, \beta \geq 0. \quad (5)$$

Using fixed Twiss parameters, ellipses of varying emittances but identical shape and orientation are drawn through every point, with the largest emittance ellipse encompassing all points. Thus, the question of a certain point being encompassed by a given ellipse is easily answered: one only has to compare the resulting emittance value for this point with the known emittance of the given ellipse.

For analytical parameters, a different criterion holds: one ellipse definition is that every circumferential point has the sum of its distances to the two focal points of the ellipse constant. Thus, whenever this sum is greater for a point than for the given ellipse, the point lies outside the ellipse. This criterion is used to identify and, consequently, eliminate the one point that gives rise to the largest emittance value of any given distribution.

The minimization process starts with a first-guess ellipse that, to a certain degree, determines the final minimum-area ellipse that encompasses all points. The symmetry-line angle (θ) of the first guess is determined by averaging the angles of all vectors from the origin to every distribution point, using the expression

$$\bar{\theta} = \frac{\sum \theta_i r_i^{REX}}{\sum r_i^{REX}} \quad (6)$$

where REX is a weighting exponent. The ellipse focal-points are located on this symmetry line, and the focal length is given by

$$F = r_{max} \cdot RFAC, \quad (7)$$

where r_{max} is the length of the longest vector and RFAC a scaling factor. The first-guess ellipse is then drawn through the largest-emittance point, as determined by the analytical parameter representation. An example of how the choice of RFAC and REX leads to different first-guess ellipses is shown in Fig. 1. The selection of the highest emittance point of any distribution may depend on the initial ellipse; however, the first-guess ellipse is drawn through the selected point in any case. We find that RFAC = 1.1 and REX = 0.8 leads to satisfactory convergence for most cases.

After determining the analytical first-guess ellipse parameters, they are converted to Twiss parameters using the transformations

$$\alpha = -A \cdot C/B, \quad \beta = A/B, \quad \gamma = A \cdot C^2/B + B/A. \quad (8)$$

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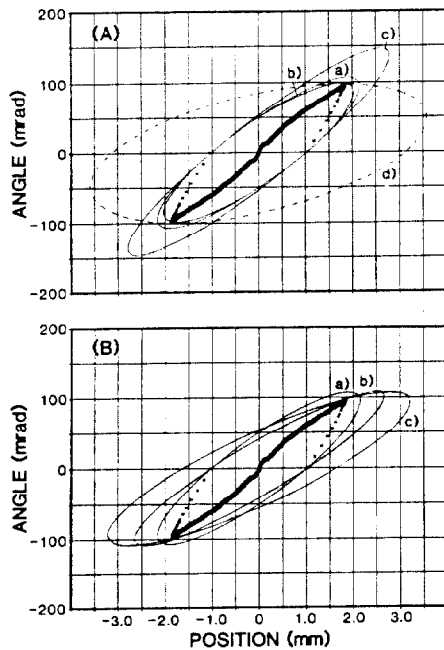


Fig. 1. Influence of parameters RFAC and REX on the first-guess ellipse. In Fig. 1(A), RFAC is equal to (a) 1.1, (b) 1.005, (c) 1.5, (d) 0.98. REX is held constant at 0.8 in all cases. For Cases (a), (b), and (c), the farthest point in the distribution tail is chosen to be eliminated. Figure 1(B) shows the influence of the parameter REX on the first-guess ellipses with RFAC = 1.1. For Curve (a), REX = 2.0 and 0.8, there being no visible difference between the ellipses. Curves (b) and (c) have REX = 0.2 and 0.1. In Case (c), a point on the distribution hump is selected for elimination whereas in Case (b), the selection seems ambiguous. Only for Case (a) with REX greater than 0.8 is the farthest point in the tail selected for elimination.

Then, starting from the first-guess ellipse, the alpha and beta parameters are systematically varied, minimizing the emittance while still enclosing the whole distribution. From a series of runs with different distributions we found that eight α and β variation cycles give converged results. It may happen that, because of the discrete character of the data, the minimization process gets locked into a local minimum and does not reach the absolute minimum. The more data points that are included, the more likely the absolute minimum will be found. The problem is usually solved by a more judicious choice of the initial angle and/or focal-point separation parameters.

After finding the minimum encompassing ellipse for the entire distribution, the code eliminates the largest emittance point and fits a new ellipse around the remaining fraction. The total current is then reduced by the current carried by the eliminated ray. The last-fit ellipse is always taken as a new first guess, and the code varies the alphas and betas to find the new minimum ellipse. In this manner, the large emittance components of the distribution are reduced as a function of beam fraction.

Results with Ray-Tracing Distributions

The minimum-ellipse procedure was used to study distributions derived from two different ion-extraction geometries calculated with the SNOW code. The geometries and their calculated trajectories are

shown in Fig. 2. The single difference between the two geometries is the width of the electrode in contact with the plasma, this width being 0.4 mm in (A) and 0.2 mm in (B).

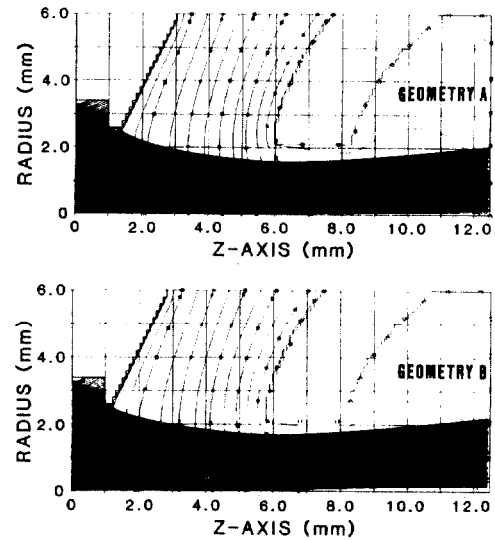


Fig. 2. Two ion-extraction geometries and their calculated trajectories from the SNOW code.

Figures (3) and (4) show the calculated distributions (100 and 64% beam fractions), the 4rms, and the minimum ellipses for the ion-extraction geometries introduced in Fig. 2. Figure 5 exhibits the 4rms and minimum-ellipse emittance versus beam fraction for the two geometries. Both the 4rms and minimum-ellipse emittances are always less for Geometry (B). For both extraction geometries, the emittance is reduced by a

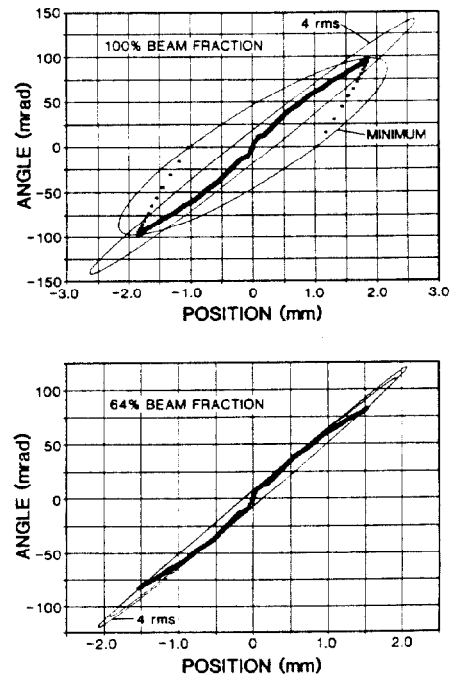


Fig. 3. Calculated distribution from Geometry (A) (as shown in Fig. 2) at the 100 and 64% beam fractions. The 4rms and minimum ellipses are also shown.

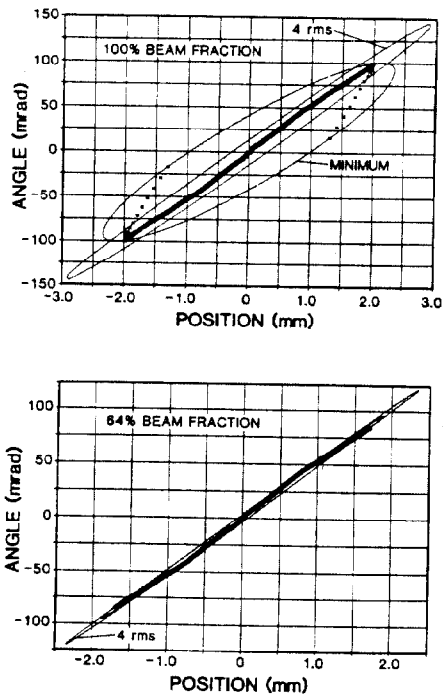


Fig. 4. As in Fig. 3, but for Geometry (B) of Fig. 2.

factor of 3-4 for only a 10% reduction in beam intensity, showing that a much brighter beam could be obtained for little sacrifice in intensity.

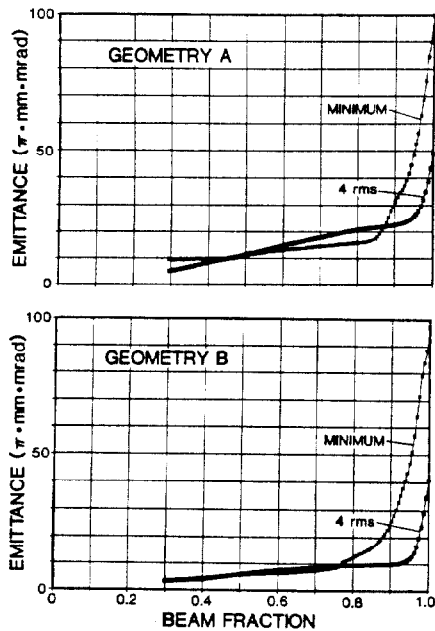


Fig. 5. The 4rms and minimum emittances plotted vs the beam fraction for the ion extraction Geometries (A) and (B).

We define a mismatch factor $\delta R/R_i$ between the rms shape parameters $(\alpha_1, \beta_1, \gamma_1)$ and the minimum-ellipse shape parameters $(\alpha_2, \beta_2, \gamma_2)$ using the notation of Guyard and Weiss⁵ as

$$\frac{\delta R}{R_i} = \left[(\Delta + 2 + \sqrt{\Delta^2 + 4\Delta}) / 2 \right]^{1/2} - 1, \quad (9)$$

where

$$\Delta = \beta_1\gamma_2 + \beta_2\gamma_1 - 2\alpha_1\alpha_2 - 2. \quad (10)$$

Figure 6 shows the mismatch factor between the rms and minimum-ellipses for the two geometries versus beam fraction. Significant mismatch is found for the larger beam fractions, but the mismatch is not bad below 0.7 beam fraction. A tentative conclusion is that to obtain maximum current transmission into a subsequent accelerator or beamline, the minimum-ellipse parameters are the optimum set to choose.

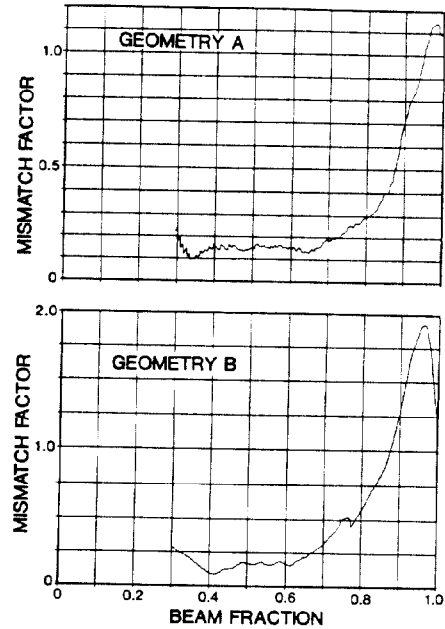


Fig. 6. Mismatch factors between the rms- and minimum-ellipses as outlined in Eqs. (9) and (10) for the (A) and (B) Geometries.

Conclusions

We have developed a procedure to calculate a minimum-area ellipse that encloses all trajectories in a two-dimensional transverse phase plane. The procedure has been integrated into a computer program that compares the minimum- and rms-ellipse results. Two distributions derived from the SNOW code have been examined. The rms- and minimum-ellipse approaches lead to the same conclusion concerning the optimum ion-extraction geometry. However, large mismatch factors were calculated between the rms and minimum ellipses. Because the minimum ellipse contains all of the particles at the given beam fraction, it may be desirable to use the minimum-ellipse parameters in matching situations to achieve the maximum transmitted current.

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