CRYSTALLINE BEAM SIMULLATIONS

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Abstract

Special program codes were elaborated for the simulation of crystalline beams on the S-LSR storage ring (Kyoto Univ., Japan) under action of different cooling methods. For the investigation of ordered proton beams, which recently were observed in first time on S-LSR with using the electron cooling, a special molecular dynamics technique was used. 2D and 3D crystalline beam were simulated for different lattice structure of S-LSR ring under action of the laser cooling

CRYSTALLIZATION CONDITIONS

The main condition for beam crystallization (orderliness) is a decrease of the particle temperature below inter-particle potential energy. This can be described by the plasma parameter:

$$\Gamma = \frac{U}{T} = \frac{Z^2 e^2}{aT} > 150 \qquad (1)$$

where U and T - potential energy and temperature of ion beam. Ze - charge of particles, a - average interparticle distance. 3D crystals are obtained for $\Gamma \geq 150$.

The next condition is related to the optical structure of the storage ring. The storage ring must be alternating-gradient (AG) focusing, and the energy of the beam must be less than the transition energy of the ring [1]:

$$\gamma < \gamma_T \qquad (2)$$

Another condition defines the periodicity of the ion storage ring. The ring lattice periodicity should be at least 4 times as high as the maximum betatron tune:

$$4 \cdot \max\{Q_x, Q_y\} < \text{Periodicity} \qquad (3)$$

These conditions (1-3) should be satisfied for 3D crystalline beams. The shape of the crystals depends on the dimensionless linear density of particles. A linear density is defined as [2]:

$$\lambda_{ion} \equiv \frac{N}{C}\left(\frac{3\, r_{ion}}{2k\gamma_0^5\beta_0^2}\right)^{1/3} = \left(\frac{3N^3 r_{ion}}{8\pi^2 Q^2 C\gamma_0^5\beta_0^2}\right)^{1/3} \qquad (4)$$

where N – particle number in the ring, C – ring circumference, $\beta_0 = v_0/c$ – relativistic factor, Q – betatron tune, r_{ion} –classical ion radius.

The λ means the line density of the beam. The crystal structure changes to higher dimensional structure, as the line density of the beam is enhanced. The 2D crystal structure extends in the weaker focusing direction of the ring. In this simulation, an ideal tapered cooling force is supposed.

MOLECULAR DYNAMICS TECHNIQUE

Since the simulation of the crystalline beams is time consuming, we approximate the situation by using a periodic boundary condition, or Molecular Dynamics (MD) technique. When ions of charge $q = Ze$ are put at position $(s, r)=(0,0),(\pm L,0),(\pm 2L,0),\ldots$ (Fig. 1) the electrostatic potential in the region $|s| < L/2$ is [3]

$$U_{sc}(s,r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} + \frac{2q}{L}\int_0^\infty \frac{J_0(kr/L)\cosh(ks/L) - 1}{\exp(k) - 1}dk\right), \qquad (5)$$

where q - particle charge, $a = \sqrt{s^2 + r^2}$ - distance between particles, $s = z - z_i$, $r = \sqrt{(x - x_i)^2 + (y - y_i)^2}$, L – MD cell size, J_0 is the Bessel function of 0^{th} order. With this MD technique, the dynamics of the ordered state (string) of ions in a storage ring usually was simulated with typically $N_p = 10$ particles.

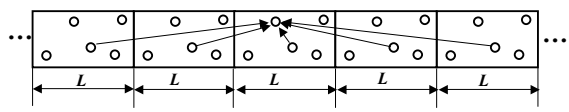

Figure 1: Periodic distribution of five particles in MD cells.

The new code based on molecular dynamics (MD) technique for the simulation of crystalline ion beams was elaborated [4]. For a first test of the validity of the new code an infinite constant focusing structure was used. Results of the calculation (Fig. 2) show good agreement with the theory of crystalline beams: transitions between different states appear for the characteristic values of λ_{ion}.

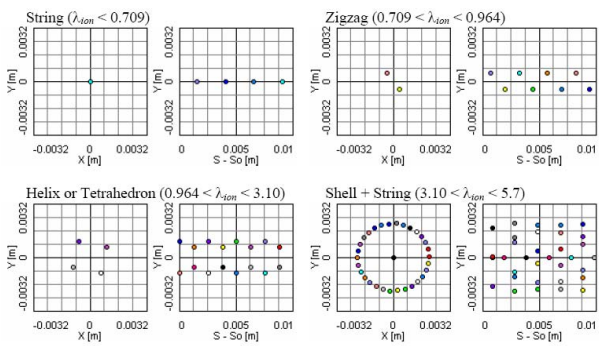

Figure 2: Crystalline beams in a straight (infinite) channel with constant focusing for different linear density.

ORDERED PROTON BEAM IN S-LSR

An approach to an ordered structure of 7MeV proton beam has been started experimentally at the ion cooler storage ring, S-LSR. By applying electron-beam cooling reducing the particle number to suppress the IBS effect, the creation of a 1D string has been studied. By the recent measurement reducing the noise level, anomalies in the fractional momentum spread and the Schottky power level for proton numbers less than several thousands have been observed which might be an indication of 1D ordering of proton beam [5].

The standard models of the intrabeam scattering can not describe the ordered state of the ion beam. Simulation results [6] with Molecular Dynamics technique show that the longitudinal component of the intrabeam scattering heating rates has a specific behaviour for the small number of particles (Fig. 3).

Left part of the IBS longitudinal component is described by the shear force which always exists in bend magnets. The boundary of this part is dependent on the particle number (dashed black-gray line on Figs. 3 and 4) and the region, where crystalline state is prohibited, is [7]:

$$\frac{\Delta p}{p} < \frac{2N^3 r_{ion}\sigma_\perp}{\pi Q^2 \gamma_0^3 \beta_0^2 C^2} \qquad (6)$$

where N – particle number, r_{ion} – classical ion radius, $\sigma_\perp=\sqrt{\varepsilon_\perp\beta_\perp}$ - transverse beam size, Q – betatron value, β_0, γ_0 – relativistic factors, C – ring circumference.

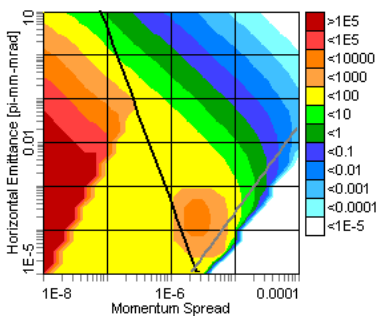

Figure 3: Intrabeam scattering rates for S-LSR ring (Molecular Dynamics, $N_p = 10^3$). Black line – ordering criteria Γ_2, gray line – temperature equilibrium, dashed black-gray line – shear force boundary (6).

The ordering criteria Γ_2 [7] describes the transition to the ordered state when a relaxation between transverse and longitudinal degrees of freedom is switched off. This region is placed below the following condition (black line on Figs. 3 and 4) [6]:

$$\left(\frac{\Delta p}{p}\right)^2 < \frac{r_{ion}}{\pi\beta_0^2\sigma_\perp} \qquad (7)$$

The phase diagram (Fig. 3) has a specific peculiarity: there is a region of high heating rate value surrounded by regions where the heating rate is sufficiently less. Origin of this "island" can be explained from the phase diagram calculated with the Martini model (Fig. 4) if we assume that in the region between conditions (6) and (7) the IBS process is suppressed due to the beam ordering (triangle yellow region on Fig. 4). The IBS island takes a place due to different angles on the phase diagram between ordering criteria and the lines of equal heating rates.

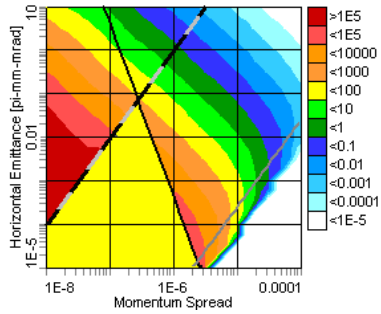

Figure 4: Longitudinal component of intrabeam scattering rates for S-LSR ring (standard model, $N_p = 10^3$).

SIMULATION OF CRYSTALLINE BEAMS

The lattice of S-LSR has been designed by incorporating the condition (1)–(3). Although the laser cooling has effect in longitudinal direction only, 3D cooling can be reached by using coupling among the three degrees of freedom. Dynamical coupling between the horizontal betatron motion and the synchrotron motion can be created by an rf cavity through dispersion. Coupling between the horizontal and vertical betatron motions is realized by the solenoid of the electron cooler. In order to maximize the coupling effect, the tune values must satisfy the difference resonance conditions: $\nu_x - \nu_s$ = integer, $\nu_x - \nu_y$ = integer.

One of the operating points satisfying this condition is (ν_x,ν_y,ν_s) = (2.07, 1.07, 0.07) . 1D and 2D ordered structures are expected for normal operation of S-LSR [8] (Fig. 11). 3D crystalline beams, however, cannot be expected due to the shear. Similar results are shown by a multi-particle simulation using a realistic laser cooling models [9].

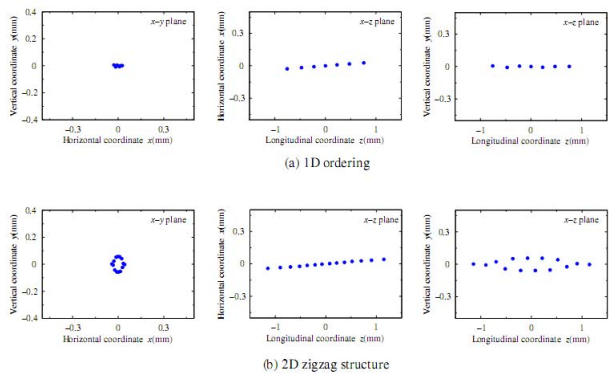

Figure 5: Simulation of 1D and 2D crystalline beams expected in S-LSR.

The idea of Dispersion-free operation to overcome shear is to use crossed electric and magnetic fields as a defection field of the circulating beam. Since the scalar

potential of the electrostatic field accelerates or decelerates particles at the edges of the defection element, those particles having the same linear velocity at the straight section can pass through the bending section with the same angular velocity. By detailed beam-dynamics studies concerning dispersion-free storage ring, it is found that a dispersion-free ring satisfying the formation and maintenance conditions for beam crystallization can be realized at S-LSR with the operating point of (ν_x,ν_y,ν_s) = (2.07, 2.07, 0.07) [10].

In a dispersion-free ring, the horizontal and longitudinal motions are independent in the range of the linear approximation. Therefore, it is not possible to realize 3D cooling relying on the dispersion by a normal rf cavity. By introducing a coupling rf cavity [11], coupling between the transverse and longitudinal motions can be obtained without dispersion. For a 35 keV $^{24}Mg^{+}$ beam, the required voltage of the coupling cavity is not very high. Setting the operating point to (ν_x,ν_y,ν_s) = (2.07, 2.07, 0.07), the emittance of three directions can be reduced by the resonant coupling method, and 3D crystalline beams (Fig. 6) can be expected by avoiding shear [8]. A larger multi-shell crystalline beam, however, cannot be expected due to the blocking of emittance reduction at the resonance crossing during the cooling process.

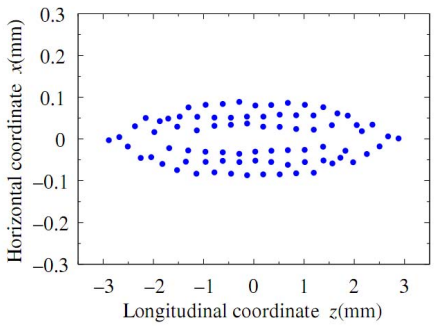

Figure 6: Simulation of 3D crystalline beam generated in the dispersion-free mode of S-LSR.

Another method to overcome shear has been known as 'tapered cooling' [12]. By applying a special longitudinal cooling force depending on the horizontal coordinate x, at the final state of cooling, we can realize the same angular velocity, even in normal storage rings with finite dispersion.

Although tapered cooling in a normal storage ring is preferable to realize multidimensional crystalline beams, a concrete method to realize tapered cooling has not yet been proposed. The laser cooling force can be somewhat tapered by slightly displacing the laser axis [13]. An accurate adjustment of the magnitude of the tapered factor, however, is extremely difficult with this method. As an alternative idea, a method using potential difference of an electric field has been suggested [14]. For this purpose, we have shown a realistic scheme of tapered cooling using crossed electric and magnetic fields of a Wien filter based on the idea to localize the interaction between the laser and the ion beam only inside the Wien filter [15].

CONCLUSION

The specific island of the IBS longitudinal component is described by different angles between lines of equal heating rates and the ordered criteria Γ_2. In real experiments when the transverse temperature of the ion beams much higher then the longitudinal one the cooling "around the IBS island" can help to reach the ordered state.

In the normal operation mode 1D and 2D ordered structure can be reached under action of the laser cooling in S-LSR. A few methods to reach the 3D crystalline magnesium beam were proposed for S-LSR ring. The dispersion-free storage rings together with a coupling cavity can generate 3D crystals avoiding the shear. Tapered cooling with Wien filter can overcome the shear, keeping lower betatron tune with finite dispersion.

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