WAKEFIELD AND RF KICKS DUE TO COUPLER ASYMMETRY IN TESLA-TYPE ACCELERATING CAVITIES *

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INTRODUCTION

In a future linear collider, such as the International Linear Collider (ILC), trains of high current, low emittance bunches will be accelerated in a linac before colliding at the interaction point. Asymmetries in the accelerating cavities of the linac will generate fields that will kick the beam transversely and degrade the beam emittance and thus the collider performance. In the main linac of the ILC, which is filled with TESLA-type superconducting cavities, it is the fundamental (FM) and higher mode (HM) couplers that are asymmetric and thus the source of such kicks. The kicks are of two types: one, due to (the asymmetry in) the fundamental RF fields and the other, due to transverse wakefields that are generated by the beam even when it is on axis. In this report we calculate the strength of these kicks and estimate their effect on the ILC beam.

The TESLA cavity comprises nine cells, one HM coupler in the upstream end, and one (identical, though rotated) HM coupler and one FM coupler in the downstream end (for their shapes and location see Figs. 1, 2) [1]. The cavity is 1.1 m long, the iris radius 35 mm, and the coupler beam pipe radius 39 mm. Note that the couplers reach closer to the axis than the irises, down to a distance of 30 mm.

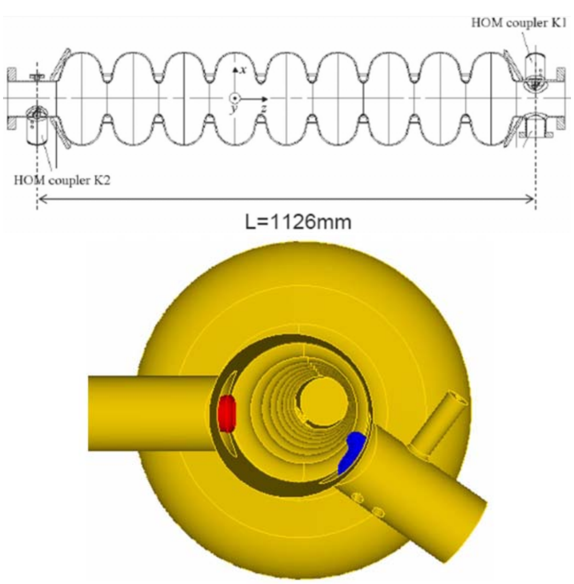

Figure 1: Sketch of an ILC cavity with the HM couplers (top). View from the downstream end of a cavity showing the FM coupler (red) and one HM coupler (blue).

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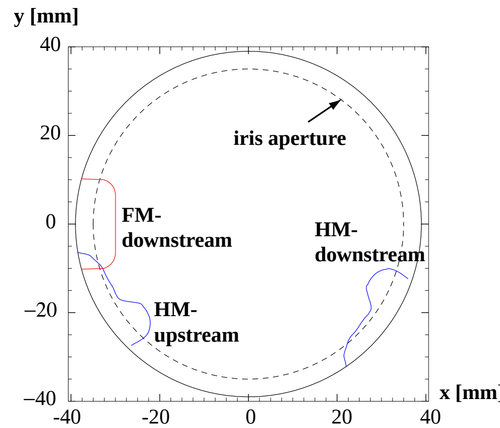

Figure 2: The profiles of the three couplers of a cavity, as seen from the downstream end. The solid circle is the coupler beam pipe, the dashed circle the iris aperture.

WAKEFIELD KICKS

We perform numerical calculations with the 3D time domain, finite difference programs ECHO[2], GdfidL[3], and PBCI[4]. Among the features in the programs are: (1) algorithms that suppress "mesh dispersion," necessary to obtain wakes for short bunches in long structures, (2) a 3D, indirect wakefield integration method, essential in problems where features—such as couplers—reach toward the axis from the beam pipe, (3) a "moving window" option, where fields are only calculated within a window in z that encloses the bunch, and (4) parallelization (GdfidL and PBCI). For the results presented here the three programs agree reasonably well.

The couplers are fully 3D objects and small compared to the size of a cavity. The length of the exciting bunch is even smaller—for the ILC the bunches are Gaussian with rms length $\sigma_z = 300$ μm. To obtain the short-range wakes numerically the mesh size needs to be $\lesssim \sigma_z/5$, which is thus quite challenging. Our calculations proceed in three steps: (1) couplers in their beam pipes, (2) one cavity with its couplers, and (3) steady-state wakes after many cavities.

Couplers in Beam Pipe

To make the calculations more tractable we begin with a Gaussian bunch with rms length $\sigma_z = 1$ mm. The first calculations were performed for the couplers in their beam pipe extended to plus and minus infinity [5]. The transverse wake is shown in Fig. 3. Note that the result is similar, whether the couplers are placed next to each other or separated by their z separation in a cavity, ~ 1 m. The shape of the wake is consistent with it being *capacitive*. That is, the (bunch) wake $\mathbf{W}_\perp \propto \int \lambda(s)\,ds$, with s longitudinal bunch coordinate and λ the charge distribution—it is (locally) independent of bunch length; the impedance $\mathbf{Z}_\perp \propto 1/\omega$, with ω the frequency. For a Gaussian bunch $\mathbf{W}_\perp(s) \propto \mathrm{erf}(s/\sqrt{2}\sigma_z)$, with erf the error function. The kick factor—the average of the wake when weighted by the charge distribution—is given by $\mathbf{k}_\perp = \frac{1}{2}\omega\mathbf{Z}_\perp$.

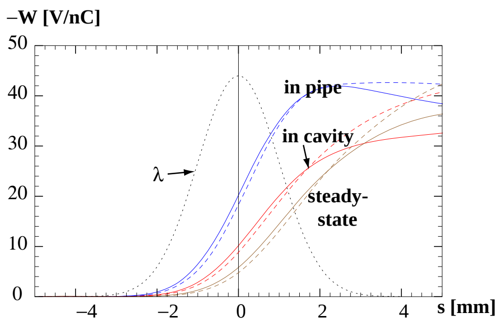

Figure 3: Plots of $-W_{x0}(s)$ (solid) and $-W_{y0}(s)$ (dashed) for couplers in pipe, in cavity, and the steady-state solution, for $\sigma_z = 1$ mm (ECHO). Dots indicate bunch shape.

For this model problem the *optical approximation* can be used to compare with [6]. This approximation is valid provided that $\sigma_z/a \ll 1$, with a the distance from the beam axis to the coupler. It predicts a capacitive wake, with the on-axis impedance given by a line integral:

$$\mathbf{Z}_{\perp 0} = \frac{1}{2\pi\omega}\int_C \phi_d(\mathbf{n}\cdot\nabla\phi_m)\,dl, \qquad (1)$$

where the integral is over the (projected) edge of the coupler, $\mathbf{n}$ is a unit normal vector pointing away from the coupler, and $\phi_d(\phi_m)$ are solutions to Poisson's equation in the beam pipe with a dipole (monopole) source on axis. For a partial iris of azimuthal extent θ and minimum radius a, centered at the top of a round beam pipe of radius b, we obtain $Z_{y0} = \frac{4}{\pi\omega a}(1-\frac{a^2}{b^2})\sin(\theta/2)$. However, to compare with the numerical results we apply Eq. 1 to the projected geometry, such as is shown in Fig. 2. The comparison between analytical and numerical results is given in Table 1 (the first row); we see good agreement.

Table 1: Wake kick on-axis (k_{x0},k_{y0}) due to coupler asymmetry, for bunch length $\sigma_z = 1$ mm, in [V/nC] (ECHO).

Case	Numerical	Analytical
Couplers in pipe	$(-21.2, -18.6)$	$(-20.8, -17.1)$
Couplers in cavity	$(-10.8, -10.0)$	$(-12.7, -7.0)$
Steady-state solution	$(-7.6, -6.8)$	

Couplers in Cavity

Numerical calculations were next performed for an entire cavity, where there are 9 cells between the upstream and downstream couplers (see Fig. 3). The wake amplitude reduces by a factor ~ 2. We believe that this is due to the partial *shadowing* of the couplers by irises (see Fig. 2). As an approximation we perform the analytical estimate as before, except the beam pipe is taken to be the iris radius, 35 mm (instead of 39 mm). We see that with irises the estimate does not work as well (see Table 1, second row).

There are many cavities in the main linac of the ILC. It is known that when going from a single cavity with beam pipes to n cavities with beam pipes, the wake per cavity decreases until some steady-state is reached; for normal cavities the distance to steady-state is on the scale of the catch-up distance, $\sim a^2/\sigma_z$ (here ~ 1 m, with a iris radius) [7]. For the ILC cavities with couplers the same behavior is expected for the on-axis transverse wake (with a transverse distance to coupler). Numerical simulations bear this out. Fig. 3 shows the after-two cavity wake (per cavity) for $\sigma_z = 1$ mm; Table 1 gives the kick factor (the third row).

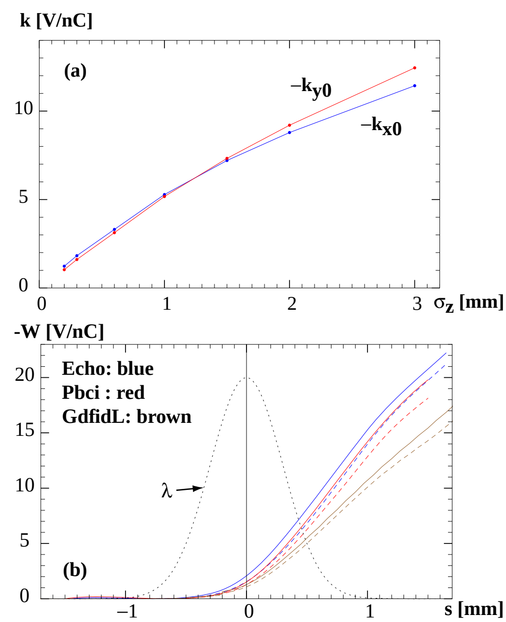

Figure 4: The steady-state solution: (a) on-axis kick factor as function of σ_z (GdfidL); (b) $-W_{x0}(s)$ (solid), $-W_{y0}(s)$ (dashed) for $\sigma_z = 300$ μm. Dots indicate bunch shape.

Numerical simulations were also performed for different bunch lengths. The steady-state kick factor appears to depend linearly on bunch length for short bunches (see Fig. 4a). This is the same behavior as the normal dipole wake of the cavities (due to the irises) for very short bunches. We can understand this by the following argument: For a periodic structure $\int \mathrm{Re}Z_{\parallel 0}(\omega)\,d\omega$ is finite ($Z_{\parallel 0}$ is the longitudinal, on-axis impedance), implying that for small s, the (longitudinal) point charge wake $W_{\delta\parallel 0}(s) = c_1$, a constant. From the Panofsky-Wenzel Theorem this implies that for small s, $\mathbf{W}_{\delta\perp 0}(s) = \mathbf{c_2}s$, with $\mathbf{c_2}$ a constant; for a Gaussian bunch $\mathbf{k}_{\perp 0} = \mathbf{c_2}\sigma_z/\sqrt{\pi}$. For a normal cavity-type impedance, where diffraction dominates, we'd expect the scale of the linear region $s \sim a^2/L \sim 100$ μm (taking $a = 30$ mm, period $L = 1$ m)[7]; for our couplers, however, where geometric optics dominates, the impedance at high frequencies should drop more quickly and the linear region of wake be more expanded—as we see in our numerical results.

For the nominal ILC bunch length, $\sigma_z = 300$ μm, and after about 3 cavities steady-state is reached (see Fig. 4b for the steady-state wakes). The kick factor $(k_{x0}, k_{y0}) = (-1.8, -1.7)$ V/nC (GdfidL), $(-2.7, -2.3)$ V/nC (ECHO), and $(-2.3, -2.0)$ V/nC (PBCI). We see reasonably good agreement, with the GdfidL results somewhat lower than the others. Finally, note that by rotating the upstream coupler by 180° with respect to the axis the kicks are reduced in amplitude to $\lesssim 0.3$ V/nC.

We have focused on the wake kick on axis due to the couplers. The next order term depends linearly on offset. For couplers in a pipe we numerically find the average kick of this component ~ 2.5 V/pC/m^2. We estimate the steady-state solution of this component to be negligible—a factor 20 smaller than the normal dipole wake of the cavities.

RF KICKS

Asymmetries in the couplers will also result in asymmetries in the main RF fields that will transversely kick the beam. The in-phase component will kick the beam rather uniformly; the out-of-phase component will kick the head and tail differently, resulting in emittance growth.

RF fields and RF kicks were computed using the programs OMEGA3P[8], MAFIA[9], and HFSS[10]. The RF fields in the coupler regions are orders of magnitude smaller than those in the main parts of the cavity. To improve the accuracy in calculating the RF kicks, the computational domain was limited to the coupler region and one cavity cell. The RF kicks of the upstream end-group (one cell plus one HM coupler) and the downstream end-group (one cell plus the FM and one HM coupler) were calculated separately. The fields in the upstream group are of standing wave type, making the field and kick calculations straightforward. The fields in the downstream group, however, are partially standing and partially traveling waves. The content of the traveling wave portion depends on the cavity gradient, the external coupling of the input coupler, and the beam current (beam loading).

In our calculation we assume a beam current of 11 mA, a loaded gradient of 31.5 MV/m, on crest acceleration, and critical coupling for the FM coupler (which implies $Q_{ext} \approx 3.5 \times 10^6$). To reach this coupling the center conductor of the FM coupler needs to intrude into the beam pipe by ~ 6 mm. Our calculated RF kicks using OMEGA3P are given in Table 2. The results of the other two programs are in essential agreement.

Table 2: RF kick on-axis due to coupler asymmetry in [kV]. $\mathrm{Re}(V)$ is the in-phase, $\mathrm{Im}(V)$ the out-of-phase kick.

Region	$\mathbf{V}_x$	$\mathbf{V}_y$
Upstream	$-1.82 + 0.22i$	$-1.29 - 0.11i$
Downstream	$-0.79 - 1.62i$	$+1.15 + 0.28i$
Total	$-2.61 - 1.40i$	$-0.13 + 0.17i$

EFFECT ON BEAM

What is the effect of the coupler kicks on the beam in the ILC? The wake kick, to first order, is independent of offset of the beam, yet depends on longitudinal position within the bunch. A simple model is the equation of motion in y

$$d^2y(z,s)/dz^2 + y(z,s)/\beta^2 = e^2NW_{y0}(s)/E, \qquad (2)$$

with z position in the linac, β the (constant) β function, eN the bunch charge, and E the energy. The solution is $y = \beta^2 e^2 NW_{y0}(s)/E$ plus free betatron oscillation. Once injected, particles will perform free betatron oscillation about different centers, depending on s. However, it is no real wake effect—the particles do not interact through their motion. After injection, the emittance will grow, oscillate, and partially damp through adiabatic damping. Residual emittance growth is due to energy spread in the beam, and because induced betatron oscillations can excite the normal wake. Let us take ILC parameters $eN = 3$ nC, $\beta = 68$ m, normalized emittance $\epsilon_n = 2 \times 10^{-8}$ m, and initial and final energies of 10 and 250 GeV. A head-to-tail wake of 4 V/nC/m (for $k_y = 2$ V/nC/m) implies that at 20 GeV the beam is spread out about maximally—by about $0.3\sigma_y$, and at the end of the machine by $0.08\sigma_y$. We estimate the emittance growth to be small.

As to the RF kicks, the in-phase component will maximally kick the beam, the out-of-phase component will cause emittance growth. There are approximately 10,000 cavities in the ILC linac. At the beginning of the machine the relative in-phase kick is $\Delta y_0'/\sigma_{y'} = \mathrm{Re}V_y/(E\sigma_{y'}) = 0.8$, for $\mathrm{Re}V_y = 1$ kV ($\sigma_{y'}$ is beam divergence); the relative beam growth due to the out-of-phase component, $\Delta y'/\sigma_{y'} = \mathrm{Im}V_y k_{rf}\sigma_z/(E\sigma_{y'}) = 0.006$, for $\mathrm{Im}V_y = 1$ kV (the RF wave number $k_{rf} = 27$ m^{-1}); assuming constant β both effects reduce as $E^{-1/2}$ as the beam moves down the linac.

Readers are directed to M. Dohlus, et al, MOPPO13 at this conference, for another report on this subject. One of us (K.B.) thanks K. Yokoya for helpful discussions on the impedance of periodic structures.

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