# POSSIBLE PARTICLE DISTRIBUTIONS AT THE ENTRANCE OF THE CYCLOTRON SPIRAL INFLECTOR

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*Abstract*

The transverse particle distribution of the ion beam produced in the Electron Cyclotron Resonance Ion Source (ECRIS) is considered. It is shown that the beam emittance at the entrance of the cyclotron spiral inflector is strongly dependent on directions of the both ECRIS and cyclotron magnetic fields. The changing of the beam Root Mean Square (RMS) emittance and bunch lengthening in the spiral inflector for every considered distribution are obtained in the computer simulation.

## INTRODUCTION

During axial injection into cyclotron by using the spiral inflector it is desirable to have the beam matched with longitudinal magnetic field at the inflector entrance. It means the beam moving in the longitudinal magnetic field without envelope oscillations. The magnitude of this field is equal to one in the cyclotron center. In this case the particle losses in the inflector will be absent if the transverse beam dimensions are less than inflector gap.

In analysis of the possible steady state particle distributions the axial symmetric case is considered. It is valid for beam focused by solenoidal lenses and double focusing analyzing magnet with edge angles closed to 26.5$^{o}$.

The violation of the rotation symmetry due to hexapolar fields of Electron Cyclotron Resonance Ion Source (ECRIS) and bending magnet is not discussed.

The RMS beam emittances of found distributions defined for each transverse plane are strongly dependent on relative direction of both ECRIS and cyclotron magnetic fields. This is explained by particle rotation (or its absence) around the longitudinal axes.

The changing of beam RMS emittance and bunch lengthening during transportation in DC-350 [1] spiral inflector is computed with the help of CASINO [2] code for every considered distribution. 3D electric field map calculated by means of RELAX3D code [3] is used in this simulation.

## INVARIANTS AND MATCHING CONDITIONS

In the axial symmetric case the average angular momentum is constant along the beam line:

$$M = j + \frac{B}{B\rho}\varepsilon_{rms}\beta = const \qquad (1)$$

Here $B$ – is longitudinal magnetic field, $B\rho$ – magnetic rigidity of the reference particle, $\varepsilon_{rms}$ – RMS beam emittance, $j$ – average mechanical angular momentum, $\beta$ – Twiss beta function.

Averaging in formula (1) is performed with beam distribution function $f$. For example, for discrete particles distribution average mechanical angular momentum $j$ is defined as follows:

$$j = \overline{xy'} - \overline{yx'} = \frac{1}{N}\sum_{i=1}^{N}(x_i y_i' - y_i x_i') \quad , \qquad (2)$$

where $N$ – is number of particle per unit beam length.

In the axial symmetric case the RMS emittance $\varepsilon_{rms}$ is constant in the presence of longitudinal magnetic fields [4]:

$$\varepsilon_{rms}^2 = \varepsilon^2 - j^2/4 + M^2/4 = const \qquad (3)$$

$$\varepsilon^2 = \overline{x^2}\,\overline{x'^2} - (\overline{xx'})^2 = \overline{y^2}\,\overline{y'^2} - (\overline{yy'})^2 \qquad (4)$$

It should be noted that RMS emittance $\varepsilon$ defined by standard way is not constant for nonzero longitudinal magnetic field. In the absence of longitudinal magnetic field $j \equiv M$ and emittance $\varepsilon_{rms} \equiv \varepsilon$.

Beta function $\beta$ is connected with RMS beam dimensions by equality $\varepsilon_{rms}\beta = \frac{1}{2}\left(\overline{x^2} + \overline{y^2}\right)$. With this definition the matching conditions at the inflector entrance have the following form:

$$\beta = 2\rho_M = \frac{2B\rho}{|B_0|} \quad ; \quad \alpha = -\frac{1}{2}\beta' = 0 \qquad (5)$$

Here $\rho_M$ is magnetic radius of the spiral inflector, $B_0$ – magnetic field in the cyclotron center.

The invariants (1,3) and matching conditions (5) give possibility to find the second order moments of beam distribution function at the inflector entrance.

## SECOND ORDER MOMENTS

The matching conditions (4) are valid in the coordinate frame rotating with Larmor's frequency around the longitudinal axis [5]. Thereby in the lab frame the second order moments at the inflector entrance have the following form:

$$\overline{x^2} = \overline{y^2} = 2\rho_M\varepsilon_{rms} \quad ; \quad \overline{xx'} = \overline{yy'} = 0 \qquad (6.1)$$

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$$\overline{x'^2} = \overline{y'^2} = \frac{\varepsilon_{rms}}{\rho_M}\left(1 - Sgn(B_0 M)\frac{|M|}{2\varepsilon_{rms}}\right) \qquad (6.2)$$

$$\overline{xy'} = -\overline{yx'} = \frac{M}{2}\left(1 - \frac{2\varepsilon_{rms}}{|M|}Sgn(B_0 M)\right) \qquad , \quad (6.3)$$

where $Sgn(x)$ is signum function. In accordance with formula (3) ratio $2\varepsilon_{rms}/|M|$ is equal to:

$$\frac{2\varepsilon_{rms}}{|M|} = \sqrt{1 + \frac{4\varepsilon_0^2 - j_0^2}{M^2}} \cong 1 + \frac{1}{2}\frac{4\varepsilon_0^2 - j_0^2}{M^2} \qquad (7)$$

Here subscript “0” denotes the moments evaluated at extraction hole of ECRIS.

Because of small value of ion velocities in the ECRIS plasma the average angular momentum $M$ is defined by magnetic flux through the extraction hole. Thereby the sign of momentum $M$ coincides with one of the ECRIS magnetic field $B_{ECR}$ and ratio $\frac{4\varepsilon_0^2 - j_0^2}{2M^2} = \kappa << 1$. It may be shown [5] that value of $\kappa$ is positive.

## POSSIBLE PARTICLE DISTRIBUTIONS

Arbitrary function $f(I)$ of bilinear form $I$ connected with the second order moments (6) is the solution of Vlasov equation. The form $I$ is defined as follows [4]:

$$I = Y^T\left(M^{II}\right)^{-1}Y \qquad ; \qquad M^{II} = \overline{YY^T} \qquad , \quad (8)$$

where $Y^T = (x, y, x', y')$ and superscript “$T$” denotes transpose vector or matrix.

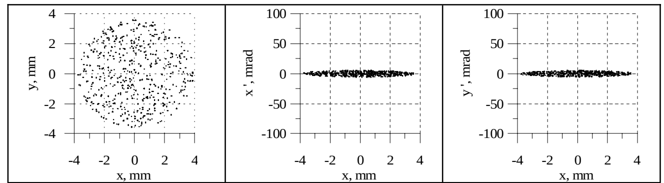
Let the positive direction of the magnetic field coincides with one of the reference particle in the beam line. Two general distributions exist in accordance with relative direction of the ECRIS and cyclotron magnetic field. In all cases the beam has a round form with RMS dimensions (6.1).

### *1. Sgn($B_{ECR}B_0$)=1*

For the first type of distribution the magnetic fields of ECRIS and cyclotron have the same direction in comparison with one of the reference particle. The moments (6.2, 6.3) have the following values:

$$\overline{x'^2} = \overline{y'^2} = \frac{\varepsilon_{rms}}{\rho_M}\frac{\kappa}{1+\kappa} \qquad ; \qquad \overline{xy'} = -\overline{yx'} = -\frac{M}{2}\kappa \qquad (9)$$

In this case RMS emittance $\varepsilon$ (4) for each transverse phase plane (x,x′) and (y,y′) is evaluated as $\varepsilon \cong \sqrt{2\kappa}\varepsilon_{rms} << \varepsilon_{rms}$. The average rotation of the particles around the longitudinal axis arising in ECRIS magnetic field disappears almost completely. The particles distribution of K-V type in the various phase plane calculated for $\varepsilon_{rms}$ = 50 π mm×mrad and $\rho_M$ = 35 mm are shown in Fig.1.




| Plane(x,y) | Plane(x,x′) | Plane(x,y′) |
|---|---|---|

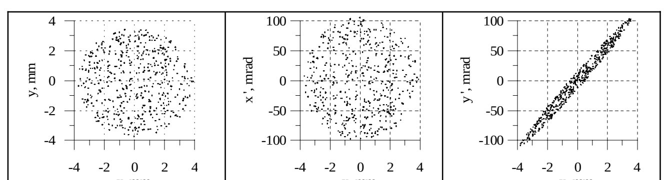
Figure 1: K-V type distribution in various phase planes. $Sgn(B_{ECR}B_0) = 1$

### *2. Sgn($B_{ECR}B_0$)=-1*

For the second type of distribution the magnetic fields of ECRIS and cyclotron have the opposite direction in comparison with one of the reference particle. The moments (6.2, 6.3) have the following values:

$$\overline{x'^2} = \overline{y'^2} \cong \frac{2\varepsilon_{rms}}{\rho_M} \qquad ; \qquad \overline{xy'} = -\overline{yx'} \cong M \qquad (10)$$

In this case RMS emittance $\varepsilon$ (4) for each transverse phase plane (x,x′) and (y,y′) is evaluated as $\varepsilon \cong 2\varepsilon_{rms}$. The average mechanical angular momentum $j$ of the particles arising in ECRIS magnetic field is doubled. Unlike the previous case the rotation of particle around the longitudinal axis intensifies. K-V type of particle distribution in the various phase planes calculated for the same values of $\varepsilon_{rms}$ and $\rho_M$ are shown in Fig.2.




| Plane(x,y) | Plane(x,x′) | Plane(x,y′) |
|---|---|---|

Figure 2: K-V type distribution in various phase planes. $Sgn(B_{ECR}B_0)$ = -1

## SPIRAL INFLECTOR

The influence of type of distribution and value of RMS beam emittance $\varepsilon_{rms}$ on dynamics of the ions in the spiral inflector has been considered. The spiral inflector of DC-350 cyclotron [1] – new cyclotron designed at the Flerov Laboratory of Nuclear Reaction of the Joint Institute for Nuclear Research has been taken as example. 3D electric field map has been calculated by means of RELAX3D code [3]. The beam dynamics simulation has been performed by using CASINO program code [2].

The horizontal and vertical RMS beam emittances and bunch length at the exit of spiral inflector have been determined in this simulation. In CASINO program the initial RMS bunch length $S_0$ is put to zero. Thereby the final RMS bunch length $S_f$ after passing through inflector may be evaluated by using formula:

$$S_f^2 = S_0^2 + S^2 \quad , \qquad (11)$$

where $S$ is the bunch lengthening in the spiral inflector.

The dependences of exit RMS values of horizontal (Er) and vertical (Ez) emittances and bunch lengthening (S) on the RMS emittance at entrance of spiral inflector $\varepsilon_{rms}$ for the first type of distribution are shown in Fig.3.

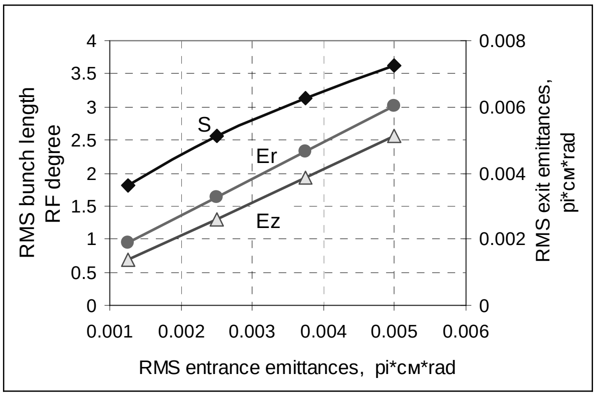



Figure 3: Horizontal (Er) and vertical (Ez) RMS emittances and RMS bunch lengthening (S) *vs* $\varepsilon_{rms}$. $Sgn(B_{ECR}B_0) = 1$

For the first type of distribution values of emittance at exit of the spiral inflector coincide approximately with initial one of invariant RMS emittance $\varepsilon_{rms}$.

The same dependencies for the second type of distribution are shown in Fig.4.

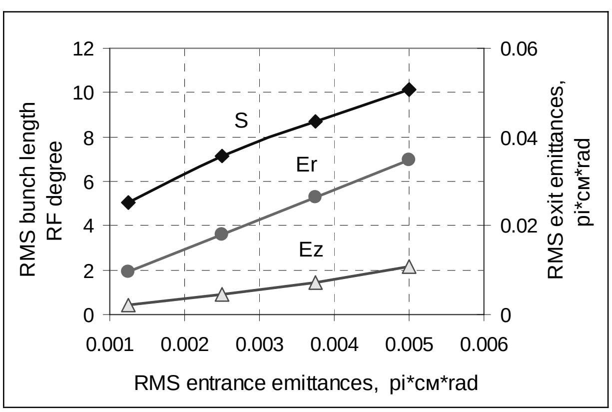



Figure 4: Horizontal (Er) and vertical (Ez) RMS emittances and RMS bunch lengthening (S) *vs* $\varepsilon_{rms}$. $Sgn(B_{ECR}B_0) = -1$

For the second type of distribution vertical emittance in two and horizontal – almost in ten times greater than in previous case. Also the bunch lengthening is about three times greater as compared with one for the first type of distribution.

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