BEAM ACCELERATION STUDIES OF PROTON NS-FFAG

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Abstract

Resonance crossing during beam acceleration in a Non-Scaling FFAG could cause beam blow-up when the acceleration rate is slow. This could potentially constraint the lattice design of proton NS-FFAG. Therefore, an understanding of the beam dynamics of resonance crossing acceleration is crucially important to establish the design strategy of proton NS-FFAG. This paper describes the beam blow-up process of an NS-FFAG and discuss the blow-up rate quantitatively.

INTRODUCTION

Recently, Fixed Field Alternating Gradient(FFAG) accelerators has received attention due to their potential of varied applications from fundamental science to practical applications like cancer therapy. For one variatiety of FFAG, Non-Scaling(NS) FFAG, intensive R&D activities are being promoted worldwide[1]. This type of FFAG was originally proposed as a muon accelerator for a neutrino factory, but its simple structure and capability of rapid beam acceleration provide prospects of various practical applications like particle therapy and accelerator driven system. With this motivation, various lattice designs are proposed[2, 4].

In a NS-FFAG, the betatron tune varies largely during acceleration. For muon acceleration, such tune drift can be overcome with extremely fast acceleration. Actually, in muon acceleration, beam reaches the final energy within 20 turns. However, for practical applications, the accelerating particles could be non-relativistic particles such as proton or heavy ions. In such cases, more than 1000 turns of acceleration are required to reach the final energy. With such a slow acceleration rate, typically two orders of magnitude slower than the case of muon acceleration, the resonance crossing during an accelerating process could cause serious beam blow-up. Therefore, in order to establish reliable design criteria for a slow acceleration NS-FFAG, an understanding of the beam dynamics of resonance crossing is crucially important. In the following, the tracking study of resonance crossing acceleration is described and the implications for lattice design in a slow acceleration NS-FFAG are also discussed.

FORMALISM

R. Baartman proposed a simple formula to evaluate the amplitude growth rate during resonance crossing[3]:

$$\frac{\Delta(A^{2-m})}{m} = \frac{\pi}{2^{m-1}} b_{n,m} \sqrt{\frac{m}{Q_\tau}} \tag{1}$$

$$b_{n,m+1} = \frac{1}{n}\frac{\bar{R}}{\bar{B}}\frac{1}{m}\frac{\partial^m B_n}{\partial x^m}, \tag{2}$$

where A,m,B_n and Q_τ mean amplitude (m), the order of resonance, $n = Q \times m$, n-th Fourier component of field error and changing rate of betatron tune per turn, respectively.

For integer resonance($m = 1, n = Q$), equation (1) is written as

$$\Delta A = \frac{\pi}{\sqrt{Q_\tau}}\frac{\bar{R}}{Q}\frac{B_n}{\bar{B}} = \frac{\pi}{\sqrt{Q_\tau}}\bar{\beta}\frac{B_n}{\bar{B}}, \tag{3}$$

where $\bar{\beta}$ means average beta of the ring. In order to suppress amplitude growth, smaller $\bar{\beta}$ is desirable. This is similar logic to that applied in muon ionization cooling.

For half integer resonance ($m = 2, 2n = Q$), equation (1) is written as[1]

$$\log\frac{A_f}{A_i} = \frac{\pi}{2}\frac{\bar{R}}{n\bar{B}}\frac{\partial B_n}{\partial x}\frac{1}{\sqrt{Q_\tau}} \tag{4}$$

LATTICE

For the beam acceleration study, the lattice of a tune-stabilized linear NS-FFAG proposed by C. Johnstone was employed [4]. It employs wedged shaped combined function quadruple magnets, which provide both chromatic correction and further tune stabilization. Based on the design, lattice building was carried out using ZGOUBI [5]. The tune obtained with ZGOUBI is shown in Figure1. Apart from the vertical tune of low momentum region (below 300MeV/c), the tune is well reproduced when compared to the original design.

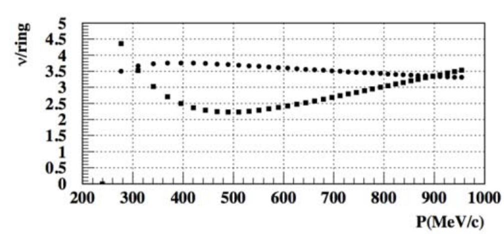

Figure 1: Tune footprint of test lattice.

BEAM ACCELERATION SIMULATION

Using the test lattice, acceleration study was carried out. The energy gain per turn is more than 200keV/turn. With this energy gain, the beam can be accelerated within 1ms

[1] For the case of $m = 2$, the right hand side of equation (1) becomes $\Delta(\log A)$.

from 30MeV to 230MeV. This implies 1kHz repetition rate for proton therapy application.

In the study, bending field errors and field gradient errors are introduced in individual magnets and 100 different error sets were examined for both errors. Bending field error was introduced through positioning errors of magnets. Figure 2 shows a typical amplitude growth during acceleration with bending field errors. In this case, the initial beam size is 5πmm mrad(normalized). 36 particles on an ellipse were tracked and the maximum amplitude among them was taken as the amplitude of the beam at the moment.

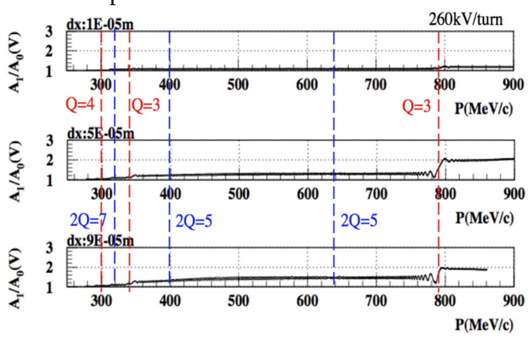

Figure 2: Typical acceleration process with various positioning errors. Vertical axis is the relative ratio of the normalized amplitude.

Beam is clearly perturbed at integer resonances and the correlation between the magnitude of portioning error and amplitude growth is also apparent. To understand the dynamics quantitatively, a $microscopic$ tracking study, in which tracking is carried out just around the resonance, was carried out.. In the following section, three major resonances are discussed. These are integer resonance, half integer resonance, and structure resonance.

Integer Resonance Crossing

In the test lattice, the integer resonances are crossed three times ($Q = 4, 3$). Figure 3 shows a typical amplitude growth process at the integer resonance crossing ($Q = 4$).

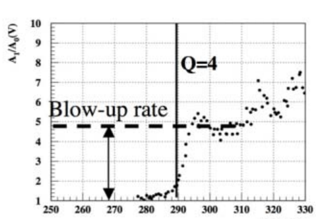

Figure 3: Typical amplitude growth in integer resonance crossing ($Q = 4$).

To evaluate the process quantitatively, equation (3) is rewritten as

$$R \equiv \frac{1}{2}\sqrt{\frac{\beta}{\epsilon}}\Delta\epsilon\frac{\sqrt{Q_\tau}}{\sigma_{pos}} = \pi\frac{\bar{R}}{Q}\frac{B_{U:N}}{\bar{B}}, \quad (5)$$

where σ_{pos} means positioning error of magnet, $B_{U:N}$ means the n-th Fourier component of field error for unit length positioning error, respectively. The right hand side of equation (5) is an intrinsic parameter of the lattice. On the left hand side,R is a combination of design parameters such as acceptable emittance growth, accelerating rate. R was evaluated for various energy gains and positioning errors for two integer resonances($Q = 4$ at 300MeV/c, and $Q = 3$ at 340MeV/c). In evaluating R, the maximum amplitude growth among 100 different error configurations was taken. Figure 4 shows the results. This shows that the amplitude growth is well evaluated by Baarman's Formula (equation (3)). With a realistic condition, say, positioning error($\sigma_{pos} \sim 100\mu m$ and energy gain $\sim$ 200kV/turn), an emittance blow-up more than factor of 40 is expected for a 5πmm mrad (normalized) beam for a single integer resonance crossing of the test lattice.

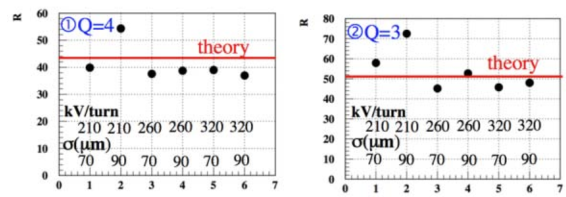

Figure 4: Emittance growth parameter of integer resonance crossing: (1) $Q = 4$, (2) $Q = 3$.

Structure resonance Crossing

In the test lattice, vertical tune crosses $2Q = 7$ resonance. Since the periodicity of ring is 14, the resonance is actually a structure resonance ($4Q = 14$), which could potentially be a problem with higher order multipole components of fringing field. Figure 5 shows a typical amplitude growth of the structure resonance crossing.

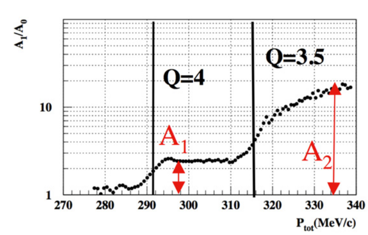

Figure 5: Typical amplitude growth in structure resonance crossing ($4Q = 14$).

A similar study was also carried out at the $2Q = 5$ resonance. However, at $2Q = 5$ resonance, no significant emittance growth was observed. In fact, the positioning error of wedge-shaped combined function magnet can directly generate field gradient error, typically $O(10^{-4})$ for $100\mu m$ of positioning error. This level of field gradient error will not excite observable half integer resonance within the range of examined parameter. The amplitude growth of half integer resonances excited by field gradient error is to be discussed in the nest section.

The resonance has a threshold amplitude shown in Figure 6. This figure shows the amplitude correlation before and after crossing the resonance of $2Q = 7$.

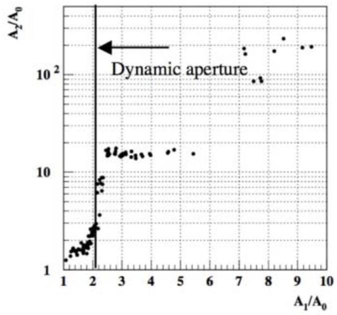

Figure 6: Amplitude correlation between before and after structure resonance crossing.

Half Integer Resonance Crossing

Tracking study was carried out for $2Q = 5$ resonance, introducing field gradient error into individual magnets. Then, amplitude growth is evaluated in a similar manner with the integer resonance crossing study. Positioning error was not introduced in the study.

For a half integer resonance, equation (4) can be rewritten in similar manner as equation (5)

$$R \equiv \log \frac{A_f}{A_i} \frac{\sqrt{Q_\tau}}{\sigma_{fld}} = \frac{\pi}{2} \frac{\bar{R}}{n\bar{B}} \frac{\partial B_{U:n}}{\partial x} \quad (6)$$

The amplitude growth parameter, R was evaluated for various conditions. Figure 7 shows the results and comparison with the theoretical value. The constant nature of the growth parameter is confirmed. However, the matching with the theoretical value is not as good as the case of integer resonances. One possible source of discrepancy is the floppy nature of beta function at the half integer resonances. This could cause large errors in calculating the amplitude after the resonance crossing. At the moment, the investigation of the error source remains an item for future study.

For single half integer resonance crossing, Under a modest field gradient error, $\Delta B_1/B_1 = 2\times10^{-3}$, the emittance blow up is less than factor of 2 for the case of 1kHz repetition rate. In particular therapy application, the final beam size is typically 10πmm mrad. Therefore, as long as injection beam is sufficiently small, say less than 2π mm mrad[6], the crossing of a single half integer resonance is acceptable.

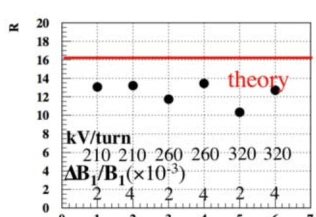

Figure 7: Emittance growth parameter of half integer resonance crossing: $2Q = 5$.

IMPLICATIONS FOR THE LATTICE DESIGN OF PROTON NS-FFAGS

Since the resonance width varies for individual lattices,similar tracking study should be carried out for individual lattice. Recent study of beam acceleration of NS-FFAG suggests the necessity of higher order field components to sustain a sufficiently large dynamic aperture in slow acceleration[7]. Thus, the results of the above study based on the tune stabilized NS-FFAG lattice could suggest some general criteria for slow acceleration NS-FFAG:

1. Integer resonances : Baartman's formula can evaluate the integer resonance blow up rate. Multiple integer resonance crossing must be avoided. Single integer resonance crossing might be acceptable, depending on the application, available accelerating field and tolerance. However, for particle therapy application, for which final emittance should not exceed 10π mm mrad, even single integer resonance would be unacceptable.
2. Half Integer resonances : Baarman's formula can somehow explain the phenomenon. Considering the beam size requirement for particle therapy applications, single half integer resonance crossing would be acceptable if the field gradient quality can be controlled to the level of $\Delta B_1/B_1 \leq\sim 1\times10^{-3}$ for 1kHz repetition rate
3. Structure resonances : These should be circumvented. Structure resonance could be a strong resonance given the existence of higher order multipole field and could largely reduces allowable tune drift width.

ACKOWLEDGEMENT

We greatly appreciate the helpful advises of, and discussion with Dr. F. Meot, LPSC, France, Dr. C. Johnstone, FNAL, USA and Dr. S. Machida, RAL, UK. This work is supported by BASROC and is carried out by the collaboration under EPSRC grant number EP/E032869/1.

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