# ADIABATIC MATCHING IN PERIODIC ACCELERATING LATTICES FOR SUPERCONDUCTING PROTON LINACS

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*Abstract*

Superconducting proton linacs with multicell cavities are split in sections (using cavities with different geometrical length) with a spatially periodic lattice (typically a focusing doublet and 2-4 cavities in a lattice period) and slowly varying focusing and acceleration parameters. The usual matching procedure for a constant parameters lattice, namely periodic conditions for transverse and longitudinal Twiss parameters, gives poor results especially in presence of strong space charge forces. A novel matching procedure has been devised, valid for adiabatic variations of the beamline and beam parameters. This procedure gives smooth beam envelopes variation along the machine, as well smooth phase advance per period. Example will be given for the case of the superconducting TRASCO linac. The procedure has also been tested for a non-spatially periodic structure, like a DTL linac.

## 1 INTRODUCTION

Superconducting proton linacs are being built or are in the design stage for a large variety of applications: spallation neutron sources, radioactive beams, waste transmutation, neutrino beams [1-4]. All have in common the following properties:

- use of elliptical multicell cavities (from 4 to 6 cells)
- spatially periodic lattices with 2-4 cavities and focusing quadrupole doublets
- two or more section each with tens of periods and one only kind of cavities

For the case of the TRASCO linac [3], operating at 704.4 MHz, there are three sections, each having 14-16-12 periods covering the energy range 90-190 MeV, 190-430 MeV and 430-1000 MeV, respectively. The sections have 2 cavities (5 cells, β=0.50), 3 cavities (5 cells, β=0.68) and 4 cavities (6 cells, β=0.86). Accelerating fields are 8.50, 10.25, 12.3 MV/m, respectively, and the synchronous phase has been set at –25 RF degrees. The transverse focusing is provided by a periodic array of room temperature quadrupole doublets.

The basic design of these linacs has been analyzed with linear codes (TRACE3D [5], TRACE_WIN [6], DoLinac [7]), which solve the rms equations assuming linear optics and linear space charge field. The envelopes of the beam are calculates assuming $\sqrt{5}$ rms values. The beam is matched for each section of the linac and the matching between the sections is done using 4 quadrupoles and two sets of cavities (either by varying the field or the synchronous phase) on the section boundary.

## 2 BEAM MATCHING IN SPATIALLY PERIODIC LATTICES

The linear codes solving the rms equation normally use a non canonical phase space, namely $(x,x')$,$(y,y')$,$(z,\delta)$ where $x$, $y$ are the transverse dimension, $x'=dx/ds$, $z$ is the bunch length and $\delta$ is the relative momentum spread. For input and output normally the longitudinal beam is specified in the canonical space ($\Delta\varphi$, $\Delta E$).

In each section the beamline is spatially periodic but the acceleration rate, the focusing (from cavities and quadrupoles) and the space charge effect are slowly varying from period to period.

The matching of the beam is particularly important since it is well known that the formation of the beam halo is related to the mismatch of the beam core [8].

### 2.1 Periodic Matching procedure

The standard matching procedure of the beam on each section is to use the first lattice period and to assume an ideally periodic structure with acceleration.

In the non-canonical phase space each submatrix has determinant less than one because of the acceleration. One usually assumes that the Twiss parameters of the beam ($\alpha$, $\beta$) are periodic and because of the acceleration the geometric emittance decrease provides a damping of the beam envelopes.

The procedure is here given for the $(x, x')$ phase space, calling $R$ the matrix of the first period (including the effects of the linear space charge). The phase advance μ and the corresponding matched beam (periodic Twiss parameters $\alpha$, $\beta$) are calculated as

$$\begin{cases} \cos(\mu) = \dfrac{R_{1,1} + R_{2,2}}{2\sqrt{\det R}} \\ \alpha = \dfrac{R_{1,1} - R_{2,2}}{2\sqrt{\det R}\,\sin(\mu)} \\ \beta = \dfrac{R_{1,2}}{\sqrt{\det R}\,\sin(\mu)} \end{cases}$$

The matching procedure is obviously iterative and must be done in the full 6D phase space since the space charge is dependent of the beam sizes $x$, $y$, $z$.

Once the matched beam is found the beam is transported through all the section and the beam envelopes as well the phase advance for each period can be computed from the knowledge of the transfer matrix of each period and the beam parameters at the beginning and end of the period. For example, in the $x$, $x'$ phase space, calling $R$ the transfer matrix, $\varepsilon_1$, $\varepsilon_2$ the geometrical emittance and $\alpha_1$, $\beta_1$, $\alpha_2$, $\beta_2$ the beam parameters at beginning and end of the period one has

$$\cos(\mu) = \frac{\beta_1 R_{1,1} - \alpha_1 R_{1,2}}{\sqrt{\det R}\sqrt{\beta_1 \beta_2}}$$

This formula will be used in the following to compute the effective phase advance experienced by the beam along the beamlines.

The phase advance and the beam envelopes for the first section of the TRASCO linac (90-190 MeV) for a current of 30 mA are presented in Figure 1. Even if the beam envelopes look smooth, the phase advances show oscillations, which are indications of a mismatched beam.

The results of this matching procedure are worst the smaller the beam energy and the higher the space charge effect. For higher currents or particular cases the mismatching is directly visible also from the envelopes.

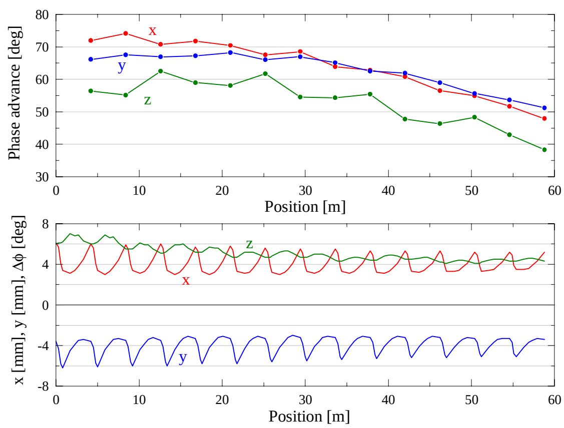


Figure 1: Phase advances and beam envelopes for a 30 mA beam in the first section of the TRASCO linac. The beam is matched with periodic conditions.

## 2.2 Adiabatic Matching procedure

As already mentioned, the procedure presented in the previous paragraph could be satisfactory only in the high energy limit owing to the reduced space charge forces, small relative energy increment and small transverse defocusing effects of the cavities.

The assumption of a really periodic structure, which implies the same transfer matrix for each period, is generally not verified. What instead can be possibly assumed is that the focusing properties of the lattice (in all planes), as well of the space charge effects, change smoothly from period to period.

In analogy with the standard SVEA (slowly varying envelope amplitude) approximation, applied to harmonic oscillators with slow varying parameters, we have devised a novel matching procedure that uses the first two periods of the spatially periodic section. The use of two periods is necessary in order to detect the smooth change of the focusing parameters.

Indicating the beam parameters $\varepsilon, \alpha, \beta$ with the subscript 1,2,3 corresponding respectively to the beginning of the first period, boundary of the two periods, end of the second period, the iterative procedure requires that:

$$\begin{cases} \alpha_3 - 2\alpha_2 + \alpha_1 = 0 \\ \dfrac{\varepsilon_3 \beta_3}{\varepsilon_2 \beta_2} = \dfrac{\varepsilon_2 \beta_2}{\varepsilon_1 \beta_1} \end{cases}$$

i.e. a smooth variation of the beam envelopes and of the tilt of the beam ellipse.

For the same case of Figure 1 (namely identical values of the quadrupoles gradients and of the cavity parameters) the phase advance and the beam envelopes are presented in Figure 2. As it can be easily seen, the phase advances are now very smooth indicating a very good matching of the beam to the adiabatically varying structure. The mismatch factors (as defined in TRACE3D) between the two cases are 1% in the transverse planes and 4% in the longitudinal plane. We point out that this mismatch factor is strongly dependent on beam current: higher currents will lead to greater values of the mismatch factors.

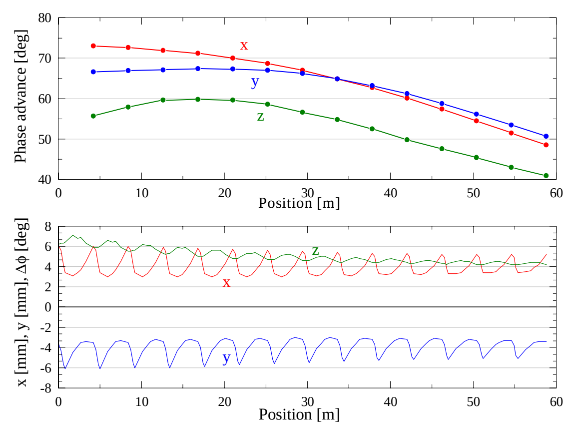


Figure 2: Phase advances and beam envelopes for a 30 mA beam in the first section of the TRASCO linac. The beam is matched with the adiabatic procedure.

## 2.3 Beam phase advance as indicator of a matched beam

As a general note, we can state that the smoothness of the beam phase advance per period along the beamline is the real indicator of a matched beam, since, by definition, the beam phase advance integrates, through the $\beta$-function, the entire beam evolution inside the lattice period. Small values of the mismatch factors, which can be hardly seen from the beam envelopes, result in evident oscillations of the beam phase advances.

We performed also several tests to check the validity of this adiabatic procedure for beam matching, i.e. the uniqueness of the solution with respect to:

- choice of the period definition (period start before/after the quadrupoles, in front of cavities, etc.);
- backward tracking (in this case we apply the procedure at the two last section periods and we trace the beam backwards).

All these tested cases resulted in the same matched beam solution, with a mismatch factor below 0.1%.

A somewhat similar procedure, at least from the conceptual point of view, has been implemented in TRACE_WIN [6] where the matched beam is calculated by minimizing the second order derivative of the phase advance over a number of lattice periods (usually 10-15) of the full section.

Indeed, multiparticle non-linear simulations codes confirmed that these smooth matching procedures result in minimization of core envelope mismatches.

## 3 NON PERIODIC LATTICES

The periodic matching procedure is usually applied also to non-spatially periodic structure (for example, DTL lattices where the cell length increases with the particle velocity).

As an extreme example we can consider a DTL tank at low energy where space charge, acceleration rate and spatial variation of the period are significant. The case presented in Figure 3 refers to a SNS like DTL tank, operating at 402.5 MHz with an average accelerating gradient of 1.5 MV/m and accelerating a 50 mA beam from 2.5 to 7.5 MeV. The focusing period is rather long since the quadrupoles inside the drift tubes are arranged in the order FFODDO.

Figure 3 shows the beam phase advances and envelopes for this case. The oscillations of the phase advances are strong, particularly in the longitudinal plane. The oscillations are also evident in the behaviour of the longitudinal envelope.

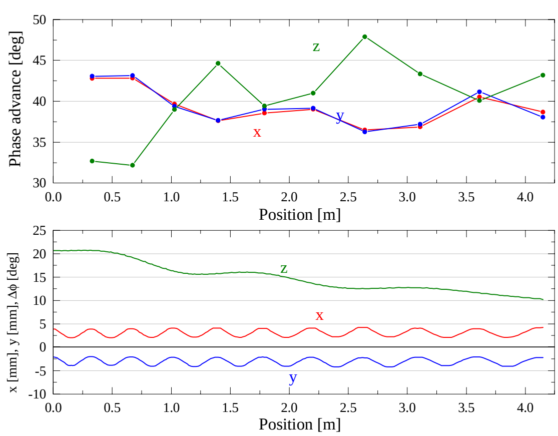


Figure 3: Phase advances and beam envelopes for a 50 mA beam in a DTL tank (from 2.5 to 7.5 MeV). The beam is matched with periodic conditions.

The adiabatic matching procedure has been investigated also in this case. In principle this procedure can deal with any smooth variation of the focussing properties of the beamline, which are embedded in the transfer matrix. We just assume that the periods of the structure are self-similar (same order of elements, with drifts of increasing length) and that the phase advance is a smooth function of the periods.

Figure 4 shows the result of applying the adiabatic matching procedure to the same beamline of Figure 3. As it can be clearly seen from the figure both the phase advances and the beam envelopes are indeed very smooth.

The mismatch factors between the two cases are approximately 3% for the transverse planes and 7% in the longitudinal plane.

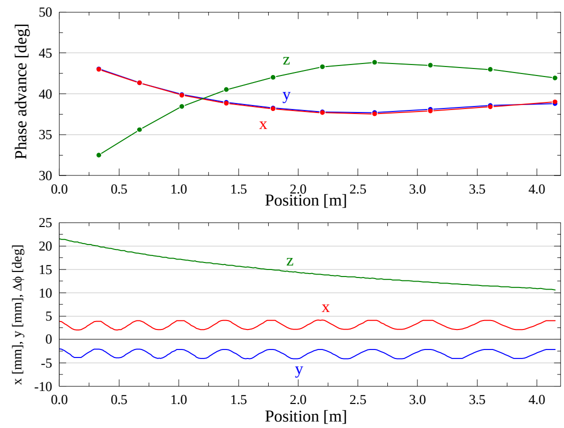


Figure 4: Phase advances and beam envelopes for a 50 mA beam in a DTL tank (from 2.5 to 7.5 MeV). The beam is matched with the adiabatic procedure

## 4 CONCLUSION

A beam matching procedure for superconducting linac with periodic structures with slow varying focussing parameters has been successfully implemented. The procedure has been confirmed to work also in the case of non-periodic structures, like DTL linac. The smoothness of the beam phase advances has also been identified as a key property of the matched beam.

## 5 REFERENCES

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