HEATING RATE OF HIGHLY SPACE-CHARGE-DOMINATED ION BEAMS IN A STORAGE RING*

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Abstract

We investigate the heating process of highly space-charge-dominated ion beams in a storage ring, using the molecular dynamics (MD) simulation technique. To evaluate the heating rate over the whole temperature range, we start from an ultra-low-emittance state where the beam is Coulomb crystallized, apply perturbation to it, and follow the emittance evolution. When the ring lattice is properly designed, the heating rate is quite low at ultralow temperature because random Coulomb collisions are suppressed [1]. It gradually increases after the ordered state is destroyed by perturbation, and comes to a peak when the beam reaches a liquid phase. The dependence of the heating behavior on the beam line density, energy and betatron tune is explored systematically. The effect of lattice imperfection on the stability of crystalline beams is also confirmed.

INTRODUCTION

The quality of a charged-particle beam can be improved by introducing dissipative (cooling) interactions into the system. If the cooling force is sufficiently strong, the beam finally exhibits an ordered structure at the space-charge limit. Such an ultimate state is known as a *crystalline beam* [2-4]. The crystalline state is quite stable and thus lasts long even after the cooling force is removed. It is, however, also true that in a non-dissipative situation, the ordered configuration is eventually destroyed due to the periodic nature of the external focusing field [1, 4].

An understanding of heating mechanisms in a highly space-charge-dominated beam is practically important for beam cooling experiments. For instance, the reachable beam emittance can be limited by collisional heating (intrabeam scattering). It is actually impossible to reach a crystalline state unless the cooling rate exceeds the heating rate due to interparticle Coulomb collisions. Wei et al. have revealed, in a previous numerical study [1], that the heating behavior of a beam at extremely low temperature is essentially different from that at high temperature; in an ultra-low temperature range, random Coulomb collisions start to be less dominant as the phase-space density of the beam becomes higher.

In the present paper, we try to provide more extensive information of the heating behavior of highly space-charge-dominated ion beams in a storage ring. First, we briefly summarize some requirements for the formation and maintenance of a crystalline beam. We then show results of systematic MD simulations that clarify the dependence of the collisional heating rate on various parameters including the betatron tune, beam current, beam energy and lattice imperfection.

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SIMULATION PARAMETERS

In order to evaluate the heating rate over a wide range, we start from an ultra-low emittance state where the beam is crystallized. For this purpose, the operating point of a storage ring has to be chosen carefully so that the stability of the crystalline state is guaranteed. According to previous works [4, 5], we have to operate the storage ring below the transition energy for crystal formation, and also set the bare betatron phase advance less than 127 degrees per lattice period for crystal maintenance. If one of these conditions is broken, it is no longer possible to make a crystal; the beam is inevitably heated at some low temperature by an instability mechanism different from Coulomb collisions.

Two model rings have been considered in our MD study, both of which satisfy the above conditions. One of them is S-LSR now in operation at Kyoto University [6]. In the following, we only show MD results on this existing lattice because no essential lattice dependence has been found so far. The main simulation parameters of S-LSR are listed in Table 1.

MOLECULAR DYNAMICS SIMULATION

Emittance Increase of a Highly Space-Charge-Dominated Beam

As mentioned above, the heating rate is very low in an ideal crystalline state because random Coulomb collisions are almost suppressed [1]. It thus takes long time until the ordered structure begins to be melted. To save the computing time, we perturb a crystalline beam with a random kick at the beginning of each simulation.

Figure 1 shows a typical time evolution of the transverse normalized root-mean-squared (rms) emittance of a coasting crystalline ion beam circulating in S-LSR. The projected emittances increase gradually due to Coulomb collisions. Around the 11500th turn, the beam

Table 1: Main simulation parameters.

Ion species	$^{24}Mg^{+}$
Kinetic energy	35 keV ~ 1 GeV
Design superperiodicity N_{sp}	6
Ring circumference	22.56 m
Bare betatron tune $\nu_x = \nu_y$ (= ν_0)	1.44 ~ 1.80

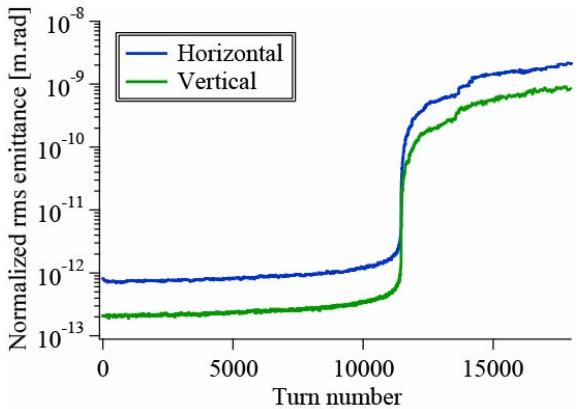

Figure 1: Time evolution of the normalized transverse rms emittance. The horizontal and vertical bare betatron tunes have been set at 1.44. The line density of the beam is 6.0×10^5 m^{-1} where the beam forms a three-shell crystalline structure at ultra-low temperature, as shown in Fig. 2(a).

heats up rapidly. As shown in Fig. 2, the ordered state is roughly maintained until the sudden jump of the emittance occurs. The distribution becomes completely random right after the jump. The heating process slows down after this collapse of the ordered configuration.

Definition of the Heating Rate

To evaluate the heating rate of a beam, we first take an average of transverse emittance every turn as $\varepsilon(n)=[\varepsilon_x(n)+\varepsilon_y(n)]/2$ where n is the turn number, and $\varepsilon_{x(y)}$ is the horizontal (vertical) normalized rms emittance averaged over one turn. The average emittance increase rate per lattice period C_{period} is then estimated from

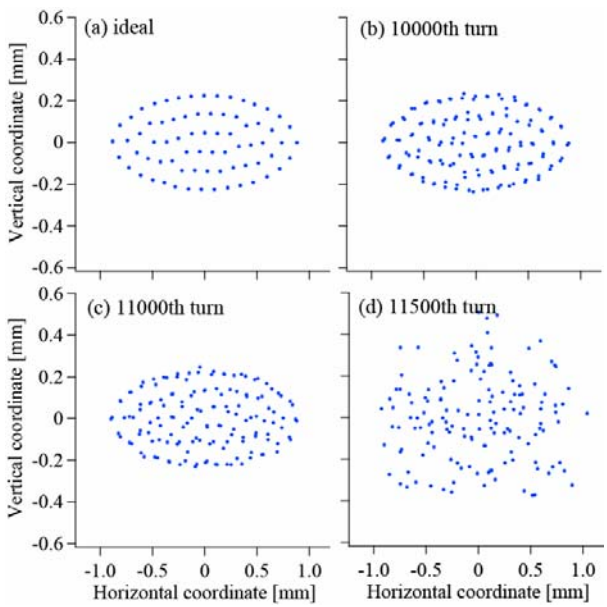

Figure 2: Cross-sectional view of the beam observed in the simulation of Fig. 1. (a) Ideal crystalline state at the beginning, (b) at 10000th turn, (c) at 11000th turn, and (d) at 11500th turn. Each dot corresponds to a single $^{24}Mg^+$ ion circulating in S-LSR at the kinetic energy of 35 keV.

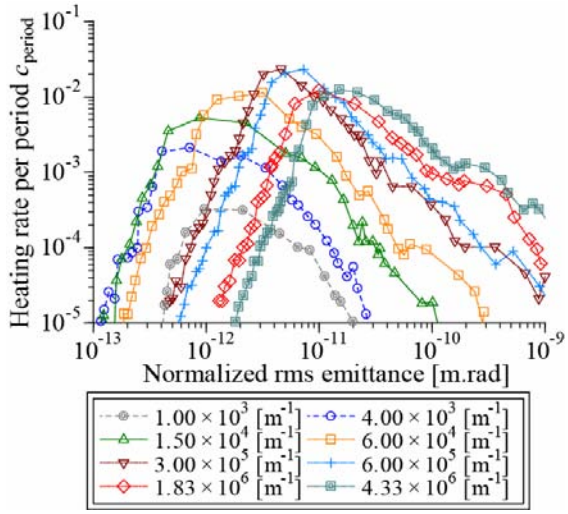

Figure 3: Heating rate of the beams vs. the normalized rms emittance. We have varied the beam line density over a wide range (from 10^3 to over 10^6 m^{-1}). When the line density is below 8.6×10^3 m^{-1}, a *string* crystalline beam is formed at ultra-low temperature where ions are aligned along the design orbit at the same intervals. The structural transition from a zigzag to a shell crystal takes place at 3.5×10^4 m^{-1}.

$$C_{\mathrm{period}}=(C_{\mathrm{turn}})^{1/N_{\mathrm{sp}}}=\left[\frac{\varepsilon(n+1)}{\varepsilon(n)}\right]^{1/N_{\mathrm{sp}}}, \quad (1)$$

where C_{turn} is the emittance increase rate per turn and N_{sp} is the design superperiodicity of the ring. Clearly, C_{period} is always larger than unity while the emittance increases. The heating rate c_{period} per lattice period is obtained by subtracting unity from Eq. (1):

$$c_{\mathrm{period}}=C_{\mathrm{period}}-1. \quad (2)$$

c_{period} is always positive in the present simulations.

Dependence on the Line Density of the Beam

Figure 3 shows a typical result of MD simulations assuming beams of different line densities. The evolutions of the heating rate have been plotted as a function of normalized rms emittance. At sufficiently low line density where the corresponding crystalline structure is a string, the heating rate clearly becomes lower as we decrease the line density. This might give a qualitative explanation to the one-dimensional ordering phenomena of electron-cooled ultra-low-intensity ion beams, observed in several storage rings [7-9]. By contrast, no clear correlation between the *peak* heating rate and line density is visible when the beam is sufficiently intense. Note also that the location of the peak moves toward the higher temperature side with the increase of line density. As a result, the heating rate at a particular emittance grows with line density on the high temperature side while the tendency is revered on the low temperature side. To reach a crystalline state, we need a dissipative force strong enough to overcome this heating-rate mountain [10]. We have confirmed that the heating behavior of a bunched beam is similar to the present case.

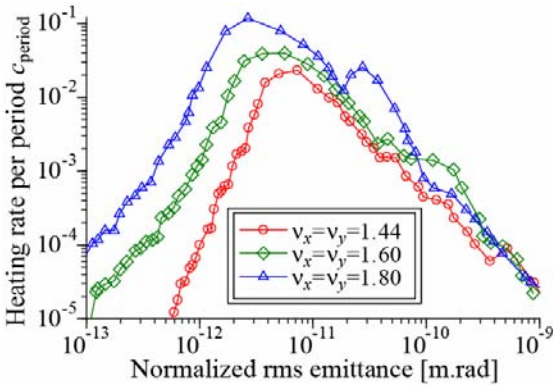

Figure 4: Tune-dependence of the heating rate. The beam line density has been fixed at 6.0×10^5 m^{-1} in all three cases.

Dependence on the Betatron Tune

The tune-dependence of the heating rate has also been studied systematically. Three MD results are shown in Fig.4 where the phase advance per lattice period has been chosen below 127 degrees in all three cases to meet the stability requirement. Obviously, the heating-rate mountain becomes higher as the bare tune increases. This is because the beam is more strongly compressed in real space with an increasing tune and, thus, the rate of Coulomb collisions is enhanced.

In the two cases where ν_0 = 1.6 and 1.8, we recognize a small second peak in the high temperature side. This peak is probably due to a second-order resonance crossing that is not avoidable when the bare phase advance per lattice period exceeds 90 degrees [11].

Dependence on Beam Energy

The dependence of the heating rate on beam energy is shown in Fig. 5. The heating-rate mountain moves towards the lower temperature side with almost the same peak height, as we increase the beam energy. The heating rate at a particular emittance thus goes down on the high temperature side with increasing beam energy. The final equilibrium emittance at which a crystalline state is established is lowered at a higher energy.

Dependence on Lattice Imperfection

Strictly speaking, the superperiodicity of any storage ring is unity because imperfections of lattice elements are inevitable. We expect that the rate of Coulomb collisions should be insensitive to small error fields when the beam temperature is high. The stability of a crystalline state will, however, be affected seriously by field imperfections because they more or less break the maintenance condition mentioned above. We have actually observed that the heating rate is strongly enhanced by error fields only in the ultra-low temperature range. In the present case, the error level of less than 0.1 % is tolerable for crystal formation, which is consistent to the previous result [1]. We have also confirmed that a beam of lower line density is less sensitive to lattice errors.

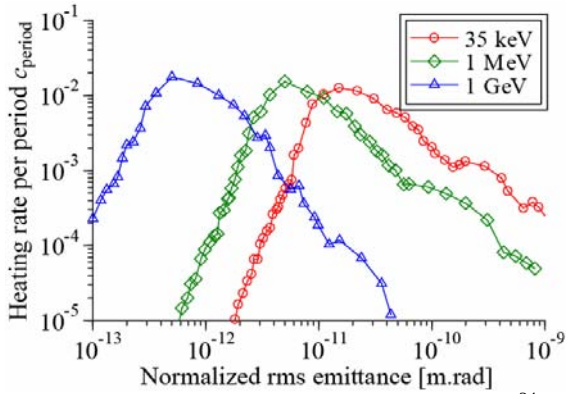

Figure 5: Heating rate vs. kinetic energy of a $^{24}Mg^+$ beam. The bare tune and the beam line density have been fixed, respectively, at 1.44 and at 4.3×10^6 m^{-1} in all three cases. The beam energy of 1 GeV is still below the transition energy (5.1 GeV) of the ring.

SUMMARY

We have explored the heating behaviour of highly space-charge-dominated ion beams over a wide temperature range. Through systematic MD simulations, we found that the heating rate curve has a peak when the lattice is properly designed. The heating-rate mountain shifts to the high temperature side as we increase the beam line density or decrease beam energy. It is, however, possible to eliminate this parameter-dependence of the peak position by plotting the heating rate as a function of the Coulomb coupling constant (instead of the normalized emittance) [12]. The peak heating rate can be lowered by choosing a smaller bare tune, which makes it easier to reach a crystalline state.

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