INJECTION AND EXTRACTION ORBITS AND TWISS PARAMETERS FOR THE EMMA RING

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Abstract

Using the FFEMMAG code, the injection and extraction orbits for the EMMA ring at a variety of injection and extraction energies together with the Twiss parameters to be used for matching have been calculated. The orbits include two kickers together with a septum at both injection and extraction. The FFEMMAG code has been used in conjunction with several scripts so as to be able to scan the parameter space of the two kicker strengths for a section of the EMMA ring. The results confirm the choice of magnet and vacuum pipe apertures as being adequate to operate EMMA from 10 to 20 MeV.

INTRODUCTION

In a FFAG accelerator the beam spirals from an orbit located to the inside of a reference curve (chosen arbitrarily) to an external orbit, so that every turn it occupies different intermediate orbits. Each orbit corresponds to a radial displacement and to a prescribed beam momentum.

There are several disadvantages of the original scaling FFAG design among which are the large aperture requirements and the relatively complicated magnets. The required aperture in the non-scaling FFAG accelerator is significantly smaller. The transverse magnetic field in the alternating gradient magnets (focusing and defocusing quadrupoles) is linear so that particles with different momenta follow orbits oscillating around the ring.

The non-scaling FFAG is a novel accelerator concept and a project of EMMA (Electron Machine for Many Applications) to demonstrate its feasibility has been recently proposed [1]. It is also designed as the scale-down model of a muon accelerator for neutrino factory. The machine is to be built in the ALICE (Accelerators and Lasers In Combined Experiments) hall of Daresbury Laboratory in the UK and ALICE itself is intended to be used as the injector.

This paper addresses some issues of beam dynamics, injection and extraction details of the EMMA ring utilizing the FFEMMAG computer code, which has been developed recently at Daresbury Laboratory.

REFERENCE TRAJECTORY AND INJECTION REQUIREMENTS

It has been shown [2] that the equations for the reference orbit can be obtained from the Hamiltonian

$$H_e(X_e, P_e; s) = -\sqrt{\beta_e^2\gamma_e^2 - P_e^2} + \frac{e}{p_0}\int \mathrm{d}X_e B_z(X_e; s). \tag{1}$$

The Hamilton's equations of motion can be linearized and subsequently solved approximately by assuming

$$P_e \ll \beta_e\gamma_e, \tag{2}$$

so that

$$P_e = \beta_e\gamma_e X_e', \qquad X_e'' = -\frac{ea_1(s)}{p_0\beta_e\gamma_e}[X_e - X_c - d(s)]. \tag{3}$$

Here $p_0 = m_0 c$, $a_1(s)$, and $d(s)$ are the focusing strength and the offset with respect to the machine polygon line of the (focusing or defocusing) quadrupole, respectively, and X_c is the distance of the polygon line from the geometric centre of the machine. Introducing the state vector $\mathbf{Z}_e = (X_e, P_e)^T$, we can write the solution to Eqs. (3) as follows

$$\mathbf{Z}_f = \mathcal{M}_e\mathbf{Z}_i + \mathbf{A}_e, \tag{4}$$

where $\mathbf{Z}_i$ is the initial value of the state vector, while $\mathbf{Z}_f$ is its final value at the exit of the corresponding element. The transfer matrix $\mathcal{M}_e$ and the shift vector $\mathbf{A}_e$ for various lattice elements can be found in a straightforward manner by solving the equations of motion (3).

Since the reference trajectory must be periodic function of the longitudinal position s, it clearly satisfies the condition $\mathbf{Z}_f = \mathbf{Z}_i = \mathbf{Z}_e$. In other words, the equation for determining the reference orbit becomes

$$\mathbf{Z}_e = (1 - \mathcal{M})^{-1}\mathbf{A}. \tag{5}$$

Here $\mathcal{M}$ and $\mathbf{A}$ are the transfer matrix and the shift vector for one period, respectively.

The basic requirements for the injection and extraction system are shown in Table 1. Taking into account the machine requirements and beam characteristics, fast injection and extraction schemes using two kickers and a septum are adopted. Provided the available space and aperture requirements, injection and extraction for EMMA have proven to be rather challenging.

DESCRIPTION OF THE FFEMMAG PROGRAMME

The computer code FFEMMAG implements the basic features of the synchro-betatron formalism suitably modified to study non scaling FFAG accelerators [2]. The method is developed starting from first principle. Its milestone is the full three-and-a-half-degree-of-freedom Hamiltonian describing the motion of a charged particle in the guiding and accelerating electromagnetic fields of the most general configuration. The Hamiltonian is written in the

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Table 1: Requirements for the injection and extraction system of the EMMA ring

Specification	**Value**
Injection energy	10 – 20 MeV (variable)
Extraction energy	10 – 20 MeV (variable)
Acceptance	3π mm rad (normalized) with painting allowance
Range of tune variation	14.47 – 5.30 (Horizontal)
Kicker rise time	30 ns (beam evolution period of approximately 55.45 ns)

natural coordinate system associated with a particle trajectory (reference orbit) for a fixed momentum/energy. Therefore, finding the reference orbit is an important task and can be considered as the initial stage of the design of the EMMA ring. The formalism allows to further single out by orders of magnitude various linear and nonlinear contributions to the particle motion by a sequence of properly chosen canonical transformations.

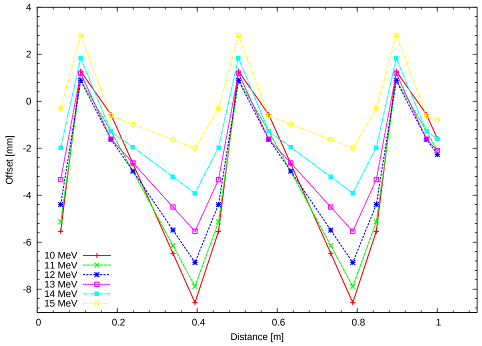

Figure 1: Typical reference trajectory for energies in the range 10 – 15 MeV for two 0.394481 meter long EMMA cells.

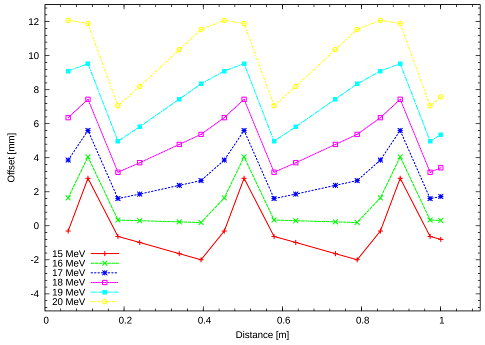

Figure 2: Typical reference trajectory for energies in the range 15 – 20 MeV for two 0.394481 meter long EMMA cells.

The first step of the method comprises in the calculation of the reference orbit. For a given energy, it must be a periodic function of the particle location along the circumference of the ring with periodicity equal to the length of one DOFO (or respectively FODO, depending on which one has been chosen as a basic lattice configuration) cell. To find the reference trajectory, the exact relativistic equations of motion following from (1) are integrated numerically by imposing periodic boundary conditions, which assures the periodicity of the solution thus obtained. Typical reference trajectories in the 10 – 20 MeV energy range are shown in Figures 1 and 2.

Having determined the reference trajectory, one can adjust the optimal strengths of the injection and extraction kickers for a variety of impact parameters (orbit location and angle with respect to the reference coordinate system) after/before the septum magnet. The FFEMMAG code includes a block which performs these calculations in detail. Some good and practically feasible cases of reasonable physical apertures and kicker strengths are shown in Figures 3 and 4.

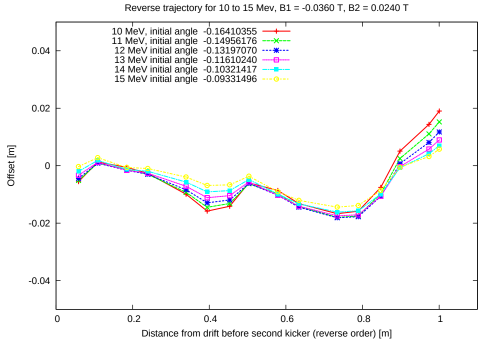

Figure 3: Typical injection trajectory for energies in the range 10 – 15 MeV at suitable kicker strengths.

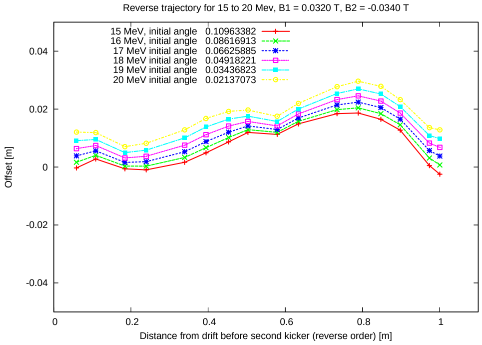

Figure 4: Typical injection trajectory for energies in the range 15 – 20 MeV at suitable kicker strengths.

The next step includes the definition of dispersion to arbitrary order in terms of powers of the energy deviation. For a particle with an accurately specified design momentum, the position vector is precisely given by the reference trajectory plus betatron oscillations (linear and nonlinear) superimposed. The latter comprise the transverse phase space. For off-momentum particles additional terms in the horizontal component of the position vector appear, which are proportional to the successive powers of the deviation of the actual energy with respect to the design energy. These terms specify the dispersion functions to all orders. The FFEMMAG code calculates the zero-order (conventional) dispersion function and the first-order dispersion function. These are used later to compute the corresponding phase slip coefficients characterizing the acceleration.

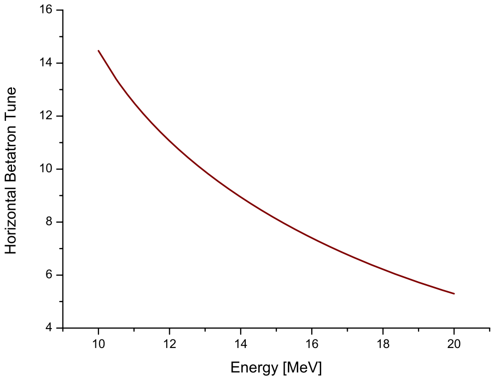

Figure 5: Horizontal betatron tune as a function of energy.

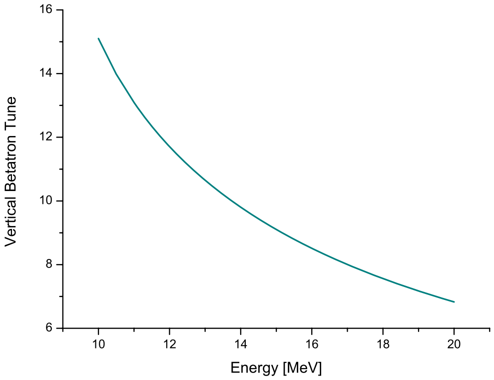

Figure 6: Vertical betatron tune as a function of energy.

Further, the dynamics of the Twiss parameters describing the betatron motion have been worked out in detail. These crucially depend on the energy and on the characteristic shape of the reference trajectory. The FFEMMAG code contains routines, which calculate the horizontal and vertical Twiss parameters (alpha and beta function) for all intermediate energies from injection to extraction as well as the corresponding horizontal and vertical betatron tunes. The dependence of the horizontal and vertical betatron tunes on the energy is shown in Figures 5 and 6.

The path length in an FFAG arc is often well approximated as a quadratic function of energy (see Figure 7 for an example obtained using the FFEMMAG programme). A novel issue which has no analogue in other cyclic accelerators is the acceleration process. Since an FFAG lattice is in principle highly dispersive, higher orders of phase slippage must be taken into account. The acceleration process is described by a longitudinal Hamiltonian, which contains terms proportional to the zero-order (conventional phase slip factor) and first-order phase slip factor. It usually suffices to take into account only terms to second order in the energy deviation as suggested by Figure 8.

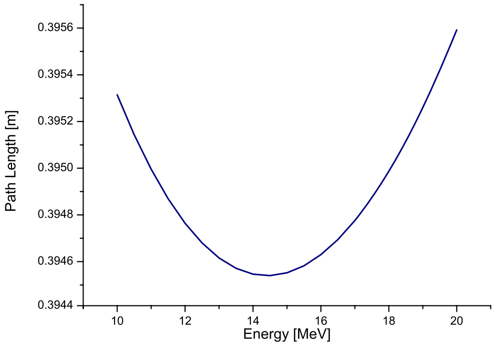

Figure 7: Path length as a function of energy for a single 0.394481 meter EMMA cell.

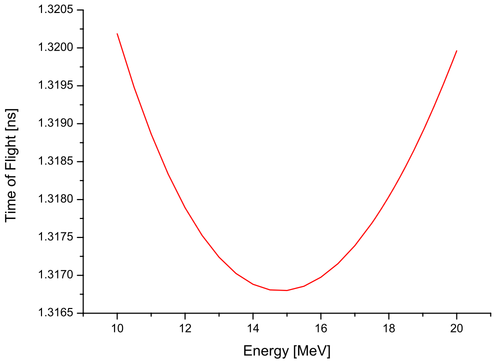

Figure 8: Time of flight as a function of energy for a single 0.394481 meter EMMA cell.

CONCLUDING REMARKS

The FFAG accelerator EMMA aims to demonstrate the feasibility of the non scaling FFAG concept. Due to the diversity of tasks to be fulfilled during its future commissioning, the injection system must be flexible enough and able to handle a broad variety of injection conditions. To investigate the practicability of the injection and extraction system, a tracking study using FFEMMAG has been carried out. After a brief description of the programme some results concerning the beam dynamics and injection/extraction matching have been shown. These results indicate that the latter is feasible within reasonable aperture and strength specifications of septa and kicker magnets.

REFERENCES

[1] CONFORM website, http//www.conform.ac.uk

[2] B.D. Muratori, S.L. Smith and S.I. Tzenov, These Proceedings.