DIFFUSION CAUSED BY BEAM-BEAM INTERACTIONS WITH COUPLINGS

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Abstract

We discuss the diffusion caused by beam-beam interactions in circular accelerator. Beam-beam interaction is strong nonlinear system and has many degree of freedom. This system in multidimension has a diffusion nature. If there is a linear coupling error such as x-y coupling, dispersion and crossing angle in this system, nonlinear diffusion are enhanced with the enlargement of beam size and the degradation of luminosity. we have estimated the diffusion rate with linear coupling by weak-strong simulation.

INTRODUCTION

We consider a beam colliding with the other beam in e^+e^- circulator accelerator. When the beam-beam force is negligible, synchrotron radiation plays an important role on determining the beam distribution. The diffusion by synchrotron radiation determines the natural emittance due to the balance of the quantum excitation and the radiation damping The beam size is obtained as the ratio of the diffusion rate and damping rate,

$$\sigma^2 = \frac{D\tau_y}{2T_0}. \tag{1}$$

When there is a linear coupling error, the large luminosity degradation is observed by strong-strong and weak-strong simulation as shown in figure 1. We discuss here the reason why the degradation is so large.

Nonlinear dynamical system in multidimension has a diffusion nature. We are considering the luminosity degradation is due to its diffusion nature [1, 2]. In this paper we have studied the diffusion process which is seen in beam-beam interaction in e^+e^- circular colliders by weak-strong beam-beam simulation code. Although the luminosity degradation by weak-strong simulation in figure 1 is smaller than that by strong-strong simulation, we are considering some essential part of the degradation is included in the weak-strong simulation. To estimate the diffusion rate caused by the beam-beam interaction, we remove the synchrotron radiation in weak-strong simulations.

DIFFUSION

For a physical system with stochastic kick, we have so-called the diffusion equation for the particle distribution,

$$\frac{\partial}{\partial t}\Psi(x,t) = B\frac{\partial^2}{\partial x^2}\Psi(x,t), \tag{2}$$

where B: diffusion coefficient is assumed to be a constant. For initial condition, $\Psi(x,0) = \delta(x - x_0)$, the solution is given as

$$\Psi = \frac{1}{\sqrt{4\pi Bt}}\exp\Big[-\frac{(x-x_0)^2}{4Bt}\Big]. \tag{3}$$

This equation means that $\Psi(x,t)$ is Gaussian and its rms value increases as

$$\sigma^2(t) = \sigma_0^2 + 2Bt, \tag{4}$$

where σ_0 is initial rms value.

When this system has the damping mechanism of $x(t)$:

$$\frac{dx}{dt} = -Dx, \tag{5}$$

the particle distribution is obtained by Fokker-Plank equation as

$$\frac{\partial}{\partial t}\Psi(x,t) = Dx\frac{\partial}{\partial x}\Psi + B\frac{\partial^2}{\partial x^2}\Psi \tag{6}$$

An equilibrium solution of $\partial\Psi/\partial t = 0$

$$\Psi = \exp\Big[-\frac{x^2}{2B/D}\Big] \tag{7}$$

is obtained. As is well-known, the equilibrium distribution doesn't depend on initial one.

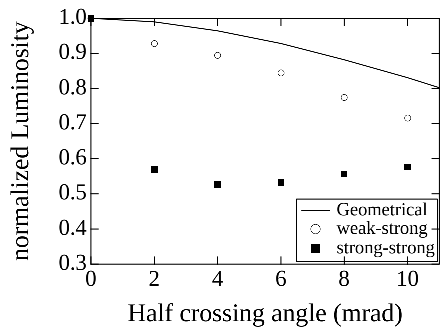

Figure 1: Normalized luminosity are plotted as a function of half crossing angle. Solid line shows geometrical luminosity. Open circle and black box shows the luminosity calculated by weak-strong and strong-strong simulation, respectively.

SIMULATION

We have estimated the diffusion rate caused by the weak-strong simulation. Beam-beam force is calculated by using a Basseti-Erskine formula [3].

A transformation of the collided bunch through the ring is calculated using a transfer matrix.

$$x(s+C) = Mx(s) \tag{8}$$

M can be parameterized as

$$M(s) = V^{-1}(s)UV(s). \quad (9)$$

The block diagonalized matrix U are characterized by tunes,

$$U = \begin{pmatrix} U_x & 0 & 0 \\ 0 & U_y & 0 \\ 0 & 0 & U_z \end{pmatrix} \quad (10)$$

where

$$U_i = \begin{pmatrix} \cos\mu_i & \sin\mu_i \\ -\sin\mu_i & \cos\mu_i \end{pmatrix} \quad (11)$$

where $\mu_i = 2\pi\nu_i$ and $i = x, y, z$. And the matrix V can be further parameterized as $V(s) = B(s)R(s)H(s)$ [4]. The matrix B normalizes the betatron motion,

$$B = \begin{pmatrix} B_x & 0 & 0 \\ 0 & B_y & 0 \\ 0 & 0 & B_z \end{pmatrix} \quad (12)$$

where

$$B_i = \begin{pmatrix} \sqrt{\beta_i} & 0 \\ -\alpha_i/\sqrt{\beta_i} & 1/\sqrt{\beta_i} \end{pmatrix}. \quad (13)$$

The matrices, R and H express x-y and longitudinal couplings, respectively.

$$R = \begin{pmatrix} \mu & 0 & r_4 & -r_2 & 0 & 0 \\ 0 & \mu & -r_3 & r_1 & 0 & 0 \\ -r_1 & -r_2 & \mu & 0 & 0 & 0 \\ -r_3 & -r_4 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (14)$$

where $\mu^2 + r_1r_4 - r_2r_3 = 1$. And

$$H = \begin{pmatrix} (1 - \frac{|H_x|}{1+a})E & \frac{H_xS_2H_y^tS_2}{1+a} & H_x \\ \frac{H_yS_2H_x^tS_2}{1+a} & 1 - \frac{|H_y|}{1+a} & H_y \\ S_2H_x^tS_2 & S_2H_y^tS_2a & aE \end{pmatrix} \quad (15)$$

where

$$H_{x(y)} = \begin{pmatrix} \zeta_{x(y)} & \eta_{x(y)} \\ \zeta'_{x(y)} & \eta'_{x(y)} \end{pmatrix}. \quad (16)$$

$a^2 + |H_x| + |H_y| = 1$ and E is a 2×2 unit matrix.

When there is a crossing angle, Lorentz transformation from Lab. frame to head-on frame is given by [5]. The linear part of it is expressed by matrix,

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \tan\theta & 0 \\ 0 & 1/\cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/\cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/\cos\theta & 0 \\ 0 & -\tan\theta & 0 & 0 & 0 & 1 \end{pmatrix} \quad (17)$$

.

The basic parameters are shown in table . The diffusion rate by synchrotron radiation is 5.4 (μm^2) for horizontal and 6.3×10^{-4} (μm^2) for vertical, respectively.

Table 1: Basic simulation parameters

	e^-	e^+
C	3016 m	
E	8 GeV	3.5 GeV
N	3.5×10^{10}	8.0×10^{10}
β_x^*/β_y^*	60/0.7 cm	
ϵ_x/ϵ_y	18 nm /0.18 nm	
σ_z	7 mm	
ν_x/ν_y	0.515/0.58	
ν_s	0.016	
$\tau_{x,y}/T_0$	4000 turn	

SIMULATION RESULTS

x-y coupling

The figure 2 shows the vertical beam sizes vs turn with x-y coupling errors. The diffusion rate by beam-beam nonlinear force is smaller than that by synchrotron radiation. The beam size grows almost linearly with turn. Diffusion of vertical beam sizes due to the x-y coupling errors are seen. The figure 3 shows the vertical diffusion rate as a function of coupling parameter r1, r2. The diffusion rate is increased monotonically by r1, r2.

The figure 4 shows the horizontal beam size vs turn for r2=0, 2, 5, 10mm, respectively. The horizontal diffusion rate is much smaller than the vertical one.

The same calculation was done for vertical dispersion error at IP. It also showed a diffusion characteristics in the vertical beam size.

Crossing angle

Similar vertical diffusion is also caused by crossing angle. Figure 6 shows the evolution of the vertical beam size for θ=0, 1, 4, 11 mrad. The horizontal crossing angle causes the vertical diffusion, so-called Arnold diffusion. The diffusion rate is also shown in figure 6.

SUMMARY

We have studied the diffusion by weak-strong simulation in oder to investigate the luminosity degradation caused by linear coupling. The diffusion rate was estimated.

If there is no linear coupling, the diffusion is small. Therefore, the synchrotron radiation plays an important role for beam-beam limit in this case.

The x-y coupling enhances the vertical diffusion with the luminosity degradation. The horizontal diffusion rate is much smaller than that by synchrotron radiation.

With crossing angle, the vertical beam size growth are observed. We are considering the enlargement of the vertical beam size is due to Arnold diffusion.

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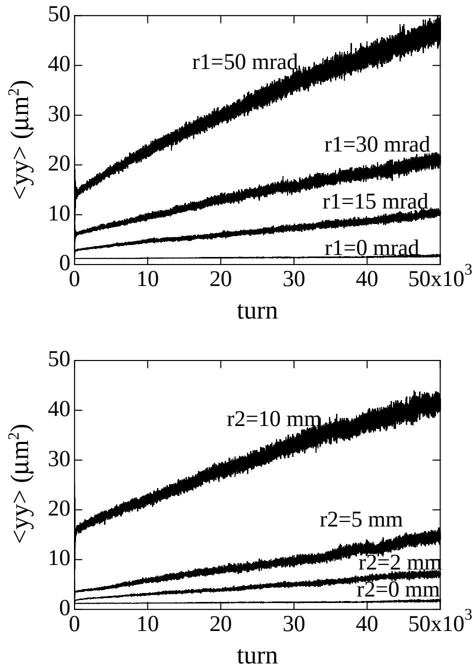

Figure 2: The evolution of the vertical beam size with x-y coupling error of r1, r2, r3, r4, respectively.

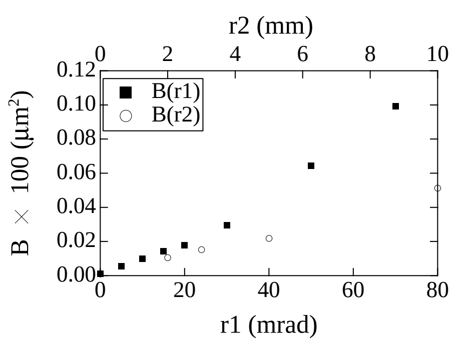

Figure 3: Diffusion rate of vertical beam size as a function of r1, r2 .

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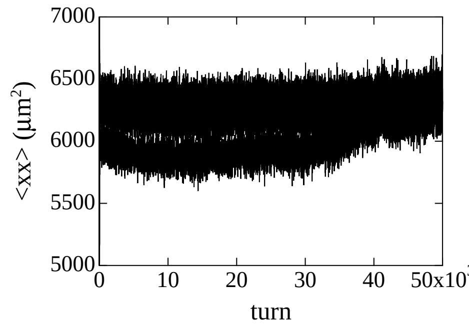

Figure 4: The evolution of the horizontal beam size for r2=0, 2, 5, 10 mm

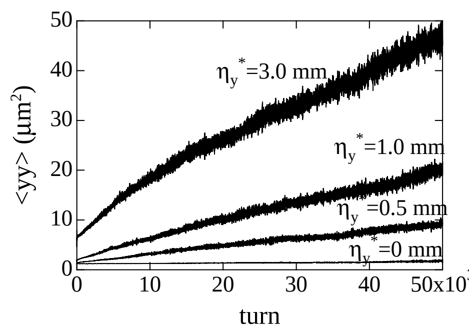

Figure 5: The evolution of the vertical beam size with the vertical dispersion error.

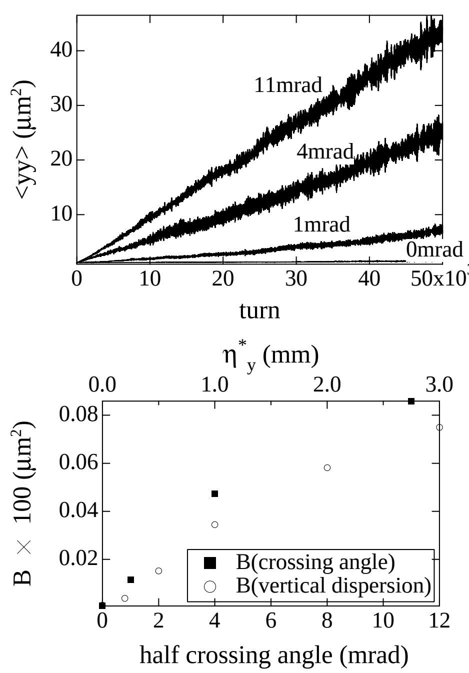

Figure 6: The evolution of the vertical beam size with crossing angle error is shown in upper graph. Diffusion rate are plotted as a function of crossing angle and vertical dispersion, respectively.