LONGITUDINAL COOLING OF A STRONGLY MAGNETIZED ELECTRON PLASMA

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Abstract

The optimal values of Q and $(\omega - \Omega_r)$ for cooling a pure electron plasma with a microwave bath have been calculated. An electron plasma which has no internal degree of freedom, cannot be cooled down below a heat bath temperature. However, the longitudinal cooling can be achieved by energy transfer from the poorly cooled parallel degree of freedom to the well cooled (by synchrotron radiation) perpendicular degree of freedom. To do this, we introduce a microwave bath to the electron plasma. A microwave tuned to a frequency below the gyrofrequency of the electron forces an electron moving towards the microwave to absorb a photon and then to move up one in Landau state. The electron loses longitudinal momentum in this process, so that the longitudinal energy can be reduced. On the basis that most of the electrons are in the ground or first excited state, we set up a transition equation and develop a FEM code. With an appropriate condition for B-field and intensity of the microwave, the cooling times for several values of Q and $(\omega - \Omega_r)$ are calculated and the optimal values are found. Applying the optimal values at appropriate times in a cooling process, the best cooling can be obtained. For an electron plasma magnetized with 10T B-field, cooling to the solid state can occur within 2 hours. Without this optimization, times were always several hours, longer than the life time of the plasma in real system.

INTRODUCTION

The concept of crystalline non-neutral plasma, regarded as a new state of matter, has been studied for a variety of fundamental and applied physics areas, including the study of space-charge-dominated beams, the study of Coulomb crystals, the realization of high luminosity ion colliders, the application to ultra-high resolution nuclear experiments and to the atomic physics research, etc. Crystallization occurs as non-neutral plasmas and beams are cooled below the transition temperature. In fact, as seen in many Penning trap experiments, the non-neutral plasmas has three different phases: fluid, fcc, and bcc[1]. The Crystallization in one dimension has been observed in the beams at the Aarhus accelerator[2], in agreement with calculations[3], and crystallization in three dimension has been observed in the ion Penning trap at NIST[4] and in dusty plasmas[5].

In high energy physics, Penning traps and antiparticle storage rings have been used for experimental tests of the CPT theorem, which predicts equivalence of various physical parameters such as masses, charge-to-mass ratio, magnetic moments, and gyromagnetic ratio for particles and antiparticles. Charged particles can be confined perfectly in an ideal cylindrically symmetric trap with a uniform axial magnetic field, which is the basic setup of the Penning trap. This approach, the use of Penning trap, has been favored and widely used because the particle can be cooled down to a temperature of the order of tens of mK. Penning traps at CERN have been used to capture antiparticles for high-resolution measurements for proton mass and for mass spectrometry of nuclei[6].

Recently, laser cooling has been the primary approach towards obtaining such ultra cool beams and plasmas. A laser that is tuned to a frequency below the resonant frequency of the ion is directed at the ionic plasma. Ions moving towards the laser beams see an upshifted laser beam, and thus can absorb the light. Subsequently they spontaneously emit a photon isotropically. Thus, in the full process, they lose momentum by recoil. This leads to cooling. Such a method naturally works only for ions, not electrons or protons, as they have the internal resonances needed for narrow absorption. For non-ionic beams, electron cooling has been used, but such cooling has not produced ultra cool beams[7].

For this reason we already investigated phase transition of strongly magnetized electron plasmas in Penning traps and we concluded that the phase transition can occur on the condition that longitudinal temperature is below a certain value irrespective of transverse temperature. Now the question is how to decrease the longitudinal temperature to the critical value. As one of the possible ways we suggest a microwave cooling method. Applying a tuned microwave into the longitudinal direction, the longitudinal energy can be reduced and then the temperature can be dropped down below the critical value. This means that the electron plasma crystallization can be achieved. In the following sections we suggest how to reduce the longitudinal temperature and the result is shown.

MICROWAVE COOLING

Two Level Equations

In the low transverse temperature limit, the longitudinal energy can be reduced by the microwave radiation. Absorption of a microwave photon in Penning trap by an electron, thus moving it up one in Landau state, can reduce the parallel energy, just as laser cooling works for ionic plasma and equilibria. Then the spontaneous radiation reduces the transverse energy, so that the transverse state move back to the original Landau state and finally the transverse temperature is the same to the heat bath temperature. From

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the entire process only the longitudinal temperature can be decreased.

The longitudinal velocity distributions during the process can be described by Master equations. With an assumption that most of particles are in the ground or first excited state, the equations are

$$\frac{\partial f_0(u_z)}{\partial t} = \sum_{k_{||}=\pm k_0} [f_1(u_z + \frac{\hbar k_{||}}{m}) D(u_z + \frac{\hbar k_{||}}{m}, u_z) - f_0(u_z) W(u_z, u_z + \hbar k_{||})] \quad (1)$$

$$\frac{\partial f_1(u_z)}{\partial t} = \sum_{k_{||}=\pm k_0} [f_0(u_z - \frac{\hbar k_{||}}{m}) W(u_z - \frac{\hbar k_{||}}{m}, u_z) - f_1(u_z) D(u_z, u_z - \hbar k_{||})] \quad (2)$$

where $f_n(p_z)$ is the distribution of transverse quantum number n. The spontaneous and the stimulated transition probabilities are defined as

$$P_{||} = \frac{2e^2\omega^2}{3c}\frac{\hbar\Omega}{mc^2} \quad (3)$$

$$w_{||} = \frac{I\sigma}{\hbar\omega_R}\frac{(\gamma/2)^2}{(\gamma/2)^2 + \Delta^2(\omega, k_{||}, u_z, v_z)} \quad (4)$$

$$D_{||} = \frac{P_{||}}{\hbar\Omega} + w_{||} \quad (5)$$

$$W_{||} = \frac{P_{||}}{\hbar\Omega} exp[\frac{-\hbar\Omega}{k_B T_\perp}] + w_{||} \quad (6)$$

with resonance conditions,

$$\Delta \equiv \omega - \omega_R \quad (7)$$

$$\hbar k_{||}(= \pm\hbar k_0) = m(v_z - u_z) \quad (8)$$

$$\hbar\omega_R = \hbar\Omega + \frac{m}{2}(v_z^2 - u_z^2). \quad (9)$$

Cooling Condition of Nonneutral Plasmas

The problem we should consider is that the cooling is very effective as long as the width is smaller than the standard deviation of the velocity profile. When a microwave with a frequency ω_R and a wave number $k_{||}$ propagates along the magnetic field in the plasma, an electron with longitudinal momentum p_z along the magnetic field experiences, because of the usual Doppler effect, a shifted frequency

$$\omega' = \omega + k_{||}\frac{p_z}{m}. \quad (10)$$

If ω' coincides with the electron cyclotron frequency Ω, then the resonant absorption of the wave energy by electrons will take place. This phenomenon is effective in the vicinity of the resonance frequency. The particles near the resonance in velocity space lose their momenta so that they move into the lower velocity space. From the resonance conditions the entire particles near the resonance are shifted into the lower velocity space. Moreover, if the resonance frequency is well-chosen, then the particles can almost lose their momentum. Thus the width of final distribution can be almost the same as the width of microwave, which means that the lowest temperature of particles can be determined by the width of microwave. This gives a condition of microwave applied to the particles. The initial width of microwave should be less than the standard deviation of initial velocity distribution[8]. The condition in mathematical form

$$\frac{\delta\omega}{k_{||}} < \frac{\sqrt{\langle p_z^2 \rangle}}{m} \quad (11)$$

where $\delta\omega = \gamma/2$ is the width of microwave, implies that the smaller width of microwave gives the lower temperature of final distribution. However if the initial width of microwave is too small, the number of particles involved with the microwave is so small that cooling time can be too long. So, in order to find the fastest way to cool the particles we should take a larger width initially. In that case the temperature is still too high to reach the critical temperature at which $\Gamma_{||} \approx 170$. Then it is necessary to take a smaller width at the moment to continue to cool the electrons down to the critical temperature.

RESULTS

Taking appropriate conditions about the initial state, the longitudinal temperature can be estimated after the state reaches an equilibrium. Our system is supposed to be immersed in a liquid helium heat bath, so that the initial temperatures (transverse and longitudinal temperatures) can be 4.2K, the temperature of liquid helium. We apply 10T as its magnetic field, $0.7 \times 10^9/cm^3$ as its number density to the plasma in the trap, and at least 10^4 as the Q factor of microwave cavity. For the number density ($0.7 \times 10^9/cm^3$), the critical temperature which gives $\Gamma_{||} = 170$ is approximately 14mK.

In order to find the best way to cool the electron plasma, we have applied various values of Q and $\Delta\omega$ ($\Delta\omega \equiv \Omega - \omega$) to the plasma. For the set of (Q,$\Delta\omega$), the cooling rates and transverse temperatures are calculated. For convenience of calculation we take the $\Delta\omega$ as

$$\Delta\omega \equiv P\frac{\gamma}{2}. \quad (12)$$

With the aid of the definition the cooling rates and transverse temperature are calculated as functions of two variables, Q and P.

The cooling rates that the plasma takes as long as Eqs. (11) is valid are calculated for various values of Q, P, and the microwave intensity $I\sigma$. From our simulations, the cooling rates are maximized at $Q = 4.0 \times 10^4$ for $P = 3.0$, $Q = 5.0 \times 10^4$ for $P = 4.0$, and $Q = 6.0 \times 10^4$ for $P = 5.0$. In table 1 the cooling rates for the three sets of (P, Q) by changing the intensity $I\sigma$ are shown.

As expected, the stronger microwave gives the faster cooling. However, the transverse temperature becomes

Table 1: The cooling times (sec) for various values of the microwave intensity $I\sigma$ and P.

	$(P, Q/10^4)$		
$I\sigma(\times\hbar\Omega)$	(3.0,4.0)	(4.0,5.0)	(5.0,6.0)
5.0	13488.4	21783.1	33791.8
10.0	7881.5	12028.4	18037.0
15.0	6014.8	8784.8	12795.2
20.0	5082.0	7167.1	10179.3
25.0	4522.2	6198.9	8612.9
50.0	3400.5	4273.1	5496.8
75.0	3023.8	3636.2	4469.2
100.0	2833.8	3318.8	3959.9

higher as the microwave becomes stronger. Then the number of particles in the second excited state is not so small that our two level assumption will not be valid any more. Therefore, the maximum intensity of microwave should be determined by the two level assumption. For 10T as its magnetic field strength, the maximum intensities are $25\hbar\Omega$ for (3.0,4.0) as $(P, Q/10^4)$, $50\hbar\Omega$ for (4.0,5.0),and $75\hbar\Omega$ for (5.0,6.0).

With the best values of P, Q, and $I\sigma$ to cool the plasma, however, it still takes too long to have the critical temperature. From the result of our simulation the cooling time is almost 6 hours for any of three cases. The time is unrealistic in experiments. The plasma profile in Penning trap with the high magnetic field cannot stand for the long time. The reason why it takes too long is explained as follows. The plasma is cooled so rapidly that the temperature reaches the value at which breaks Eqs. (11), and then the cooling becomes much slower because the high peaked central frequency of applied microwave is too far from the plasma profile. The real intensity of microwave to apply to the plasma is too weak to cool it as rapidly as before its temperature reaches the value.

For this reason we take the other way to cool the plasma more rapidly. At the moments whenever the plasma breaks the Eqs. (11), we change the central frequency and microwave width in the velocity space. This can be achieved by changing P and Q. As done initially, the best sets of P, Q, and $I\sigma$ to get a cold plasma rapidly can be chosen in the same way. In our results the P and $I\sigma$ are not changed during the whole simulation, but the Q is changed at the moments whenever the plasma breaks the condition. The best Qs are satisfied that $\sqrt{T_{||}} \times Q$ for the different values of Q and $T_{||}$ is always the same to the initial at the moments. Applying this method, we calculate the cooling times for various cases. The result is shown in table 1 where the initial Qs are picked up 4.0×10^4, 5.0×10^4, and 6.0×10^4 for their Ps. The successive Qs are determined during the simulations by the condition that $\sqrt{T_{||}} \times Q$ is the same.

In Fig 1, time evolutions are shown for $P = 3.0, Q = 4.0 \times 10^4$ and $I\sigma = 25\hbar\Omega$. Also, in table 2 the times when the Qs are determined.

As we expected, the cooling times can be reduced as much as we wanted. The fastest cooling times is more than 1 hour and less than 2 hours which is very realistic in Penning trap experiments.

Table 2: The times when Qs should be changed for various values of the microwave intensity $I\sigma$ and $P = 3.0$.

		$P = 3.0, I\sigma = 25.0$		
t (sec)	Q	α	$T_{||}$	$T_{\perp}$
0.0	40000	0.92999	4.2	4.2
2539.5	85129	1.97910	0.927294241	10.167975
3732.9	181180	4.21617	0.204713972	10.167198
4293.1	385614	9.04924	0.045192312	10.163583

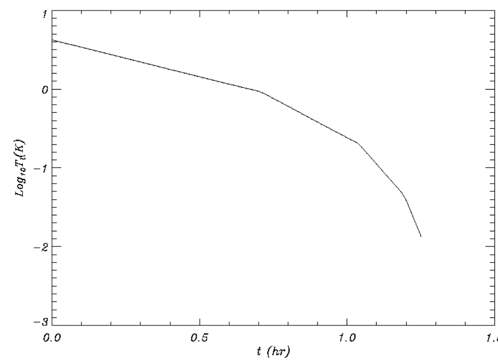

Figure 1: Time evolution of the longitudinal temperature for $P = 3.0$ and $I\sigma = 25\hbar\Omega$.

DISCUSSION

Applying microwaves into the longitudinal direction, the temperature can be decreased below the critical temperature by exchange of energy between two degrees of freedoms. With the condition that an appropriate low temperature heat bath reduces the transverse energy of the plasma continuously, the electron plasma crystallization can be achieved in Penning traps in 2 hours.

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