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BEAM CALCULATIONS FOR LAMPF MUON CHANNEL*

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I. Introduction

We will describe here calculations of the beam properties of the stopped muon channel at the Clinton P. Anderson Meson Physics Facility (LAMPF) of the Los Alamos Scientific Laboratory. We focus mainly on recent work done during 1974 which was helpful in tuning the muon channel for maximum performance and for understanding the channel behavior. Comparison with actual channel performance during this period and some notes on the computer programs used will be presented also.

II. Muon Channel Description

The general layout of the LAMPF muon channel is shown in Fig. 1. The four quadrupoles and two bending magnets immediately following the pion production target constitute the pion collector section. This section selects a band of pion momenta and transports it to the long straight section of ten quadrupoles which is called the decay section. It is here that muons arising from pion decay in flight are captured. The remaining magnets constitute the analyser section. The fourth bending magnet BM-04 is turned off when operating in the south area. When this magnet is energized, the beam is directed to the west area. The analyser section selects a desired muon momentum region.

III. Channel Characteristics

A. General Remarks. Now let us look at the three sections of the channel in more detail. Consider pions of a given momentum decaying in the decay section. The laboratory momentum spectrum for the resultant muons extends from about one-half of the pion momentum value (backward decays) to slightly greater than the pion momentum (forward decays). Between these limits the muon momentum spectrum is proportional to β_μ which means it is almost constant except for very low values of the pion momentum. Muons near the center of this momentum region correspond to large laboratory decay angles. The decay section can accept muons with decay angles of up to about only five degrees. Because the maximum laboratory decay angles are much larger than five degrees for the whole practical range of pion momenta, only extreme forward and backward decays are accepted and the muon momenta spectra widths for a given pion momentum are narrow (less than about 2 per cent for pion momenta up to 230 MeV/c).

The pion collector section consists of two bends and four quadrupoles. When properly tuned (i.e., with quadrupole gradients set to maximize pion acceptance), pions of the central momentum (momentum for which the bends are set) are scraped on the walls of the first two quadrupoles only. All pions of the central momentum that make their way past the first two quadrupoles survive all the way to the end of the decay section (unless they decay first). Pions of other momenta are scraped by various magnets along the way. With the assumed effective apertures, the peak acceptance for pions is 61 msr and the momentum spectrum width (FWHM) is 8.6 per cent. The pion collector tune which produced these results has the polarities - + B + - B in which + signifies that the quadrupole focuses horizontally and B is a bending magnet. This set of polarities was chosen because of the approximate achromaticity it provides. An achromatic tune was selected to maximize the accepted pion momentum width. By reversing some quadrupole polarities in the pion collector and reoptimizing gradient values, slightly narrower tunes were found. None of the alternate tunes could improve on the central momentum solid angle acceptance of 61 msr. Consequently, the achromatic tune is the most useful found so far. Because the width of the pion momentum spectrum is always much larger than that of the muon momentum spectrum for a given pion momentum, the actual muon momentum spectrum at the end of the decay section is approximately equal to the pion width.

Neither the south nor the west analyser has sufficient resolution to decrease noticeably the muon momentum spectrum width. The width of the muon spectrum for the best 240/132-W tune (p_π = 240 MeV/c, p_μ = 132 MeV/c, west analyser) was found to be 8.4 per cent. The resolution of the analyser is sufficient, of course, to eliminate all pions in the beam for this backward decay tune since they are double the momentum and only about 8 per cent in momentum width. Unfortunately, the analyser (either south or west version) transmits only about a fourth of the available mesons because of its limited transverse phase space acceptance.

The muon momenta spectra for muons emerging at the end of the channel for the above mentioned tune (called Tune 1 in this paper) corresponding to various pion momenta are shown in Fig. 2. For each of the pion momenta shown, the pion solid angle acceptance is plotted above the corresponding extreme backward muon momentum. We see that the number of muons arising from extreme backward decay is roughly proportional to the number of pions available for decay. This occurs because an extreme backward decay corresponds to a zero decay angle so that as the pion turns into a muon, only the momentum of the particle is changed and the transverse phase space acceptance of the decay section is not very dependent on momentum.

B. Simple Model for Muon Flux. Because of the channel characteristics described above, it is evident that a simple model of a properly tuned channel can be made which will determine the muon flux as a function of the central muon momentum (momentum for which analyser is tuned). For backward decay tunes we can use the following expression

$$N_\mu = \frac{d^2N_\pi}{d\Omega dp_\pi}\,\Delta\Omega\Delta p_\pi\, e^{-L_c/L_o}\,(1-e^{-L_d/L_o})N_b(\theta) \qquad (1)$$

where $\Delta\Omega$ is the pion solid angle acceptance (61 msr), Δp_π is the pion width (8 per cent), L_c is the length of the pion collector section (8.3 m), L_d is the length of the decay section (9.4 m), L_o is the pion laboratory decay length (0.0559 m cp_π/MeV), and $N_b(\theta)$ is the fraction of backward decay muons decaying into laboratory angles less than or equal to an angle θ. The value of this angle is not critical; a value of 5 degrees was found to work well. The decay factors in the formula above give the fraction of the pions entering the channel that decay in the decay section. The peak value of this fraction is about 27 per cent and it does not vary substantially over the range over which the muon channel will be used.(pion momenta from 100 to 250 MeV/c). The fraction of decays into less than 5 degrees rises rapidly with momentum. The peak differential muon flux can be determined by

$$\left(\frac{dN_\mu}{dp_\mu}\right)_{max} = \frac{1}{\Delta p_\pi} \; \frac{p_\pi}{p_\mu} \; N_\mu \qquad (2)$$

where p_μ is the backward or the forward, depending on the tune, muon momentum corresponding to pion momentum p_π. Neither of the above formulas is good for predicting absolute numbers of muons but is useful for showing dependence on momentum. Figure 3 shows these results normalized to their value at 94 MeV/c. The results for four south analyser backward decay tunes determined by the computer model to be described below are also shown for comparison. The simple model given by Eq. 1 and 2 is a good indicator if a new computer or experimental tune is as good as a previous tune for it enables comparison between tunes of different momenta.

The forward decay tunes, the decay factors in the expression for total muon flux must be replaced by

$$e^{-(L_c + L_d)/L_o} \qquad (3)$$

since muons generated in the pion collector are also accepted by the channel. Forward tunes thus produce a greater muon flux but are unattractive because of the large pion and electron contaminations. For this reason, only backward tunes have been studied in detail.

IV. Computer Tuning

In order to determine the output muon beam as a function of the magnet settings a computer model of the channel was used. The LAMPF muon channel had been originally designed with the help of two computer programs.[1] One was a CERN program[2] modified at Los Alamos by H. Vogel to be applicable to pions produced in a field free region. The second is a computer program developed by S. Ohnuma at Yale University. The Yale program has been further refined and was used for most of the present channel tuning calculations.

This computer program calculates properties of the beam emerging from the muon channel (both pions and muons) for given channel parameters and input pion distribution. The pion distribution at the pion production target is characterized by a uniformly populated polygon in both horizontal and vertical transverse phase spaces. The input pion momentum spectrum is described by specifying the number of pions per MeV/c into this phase space area at several discrete values of the pion momentum. Then for each pion momentum, the initial phase space polygons are traced to the end of the channel or until all the pions are completely lost by striking aperture walls. This is done by applying the appropriate linear transformation, element by element, and cutting off those portions of the transformed polygons that extend past the assumed element apertures. This cutting algorithm was taken from the CERN program.[2] The transformations assumed that the quadrupoles are ideal long lenses. For the bending magnets, the effect of edge focusing is considered and the first order correction to vertical focusing due to the finite field fall-off distance is included. The transformation matrices are recalculated for each momentum value.

The muon distributions are calculated by considering the pion polygons at the entrance of each element as sources of muons. The decay angles are taken into account by shifting the polygons in the x' and y' directions by the projection of the decay angle on the horizontal and vertical planes, respectively. The muon polygons are traced to the end of the channel similarly to the pion polygons. Only muons that emerge through a given rectangular region a given distance from the last quadrupole (the experimental target) are counted. The output of the program consists of plots of phase space and beam spot densities reported at ten values of a predetermined muon momentum range. Integrals over these distributions and muon polarization are also presented.

The computer tuning was done with the help of an optimizing version of the Yale program. We define a "tune" to be a relative maximum in the muon flux as a function of all the magnet settings. The computer program varies magnet settings (usually quadrupole gradients but also bend fields) to determine a tune in the vicinity of the starting point. Tunes are found to scale well with momentum. That is, if a set of quadrupole gradient values at a given π/μ momentum are at a local maximum in the muon flux then the same will be true approximately at a different momentum if the fields are changed in proportion to the momentum.

The optimizer operates as follows. The problem is to maximize f, the pion or muon output flux as a function of the magnet settings. This function is first determined at the initial point x_0, a vector whose components $x_0(i)$, $i = 1,2,\ldots N$ are the quadrupole gradients and bend fields of the N magnets that are to be varied. A gradient of f at the point x_0 is then estimated by calculating f at N points in the vicinity of x_0. Once the gradient is determined, two points x_1 and x_2 along this gradient direction are determined and f is evaluated there. These two points and the original point x_0 are then fitted to a parabola to determine a maximum along this direction (if any). The f is calculated at this new point which is used as the new starting point x_0 for the next iteration. It is usually necessary to vary about 10 magnets at a time (up to 20 have been varied at once with good results). With a good starting point, only a few cycles are required to achieve a relative maximum in the flux to good accuracy. Even in this case, however, computer times of more than 15 minutes on the CDC 7600 are required. Optimization of muon flux is done for a single pion momentum because of such computer time limitations. Once a tune is determined, then a full pion spectrum calculation with no optimization is done to determine beam characteristics for this tune.

Figure 4 shows the west analyser 233/128 tune which provided the greatest muon flux. The absolute value of the quadrupole gradient is plotted as a function of the magnet number (in the order that they occur in the channel). Quadrupole polarities are shown above the magnet numbers (+ means horizontally focusing). Also shown on this graph is the corresponding experimental tune. The experimental tune was determined by experimental optimizing on the actual muon channel. The fields in the magnets in the channel are controlled by an on-line PDP-11 computer which maximizes the muon flux (or other experimental signal) by varying channel magnets one at a time. Several other tunes besides the one shown in Figure 4 have been investigated. The following behavior was observed. The muon channel magnets were set to the values determined by optimizing on the computer model of the channel. Then using this starting point, the experimental optimizer found a tune near the starting point increasing the muon flux by a factor of approximately 2.5 in this process. The same behavior was noted when an experimental tune was used as the starting point for optimizing on the computer model. The various tunes investigated differed in polarities for various quadrupoles in the analyser section. All cases studied showed the good agreement between the experimental and predicted tunes shown in Fig. 4 except that there were larger deviations for the lower momentum 150/75-S tune, possibly because of the larger effect of multiple scattering for lower momenta.

V. Comparison with Experiment

The previous section pointed out the good agreement between predicted and measured tune point locations. Let us now look at channel performance at various tunes shown in Table I. These tunes were optimized by considering only one pion momentum value to save computer time. The various beam characteristics were then calculated for a full pion spectrum. Tunes 1, 3, and 4 were also done experimentally on one day and provide a good indication of tune-to-tune dependence of muon flux since variations in proton beam line tune and counting efficiencies were held relatively constant over this period. The last two columns in Table I show the good agreement between prediction and measurement in the total muon flux relative to Tune 1.

Table II shows the absolute muon flux and other properties of the muon beam for Tune 1. Both computed and measured quantities are shown. We see that the range width was predicted reasonably well. The beam spot size was predicted to within the resolutions involved which were about 3 mm for both the calculation and the measurement (by photographic film by R. L. Hutson). The absolute muon rates are not accurately predicted. The prediction is higher than the measured value by a factor of more than 2. Such factors have been observed for other tunes also. The reason for this has not been determined but the fact that other aspects of channel behavior are so well described by the computer model makes one suspect that the input pion flux estimate may be wrong. That is, the pion production cross-section or primary proton beam may not be what we have assumed.

VI. Discussion

We have seen that because of the limited angular acceptance of the decay section, and the large momentum acceptance of the analyzer the output muon momentum spectrum width is approximately equal to the accepted pion momentum spectrum width, i.e., about 8 per cent. Because of this behavior a simple model can be constructed which shows how muon flux increases with higher momentum tunes. Such an increase in muon flux with momentum and the fact that tunes scale with momentum has been verified with a computer model of the muon channel. Such a model is necessary to determining the actual quadrupole gradient values that are required.

In order to determine tunes, i.e., the location of relative maxima in the output muon flux as a function of all the quadrupole gradients, an automatic optimizer is essential because there is no obvious way to set the more than twenty magnets. The optimizer which has been added to the muon channel program has been successful in finding sets of gradient values that are good tunes and exist in the real channel. Over-all, we have seen that, except for the mysterious inability to predict absolute muon rates accurately, the computer model provides an accurate description of muon channel behavior. This situation is helpful in analysing changes in beam characteristics for changes in experimental requirements (experimental target size, shape, and location) and the retuning required since we now have some confidence in the ability of the computer model to predict actual behavior.

The possibility exists, however, that improved muon rates can be achieved by finding better tunes. Computer time limitations make an exhaustive search of the many dimension space of quadrupole gradient values impossible. One can never know for certain whether the ultimate tune has been achieved. As an indication of this situation, we note theexistence of Tunes 1 and 2 which are two different relative maxima in the muon flux for the same set of polarities, momentum, and experimental target requirements.

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Table I. Calculated and Measured Fluxes for Five Tunes.

Tune No.	Tune[1]	Analyser Polarity	Comments	Calculated Muons per Pion[2,3]	Calculated Total Muons[2,4]	Measured Total Muons
1	240/132-W	+-+B++-+	Best tune	0.287×10^{-3} (1.00)	36.6×10^{-4} (1.00)	(1.00)
2	"	"	Different mode of above	0.241 (0.84)		
3	"	-+-B+-+-		0.236 (0.82)	23.4 (0.63)	(0.61)
4	"	+-+B-+-+		0.186 (0.65)	17.9 (0.49)	(0.53)
5	150/75-S	+-+-+	South analyser lower momentum	0.0774 (0.27)	5.78 (0.16)	

1. The 240/132-W tunes were actually measured at 233/128. For west analyser tunes, fluxes are for muons into a 7.5 cm x 7.5 cm counter 0.7 m from the last quadrupole. For the south leg tune, the counter is 15 cm x 15 cm 1.5 m from the last quadrupole.

2. Numbers in parentheses are relative to Tune 1.

3. The input pion flux is one pion of the central momentum per steradian.

4. The input pion flux is one pion per MeV/c per steradian with a flat momentum spectrum.

Table II. Comparison of Calculation and Measurement for 240/132 west analyser tune (Tune 1). The measurements were done at 233/128 on negative muons. The rates and widths are for muons into a 7.5 cm x 7.5 cm counter 0.7 m from the last quadrupole.

Quantity	Units	Calculated Value	Measured Value	Difference
Peak dN_μ/dp_μ for 1 pion per MeV/c per sr	$(\mathrm{MeV/c})^{-1}$	4.75×10^{-4}		
Total muons for 1 pion per MeV/c per sr		58.3×10^{-4}		
Multiply above by $d^2N_\pi/d\Omega dp_\pi=6.2\times10^6$ to get: Total muons/μA·sec	kHz/μA	36.1	13.9	-61%
Stops in 2 gm/cm^2 target (CH_2)	kHz/μA	18.6	6.9	-63%
Range curve FWHM	gm/cm^2	2.9	3.6	+24%
Spot size horizontal dimension FWHM	cm	3.8	3.5	- 8%
Spot size vertical dimension FWHM	cm	9.8	10.2	+ 4%
Average muon polarization		0.89		

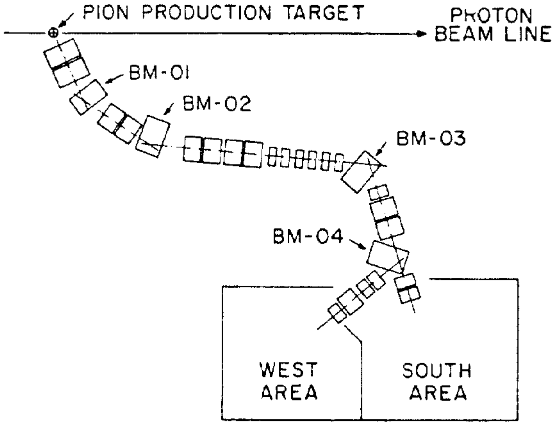

Fig. 1. The Muon Channel Consists of Four Bends and 23 Qudrupoles.

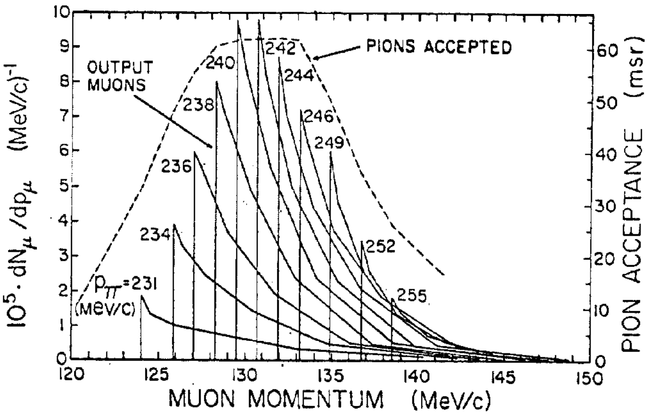

Fig. 2. Output Muon Momenta Spectra for Various Pion Momentum Values.

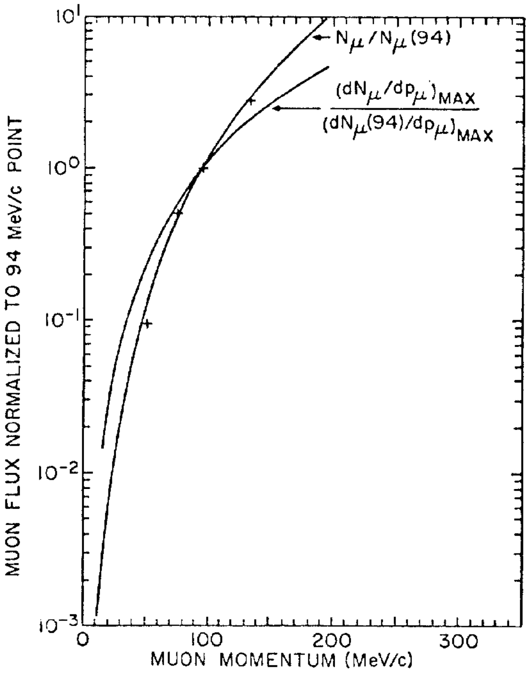

Fig. 3. Total and Peak Differential Muon Fluxes as a Function of Muon Momentum Predicted by the Simple Model. The Four DisCrete Points are Computer Model Results.

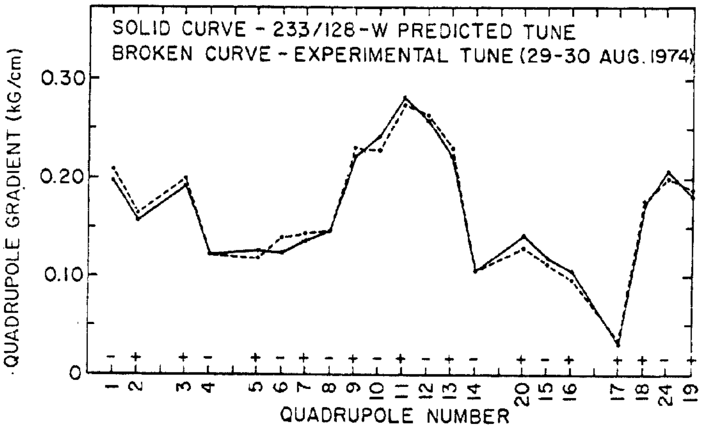

Fig. 4. Predicted and Measured 233/128 West Analyser Tunes (Tune 1).