WAVEGUIDE STRUCTURES FOR RF UNDULATORS WITH APPLICATIONS TO FELS AND STORAGE RINGS *

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Abstract

RF undulators, suggested long time ago, has the advantage of fast dynamic control of polarization, undulator strength and wavelength. However, RF undulators require very strong RF fields in order to produce radiation of the same order as conventional static devices. Very high power RF energy confined inside a waveguide or a cavity can provide the necessary RF fields to undulate the electron beam. However, the wall losses in the waveguide should be low enough to make it practically feasible as a CW or quasi CW undulator and, hence, competitive with static devices for applications to storage rings and FELs. Here we present various waveguide structures such as smooth walled and corrugated walled waveguides and various RF modes. We will show that there are some advantages in operating with higher order modes and also with hybrid modes in the corrugated guide. We will show that the RF power requirement for some of these modes will permit a quasi CW operation of the undulator, thus permitting its operation in a storage ring.

INTRODUCTION

In an undulator a highly relativistic electron beam passes through a wiggler field to produce synchrotron radiation. The wiggler field can be either magnetostatic or electromagnetic in nature. The first experimental demonstration of an RF undulator was done in 1983 by Shintake and others [1]. RF undulators require a very large amplitude electromagnetic wave to generate synchrotron radiation level comparable with static undulators. Such a large amplitude electromagnetic wave can be generated by “pumping” a RF cavity with power from a high power precise amplitude and frequency RF source.

Some of the advantages of RF undulator over static undulator are small undulator periods and larger apertures are possible, beams with larger radius and emmitance can be used; helical undulators are relatively easy with microwaves and more importantly the radiation properties such as polarization, flux and wavelength can be varied from pulse to pulse which is not possible with static undulators. However, it is very challenging to build a RF cavity that can handle tens of giga watts of RF power while keeping the wall losses low.

We are designing a microwave undulator to be used in SPEAR3 ring at SLAC. The requirement is that it should generate at least a tenth of circularly polarized radiation flux that can be generated by a proposed static undulator. The undulator should be capable of producing switchable polarization radiation energy between 700 and 900 eV.

The choice of the waveguide for the undulator cavity is restricted to have a circular cross-section as the wave should be “rotate able” to control radiation polarization. The operating RF mode should have very strong undulating RF field near the waveguide axis and weak tangential magnetic field near the guide walls to minimize wall losses. To satisfy these requirements we have considered TE_{11} and TE_{12} circular waveguide modes and balanced hybrid HE_{11} mode in a corrugated waveguide as possible candidates for the design of the microwave undulator.

We present our analysis of the three modes as applied to RF undulators in this paper.

ELECTRON MOTION AND RADIATION CHARACTERISTICS

Electron motion in a circular polarized standing wave is characterized by

$$\beta_x + i\beta_y = -\frac{K}{\gamma}\left(e^{i(k_{||}+k)z} + e^{i(-k_{||}+k)z}\right) \tag{1}$$

$$\beta_z = 1 - \frac{1+2K^2}{2\gamma^2} - \frac{K^2\cos(2k_{||}z)}{\gamma^2} \tag{2}$$

where $K = eB_0/m_0ck_{||}$ is the deflection parameter, B_0 is the peak RF magnetic field strength on the axis. γ is the relativistic factor, k and $k_{||}$ are the free space and axial wave numbers of the RF wave in the undulator cavity. β_x, $\beta_y$and β_z are the velocity components of the electron in x, y and z directions normalized to velocity of light in free space, c.

For a standing wave RF undulator the resonant radiation wavenumber k_s is given by

$$k_s = \frac{2\gamma^2}{1+2K^2}\left(k + k_{||}\right), \tag{3}$$

where we have considered interaction with only the wave component traveling in the opposite direction of electron motion.

It can be shown from [2] that the flux radiated by an electron beam with average beam current I within a bandwidth of $\frac{\Delta\omega}{\omega}$ in the central cone can be expressed as

$$F = \alpha L\frac{IK^2k_s}{2e\gamma^2}\left(J_0\left(\xi_0\right) - J_1\left(\xi_0\right)\right)^2\frac{\Delta\omega}{\omega} \tag{4}$$

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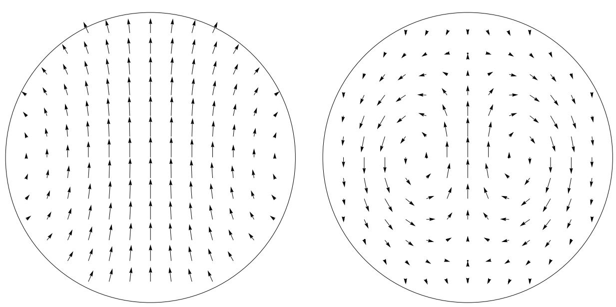
where $\alpha = e^2/(2\epsilon_0 c\, h)$ is the fine structure constant, ϵ_0 is permittivity of free space, h is Plank's constant and $\xi_0 = \frac{K^2}{1+2K^2}\left(\frac{k}{k_{||}}+1\right)$.

WAVEGUIDE AND MODE STRUCTURES

Circular Cylindrical Waveguide

The simplest waveguide structure for an RF undulator with circular polarization is a circular cross-section cylindrical waveguide. The fundamental mode in this waveguide is a TE_{11}-mode (Fig. 1(a)) and is the easiest to excite. However, as we shall see, a TE_{12}-mode (Fig. 1(b)) has lower attenuation and hence is less lossy than a TE_{11}-mode. The RF field structure for both TE_{11} and TE_{12} modes is the same near the axis of the waveguide. Note that a TE_{12}-mode would require a larger waveguide and larger RF power flowing in the waveguide than a TE_{11}-mode in order to achieve the same deflection of the electron beam. However, the overall power loss in the waveguide walls for obtaining the same radiation energy and flux would be lower for TE_{12}-mode than TE_{11}-mode. As the external RF power source needs only to compensate for the power loss in the waveguide, a TE_{12}-mode would require a lower power, and hence less expensive, RF source than a TE_{11}-mode.



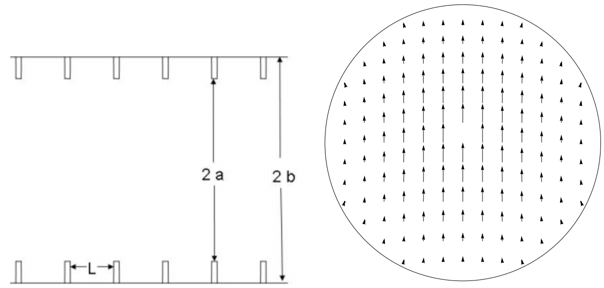
(a) (b)

Figure 1: Cross section of transverse electric field in (a) TE_{11} and (b) TE_{12} modes in a circular waveguide.

Corrugated Waveguide

A corrugated waveguide shown in Fig. 2(a) is a cylindrical waveguide with periodic corrugations along the axis of the waveguide. Due to the presence of corrugations modes in a corrugated waveguide have both electric and magnetic field components along the waveguide axis which in effect are a combination of TE and TM modes in a smooth waveguide. Therefore, modes in a corrugated waveguide are called "hybrid" modes.

The mode of interest for our study is the HE_{11} mode. When the depth of corrugation $b - a \approx \frac{\lambda_0}{4}$, the ratio of the axial electric field and axial magnetic field $\frac{E_z}{H_z} \approx jZ_0$,



(a) (b)

Figure 2: (a) Corrugated waveguide (b) Cross section of transverse electric field in a HE_{11} - mode in a corrugated waveguide.

where λ_0 and Z_0 are free space wavelength of the RF wave and free space impedance, respectively. This condition is known as the "balanced hybrid" condition when H_ϕ at $r = b$ becomes very small leading to very low losses in the waveguide wall. Another significant advantage is that the cross section RF fields (both E and H) are strongly polarized in one direction (see Fig. 2(b)). Compare the fields of circular waveguide modes (Fig. 1) and HE_{11} corrugated waveguide mode (Fig. 2(b)). We see that the electron beam placed on the waveguide axis would radiate with lower cross polarization effects with a HE_{11} than with either the TE_{11} or TE_{12} modes.

Analysis of the Corrugated Waveguide

As the modes in a corrugated waveguide are hybrid, in the region $0 < r < a$ the axial electric vector potential and magnetic vector potential for a given mode can be respectively written as

$$\begin{aligned} F_z(r,\phi) &= P_m J_m(k_r r)\, e^{m\phi} \\ A_z(r,\phi) &= P_m Z_z J_m(k_r r)\, e^{m\phi}, \end{aligned} \quad (5)$$

where k_r is the radial wavenumber inside the corrugated waveguide, P_m is the amplitude of the mode and Z_z is the axial wave impedance inside the waveguide. The RF fields can be found using Maxwell's equations which result in

$$\begin{aligned} E_z &= -j\frac{k_r^2}{k\sqrt{\mu\epsilon}} A_z \\ H_z &= -j\frac{k_r^2}{k\sqrt{\mu\epsilon}} F_z \\ E_r &= -\frac{1}{\epsilon r}\frac{\partial F_z}{\partial \phi} - \frac{\beta_z}{k\sqrt{\mu\epsilon}}\frac{\partial A_z}{\partial r} \\ E_\phi &= \frac{1}{\epsilon}\frac{\partial F_z}{\partial r} - \frac{\beta_z}{kr\sqrt{\mu\epsilon}}\frac{\partial A_z}{\partial \phi} \\ H_r &= \frac{1}{\mu r}\frac{\partial A_z}{\partial \phi} - \frac{\beta_z}{k\sqrt{\mu\epsilon}}\frac{\partial F_z}{\partial r} \\ H_\phi &= -\frac{1}{\mu}\frac{\partial A_z}{\partial r} - \frac{\beta_z}{kr\sqrt{\mu\epsilon}}\frac{\partial F_z}{\partial \phi}. \end{aligned} \quad (6)$$

In the region of the corrugation ($a < r < b$), when the period L between corrugations is small compared to a free space wavelength of the RF wave, no TE_z modes can exist. Hence, $F_z = 0$ in region $a < r < b$. Applying the boundary condition that E_z is continuous at $r = a$, the axial magnetic vector potential in region $a < r < b$ is given by

$$\begin{aligned} A_z^> &= jP_m \frac{\sqrt{\mu\epsilon}}{k} J_m(k_r a) \\ &\times \left(H_m^{(2)}(kr)\, H_m^{(1)}(kb) - H_m^{(1)}(kr)\, H_m^{(2)}(kb)\right) \\ &/ \left(H_m^{(2)}(ka)\, H_m^{(1)}(kb) - H_m^{(1)}(ka)\, H_m^{(2)}(kb)\right) \\ &\times e^{jm\phi}, \end{aligned} \tag{7}$$

where $H_m^{(1)}$ and $H_m^{(2)}$ are Hankle Bessel functions of the first and second kind. The superscript “>” in eqn7 represents the region $a < r < b$. Then all the RF fields in the region $a < r < b$ are given by,

$$\begin{aligned} E_z^> &= -j\frac{k}{\sqrt{\mu\epsilon}} A_z^> \\ H_r^> &= \frac{1}{\mu r}\frac{\partial A_z^>}{\partial \phi} \\ H_\phi^> &= -\frac{1}{\mu}\frac{\partial A_z^>}{\partial r}. \end{aligned} \tag{8}$$

Eqs. (6) and (8) are essentially the same as those presented in [3]. As there is no $H_z^>$ the contribution of the RF axial magnetic field to the waveguide wall RF losses is zero. Under balanced hybrid conditions for the HE_{11} - mode, the $H_\phi^>$ at $r = b$ tends to be very small leading to very low RF losses.

RESULTS AND DISCUSSION

The electron beam in the SPEAR3 storage ring at SLAC for which we are designing an RF undulator has a beam energy of 3 GeV and a beam current of 500 mA. The design requirements for the RF undulator are: it should produce a circularly polarized radiation with photon energy that can be varied between 700 eV and 900 eV and should produce at least a tenth of the calculated flux of the proposed BL13 static undulator. The deflection parameter, K, for the BL13 undulator is 1.07 and the radiation flux calculated at 700 eV radiation is 3.4×10^{15} photons/s/0.1 % bandwidth. The length of the BL13 undulator is 3.7 meters. To be conservative, we have optimized the dimensions of the waveguide for the three modes for $1/5^{th}$ (instead of $1/10^{th}$) of the radiated flux that the BL13 undulator can produce at 700 eV.

We have developed a mode matching code for analysis and design of the corrugated waveguide. We are in the process of optimizing the design of the RF cavity. The results presented in this paper assumes that the waveguides are uniform over the length of the cavity. The RF fields in the corrugated waveguide HE_{11} - mode was calculated using the equations presented in the analysis of a corrugated waveguide. We have assumed the distance between each corrugation to be $L = 3cm$ and the corrugation ratio $a/b = 0.9$. The RF fields in the circular TE_{11} and TE_{12} modes are calculated using the well known field equations for a circular waveguide. The results of our optimization are tabulated below. The wall radius for the HE_{11} - mode in Table 1 is the outer radius of the corrugation, b.

Table 1: Optimized RF power flow and losses, frequency and cavity radius for the RF undulator

	TE_{11}	TE_{12}	HE_{11}
Undulator parameter K	0.71	0.68	0.68
Power flow (GW)	5.8	180	79
Power loss (MW/m)	5.07	1.6	0.326
Frequency(GHz)	2.64	2.38	2.37
Wall radius (cm)	6.5	57.7	38

From Table 1 we see that although for a TE_{11}- mode in a circular waveguide the total power needed to produce the required undulating field is only 5.8 GW, the power loss per meter is 5.07 MW which is prohibitively large. For a TE_{12}- mode, the total power in the waveguide is 180 GW which is very large compared to the TE_{11} - mode. But, the RF power loss is only 1.6 MW per meter. As the external RF source has to compensate only the power loss irrespective of the total power in the waveguide, working in the higher order TE_{12} - mode is preferable to working in the TE_{11} - mode. In the case of the corrugated waveguide HE_{11} - mode, we see that the total power in the waveguide is a comparitively moderate 79 GW to produce the needed undulating field. The power loss in the waveguide walls is less than a fourth of the loss for a TE_{12} - mode. Therefore, the corrugated waveguide operating in the HE_{11} - mode is the most promising mode for the RF undulator we are designing.

CONCLUSIONS

Our study shows that a HE_{11} - mode in a corrugated waveguide cavity is an attractive option for a microwave undulator due to its low losses and desirable field structure. A fifth of radiation flux of the BL13 static undulator with an RF loss of less than 2 MW seems possible.

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