THE 3D SPACE CHARGE FIELD SOLVER MOEVE AND THE 2D BASSETTI-ERSKINE FORMULA IN THE CONTEXT OF BEAM - E-CLOUD INTERACTION SIMULATIONS *

A. Markoviḱ, G. Pöplau, U. van Rienen, Rostock University, Germany

Abstract

In this paper the fields computed with the 3D space charge routines of MOEVE [2] are compared to those obtained by means of the Bassetti-Erskine formula [1], which is a widely used 2D approximation of the electric field of a Gaussian bunch. In particular we are interested in the transversal fields of very flat bunches as the ILC or the KEKB positron bunch. It turns out that the fields of the 2D and 3D computation coincide very well.

INTRODUCTION

Damping rings (DR) are part of the injector complex responsible for producing tiny vertical emittances critical to obtain high luminosity in the International Linear Collider (ILC). The effect of electron-cloud (e-cloud) instabilities in the positron damping ring and the proposal of suppression techniques are of great importance in the DR design. The simulation of e-cloud instabilities is one of the tools to estimate the instability threshold as well as to evaluate some mitigation techniques. In order to simulate a single bunch instability due to the e-cloud the bunch movement should be followed turn by turn until the synchrotron tune of the bunch has been resolved, this may be some thousands of turns of the bunch in the ring. At each turn along the ring the bunch interacts with the e-cloud. For the simulation it is being assumed that the e-cloud is concentrated at some n interaction points (called IPs or kicks) along the ring. At each IP it is necessary to simulate the interaction between the beam and the e-cloud.

Our goal is to simulate the interaction between the beam and an e-cloud in 3D, where both the positrons and the electrons are represented by macroparticle distributions in the six-dimensional phase space. Some simulation programs for single bunch instabilities due to an e-cloud divide the bunch into a number of longitudinal 2D slices. The e-cloud at the IP is also concentrated in a 2D slice. Each of the bunch slices enters into the e-cloud slice on successive time steps and interacts with the cloud. The transversal electric field from the bunch which perturbs the electrons in the 2D slice are often computed by the Bassetti-Erskine (BE) formula [1]. BE formula is a widely used 2D approximation of the electric field of a Gaussian bunch. In this paper the fields computed with the 3D space charge routines of MOEVE [2] are compared to those obtained by means of the BE formula. In particular we investigate the transversal fields of very flat bunches of the ILC and the KEKB positron bunch. Supposing a longitudinal Gaussian distribution of the bunches we compare the transversal fields for a certain line density of the positron bunch. The fields of the 2D and the 3D computation at a certain transversal slice of the bunch match very well.

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BASSETTI-ERSKINE'S FORMULA, COMPARISON WITH THE MOEVE POISSON SOLVER

Bassetti-Erskine's formula

Many simulation codes which investigate beam-beam interaction use the Bassetti-Erskine (BE) expression for the electromagnetic field of a Gaussian charge distribution. It is an analytical expression in terms of the complex error function which assumes that the two-dimensional charge distribution is Gaussian with dimensions σ_x and σ_y and the total charge of the 2D bunch is Q.

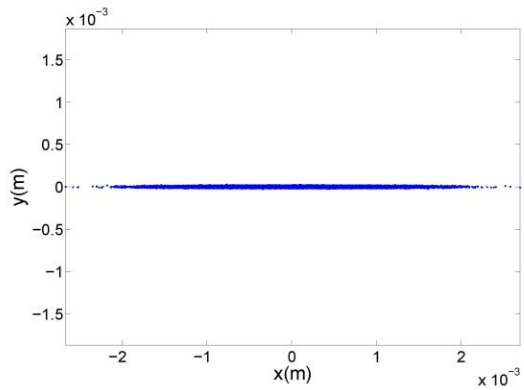

Figure 1: Transversal profile of the ILC flat beam, Gaussian distribution in every direction with $\sigma_x = 0.6$ mm, $\sigma_y = 0.006$ mm, $\sigma_z = 6$ mm.

In the following we investigate if the transversal electric fields computed by the BE formula are matching with the transversal electric fields computed by our 3D MOEVE Poisson solver. We compare the field components E_x and E_y in an infinitesimal thin transversal plane at a certain longitudinal position z in the bunch. The bunch is defined in the laboratory frame with a macroparticle distribution in the six-dimensional phase space $\Psi(x, p_x, y, p_y, z, p_z)$. Each macroparticle is located inside a grid cell and it's charge, correspondingly to it's position in the cell, is weighted on the 8 grid nodes defining that 3D grid cell, where 4 of the 8 cell nodes belong to one discrete transversal plane. For an appropriate comparison of the fields in a transversal plane at position z, the input charge Q in the BE formula should be equal to the corresponding line charge density $\lambda_b(z)$ of the bunch at z. To compare the E_x and

E_y components we choose one of the discrete transversal grid planes of the 3D discretization domain. The amount of charge Q_m deposited on that transversal plane can be considered as the total amount of charge between two neighboring transversal planes of the 3D Cartesian grid and therefore should be divided by the distance between the planes h'_z. Since MOEVE computes the fields in the rest frame of the relativistic bunch, the length h_z (defined in the laboratory frame) is multiplied by the relativistic factor γ. Hence the distance between transversal planes in the rest frame is $h'_z = \gamma h_z$ and the input charge for the BE formula is $\lambda_b(z) = Q_m/h'_z$.

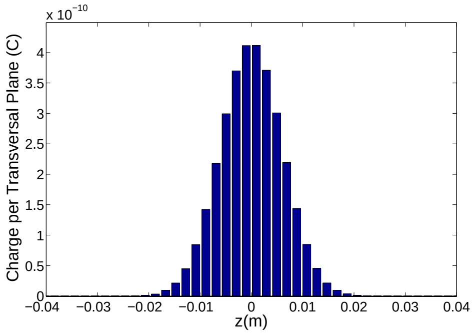

Figure 2: Discretized longitudinal charge distribution of the ILC bunch.

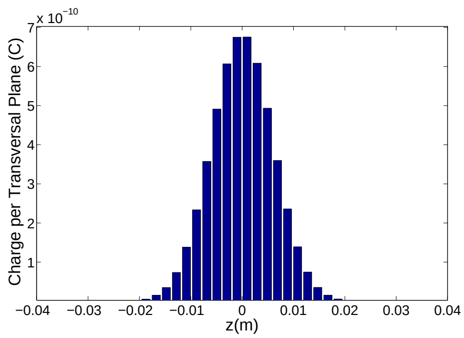

Figure 3: Discretized longitudinal charge distribution of the KEKB bunch.

Comparison: Bassetti-Erskine 2D fields vs. 3D space charge fields of MOEVE

In the following we compare the transversal fields E_x and E_y in the middle transversal plane of the ILC and KEKB positron bunches. The ILC bunch is defined as Gaussian distribution with $\sigma_x = 0.6$ mm, $\sigma_y = 0.006$ mm and $\sigma_z = 6$ mm and its charge ($Q = 3.22$ nC) discretized in the longitudinal direction is shown in Figure 2. Similarly the charge ($Q = 5.28$ nC) of the the KEKB bunch ($\sigma_x = 0.42$ mm, $\sigma_y = 0.06$ mm and $\sigma_z = 6$ mm) is discretized in the longitudinal direction as shown in Figure 3. The maximum in Figure 2 ($Q_m = 4.118 \cdot 10^{-10}$C) and Figure 3 ($Q_m = 6.7525 \cdot 10^{-10}$C) is the charge on the middle transversal plane of the ILC and KEKB positron bunch, respectively. This value can be considered as the total charge between two transversal planes. The discretization in the laboratory frame provides $h_z = 2$ mm. Since the bunch is highly relativistic (5 GeV energy for the ILC positron bunch in the damping ring and 3.5 GeV for the KEKB positron bunch), in the rest frame h_z multiplies by γ ($\gamma = 9765.7$ and $\gamma = 6836$) and yields $h'_z = \gamma h_z$. Thus we obtain the line density $\lambda_b = Q_m/h'_z = 2.1084 \cdot 10^{-11}$ C/m for the ILC and $\lambda_b = Q_m/h'_z = 4.9389 \cdot 10^{-11}$ C/m for the KEKB as input charge for the BE formula.

The field components computed by means of the BE formula were compared to the results taken from the 3D Poisson solver MOEVE on the middle transversal plane. The 3D Poisson solver assumed perfect electrical conductor boundary condition (PEC b.c.) on a circular beam pipe with a radius of $r = 5$ mm. It should be pointed out that the fields we compare are the fields in the rest frame of the bunch, the fields that the e-cloud experience in the laboratory frame are multiplied by the corresponding γ. The plots of E_x at y=0 and E_y at x=0, are presented in Figure 4 for the ILC and in Figure 5 for the KEKB bunch. The discretization is non-equidistant in y-direction with an aspect ratio of 2. The Figures 6 and 7 show E_x and E_y at the line $y = \frac{r}{2}$ for the ILC and the KEKB bunch, respectively. Figures $4 - 7$ show a very good match of BE (solid lines) and our 3D Poisson solver fields (dashed lines).

CONCLUSION

In this paper we compared the transverse space charge fields computed by the 3D Poisson solver MOEVE with those obtained by the BE formula which takes as input the corresponding line charge density $\lambda_b(z)$ and the slice dimensions (σ_x and σ_y). This approach is similar to dividing the bunch longitudinally into many transversal slices. Successively each slice interacts with the e-cloud using the BE formula. From the comparisons we get a very good matching of the computed fields. In the context of beam - e-cloud interaction simulations the conclusion is that the transverse fields acting on the e-cloud from the Gaussian bunch can be computed by the BE formula, however the bunch should be divided into a sufficient number of transversal slices. On the other hand the fields from the fast changing transversal distribution of the electrons in the cloud, during the interaction with the beam, can't be approximated well by the BE formula. The interaction of the bunch with the e-cloud can be more precisely computed by approximating the fields with the Poisson solver MOEVE. Furthermore it allows a full 3D beam - e-cloud interaction simulation.

ACKNOWLEDGEMENT

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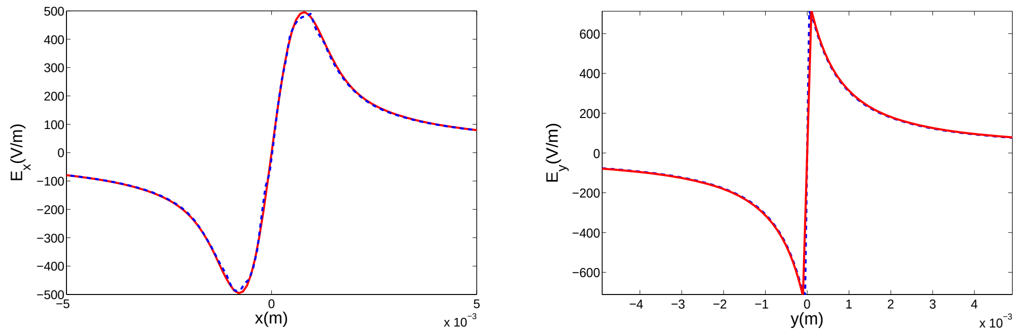

Figure 4: ILC bunch: E_x vs. x at $y = 0$ (left) and E_y vs. y at $x = 0$ (right), BE (solid) and 3D MOEVE (dashed lines).

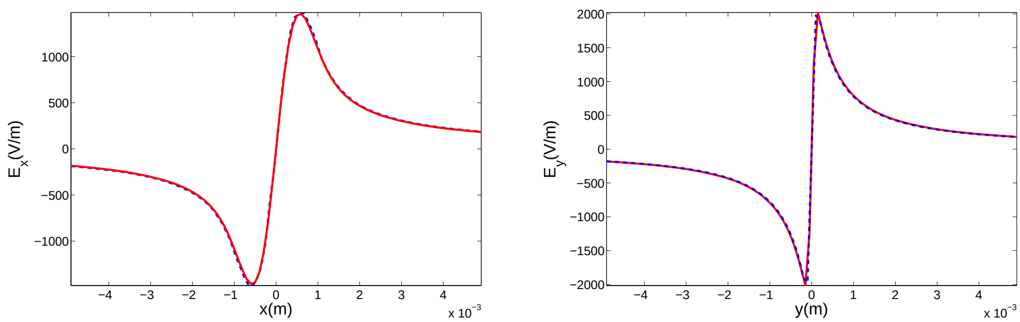

Figure 5: KEKB bunch: E_x vs. x at $y = 0$ (left) and E_y vs. y at $x = 0$ (right), BE (solid) and 3D MOEVE (dashed lines).

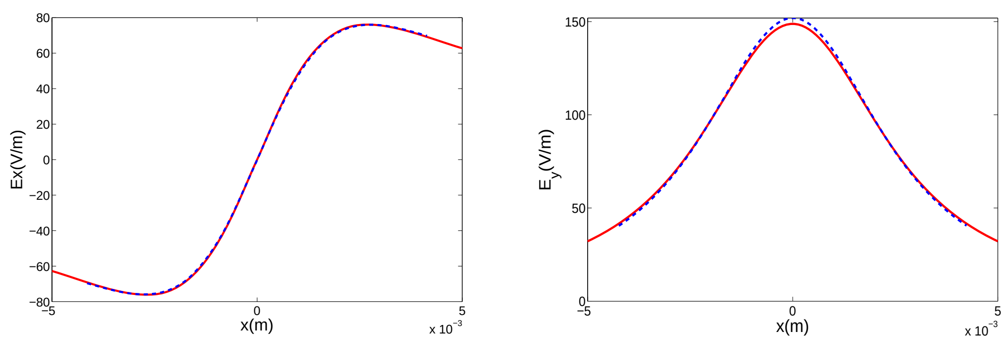

Figure 6: ILC bunch: E_x vs. x (left) and E_y vs. x (right) at $y = \frac{r}{2}$, BE (solid) and 3D MOEVE (dashed lines).

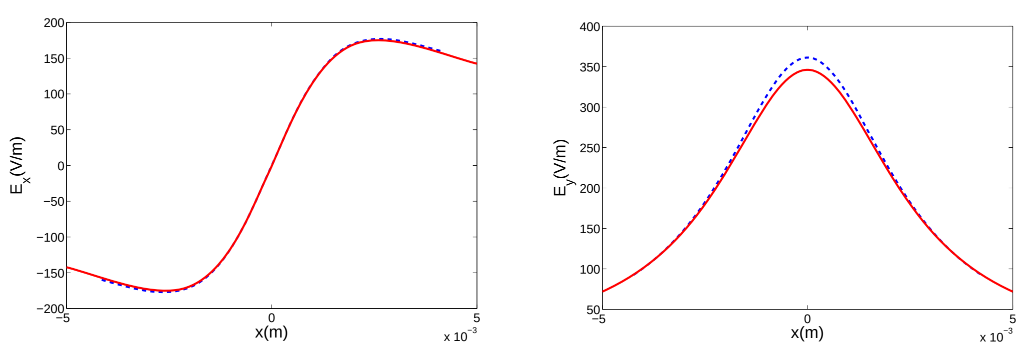

Figure 7: KEKB bunch: E_x vs. x (left) and E_y vs. x (right) at $y = \frac{r}{2}$, BE (solid) and 3D MOEVE (dashed lines).

REFERENCES

[1] M. Bassetti and G.A. Erskine. Closed expression for the electrical field of a two-dimensional gaussian charge. *CERN-ISR-TH/80-06*, 1980.

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