ADAPTIVE SPACE CHARGE CALCULATIONS IN MADX-SC*

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Abstract

Since a few years MADX allows to simulate beam dynamics with frozen space charge à la Basseti-Erskine. The limitation of simulation with a fixed distribution is somewhat overcome by an adaptive approach that consists of updating the emittances once per turn and by recalculating the Twiss parameters after certain intervals, typically every 1,000 turns to avoid an excessive slowdown of the simulations. The technique has been benchmarked for the PS machines over 800,000 turns. MADX-SC code developments are being discussed that include the re-introduction of acceleration into MADX and more advanced beam sigma calculations that will avoid code interruptions for the Twiss parameters calculation.

INTRODUCTION

With the demand for ever higher intensities [1, 2] and the SC tuneshifts already exceeding 0.5 there is a need in reliable tools for simulations which take into account the specifics of the lattice including imperfections. The PIC codes use more detailed information on the SC forces but are quite slow.

On the other hand there are codes (see references in [3, 4]) that use a predefined beam shape (usually Gaussian) for the SC kick calculations. Though this is only an approximation to the real SC distribution it greatly increases the speed making such codes suitable for simulations studies of various factors e.g. the strategy of the optics correction. It is also important that they use exact formulas for the chosen beam shape making the SC kick manifestly symplectic (at least in the 4D case), hence these codes can be referred to as ***symplectic codes***.

Depending on whether the beam sizes used for the SC calculations are fixed or updated according to the evolution of the tracking particle ensemble, the model is called either ***frozen*** or ***adaptive***. Both models are available with MADX-SC.

In the present report we discuss the MADX-SC present status and future developments.

MADX-SC CODE FEATURES

MADX-SC uses for SC simulations the BEAMBEAM elements providing kicks according to the Basseti-Erskine formula for Gaussian bunches. The simulations take a few steps:

- preparation of the lattice (at present no name longer than 8 characters is allowed),
- insertion of the BB elements using an external program which cuts – if necessary – long elements,
- finding self-consistent optics functions and beam sizes with linear SC forces,
- tracking an ensemble of particles with ***non***linear SC forces modulated by the particle synchrotron motion. The latter feature is activated with command *option, bb_sxy_update=true* before entering the TRACK module [5].

The requirement for the existence of the self-consistent closed optics limits the applicability of the present approach. The way to overcome this limitation is discussed in the last section.

Some new features were recently introduced in MADX-SC e.g. option *bucket_swap* which returns particle leaving the RF bucket at one end to the other end like it happens in a train of bunches.

Space Charge Kick

In order to represent the space charge kick accumulated over distance L_n the number of particles N_n in a fictitious colliding beam must be set as:

$$N_n = B_f \frac{N\, L_n}{C\,(\gamma^2 - 1)}\ , \tag{1}$$

where $B_f = I_{\text{peak}} / I_{\text{average}} > 1$ is the bunching factor, N is the total number of particles in the beam, C is the machine circumference and γ is the relativistic mass factor. Eq. (1) was obtained for ultra-relativistic BB elements, to work correctly with the latest versions of MADX command *option, bb_ultra_relati=true* should be issued.

Emittance Update

The tracking can be performed with either frozen or adaptive SC. Both methods have their pros and contras. The frozen SC method is faster but usually underestimates the emittance blowup and particle losses since the tunes of individual particle do not change. On the other hand, the adaptive method can strongly overestimate the blowup and the losses as the reduction in the SC tuneshift can drive new and new particles onto a resonance while in the case of frozen SC they would remain stable.

The effect of the SC tuneshift reduction will be exacerbated in simulations if the beam sizes are computed with the r.m.s. emittances which are dominated by halo particles with large amplitudes. Therefore the suppression of the halo contribution is necessary but in such a way that the core contribution was not affected.

Presently the following algorithm is implemented in MADX which uses the self-consistent optics functions to calculate the action variable values J (half the Courant-

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Snyder invariant) in the transverse planes for each particle.

The distribution in each plane is sought for as an exponential distribution (we drop the index of the plane)

$$F(J)=\frac{1}{\varepsilon}\exp(-J/\varepsilon)\,, \tag{2}$$

where ε is the geometric emittance which must be found by fitting the tracking data. For this purpose it is better to use the integrated distribution function. In the model it is

$$f(J)=\int_0^J F(J')dJ'=1-\exp(-J/\varepsilon)\,, \tag{3}$$

while in tracking

$$g(J)=\frac{1}{N}\int_0^J\sum_{k=1}^N\delta(J'-J_k)dJ'=\frac{1}{N}\sum_{k=1}^N\theta(J-J_k)\,, \tag{4}$$

where N is the number of microparticles, $\delta(x)$ is Dirac's delta and $\theta(x)$ is the step function defined here as $\theta(x)=0$ for $x\le 0$ and $\theta(x)=1$ for $x>0$.

To do the fitting the microparticles must be ordered so that $J_k\ge J_{k-1}$. Now we can introduce weight function $w(J)$ and construct penalty function

$$P(\varepsilon)=\sum_{k=1}^N w(J)\{J/\varepsilon+\log[1-g(J)]\}^2\big|_{J=J_k}\,, \tag{5}$$

Looking for the extremum w.r.t. ε we get for the particular form (4) of $g(J)$

$$\frac{1}{\varepsilon}=-\sum_{k=1}^N w(J_k)J_k\log[1-\frac{k-1}{N}]/\sum_{k=1}^N w(J_k)J_k^2\,. \tag{6}$$

In the program the weight function was chosen as $w(J)=1/(J^2+J_0^2)$ with some small J_0 to avoid division by zero. This weight function provides a moderate suppression of the halo contribution and gives the exact result for an exponential distribution $g(J)$.

Unlike the transverse dimensions the bunch length is presently determined via simple r.m.s. value.

The adaptive mode is initiated by command *option, emittance_update=true* before entering the TRACK module.

The method is quite fast but requires the knowledge of optics functions which are changing with the beam parameters. They can be periodically recomputed but this is associated with difficulties mentioned in [4].

EXPERIENCE WITH ADAPTIVE SC

Both frozen and adaptive SC models were checked against experimental data obtained at CERN PS and SPS [4]. Figure 1 borrowed from [6] shows relative emittance increase over $5\cdot 10^5$ turns at 2GeV observed experimentally at PS and in MADX adaptive simulations. The vertical tune was kept at $Q_{y0}=6.476$, the SC tuneshifts were estimated as $\Delta Q_x\approx$-0.05, $\Delta Q_y\approx$-0.07.

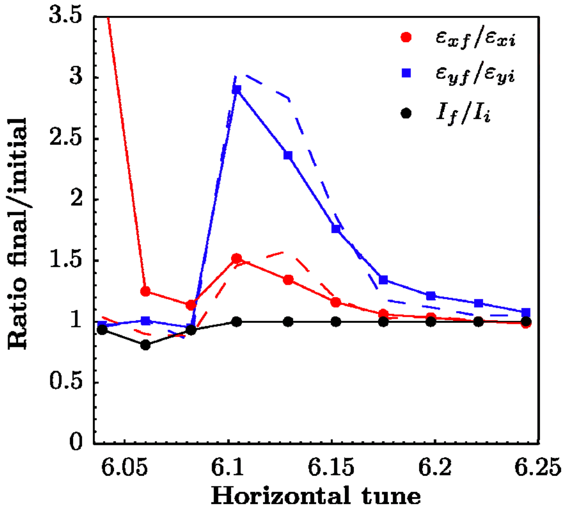

Figure 1: PS emittance evolution vs. Q_{x0}. Dashed lines present experimental results, solid lines with dots show MADX simulations with adaptive SC.

For tunes $Q_{x0}>6.1$ the agreement between the experiment and the adaptive SC calculations is better than with the frozen SC model but for tune as low as 6.037 the adaptive SC calculations predict a dramatic horizontal emittance blowup which is absent in both the experiment and the frozen SC model.

There are a number of possible explanations of this disagreement. First, the inevitable statistical fluctuations in calculated emittances produce noise in the SC forces which artificially heats the beam over that many turns.

Second, at $Q_{x0}=6.037$ it was no longer possible to perform Twiss runs during the simulations which indicate that particle motion is basically locked to the (half) integer resonance. We found a constant rate of emittance growth over time that was not present when simulations were done in the frozen mode.

In absence of stable self-consistent optics there was an appreciable mismatch in the initial distribution which excited the beam envelope oscillations. Figure 2 shows initial emittance evolution with frozen (red) and adaptive (green) SC. The envelope oscillations produced satellites of the parametric resonance extending its reach in the case of adaptive SC.

Hopefully it will be possible to study these effects in more detail in the future.

Fermilab Booster SC Simulations

A study is going on of the possibility to increase beam intensity in the Booster by as much as 50% with the same injection energy 0.4 GeV. Simulations were performed with beam parameters providing the SC tuneshifts $\Delta Q_x\approx$-0.45, $\Delta Q_y\approx$-0.55. The bare lattice tunes were chosen in the usual range $6.5<Q_{x,y0}<7.0$ so that the beam tunespread overlapped the half-integer value.

The measured optics from [7] was used but with a different strategy of the optics correction [8]. Even with corrected half-integer stopband there was strong vertical beta-beat for the total tune $Q_y\ge 6.3$.

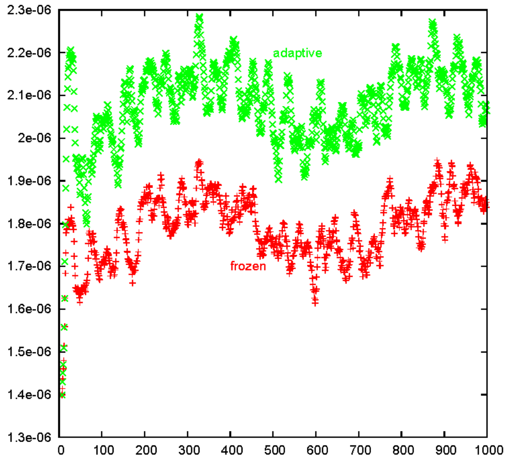

Figure 2: Horizontal emittance over first 1000 turns.

Particle tracking was performed over 1000 turns in both frozen and adaptive modes (see Fig. 3) with a small number of microparticles, $N = 2000$. The total horizontal tune was kept at $Q_x = 6.25$.

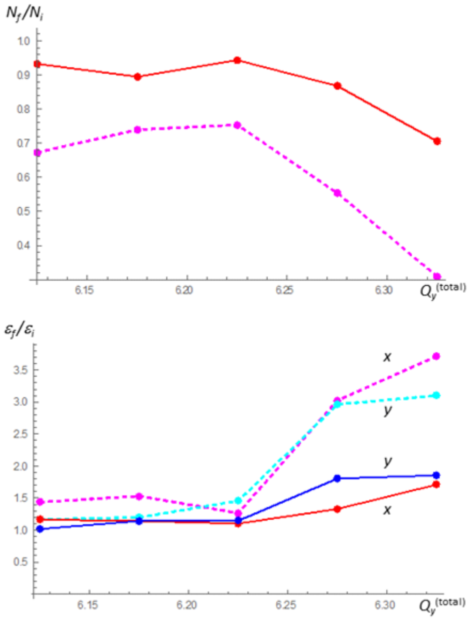

Figure 3: Relative intensity (top) and emittance (bottom) evolution over 1000 turns vs. total vertical tune in frozen (solid line) and adaptive (dashed line) modes.

In the adaptive mode both losses and emittance growth were significantly higher than in the frozen mode. As in the case of CERN PS the beam size noise and mismatch induced oscillations can be a factor. In addition there was a noticeable longitudinal mismatch providing the SC force modulation at twice the synchrotron tune, $Q_s \approx 0.1$.

But the major factor supposedly is the mentioned before fast reduction in the SC tuneshift which drive new and new particles onto the half-integer resonance – the process that can be called "suction by a resonance". Comparison of low intensity results with experimental observations shows that simulations overstate this mechanism, probably due to insufficient halo suppression in emittance calculations.

OUTLOOK

Both the level of the noise and the severity of the resonance "suction" in the adaptive SC simulations can be alleviated by a stronger halo suppression. In the accompanying report [9] a new method is presented which provides halo suppression in a non-ambiguous way and does not require the knowledge (or even existence) of stable self-consistent optics.

The method is applicable for any number of coupled degrees of freedom and yields a fitted Σ-matrix which then can be propagated around the machine with the help of linear transport matrix T as

$$\Sigma(s) = \mathrm{T}\Sigma(0)\mathrm{T}^{\mathrm{t}}, \tag{7}$$

where superscript "t" means transposition. After the new BB elements are determined the tracking will be performed as in the present version of MADX-SC.

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