SEPARATRIX FORMALIZM IN SUPER-CONDUCTING LINAC DESIGN

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Abstract.
Under design of the linear accelerator we use the accelerating systems with external synchronizing of the different groups of structures with few number of similar gaps. The phase velocity is constant along each structure and it is changed by step from cavity to cavity. Depending on the ratio between the particles and the structure phase velocities the particles are sliding down or up relatively of RF. Thus, the particles are never in synchronism with the equivalent traveling wave, and even some time they have no instantaneous longitudinal stability. But due to a proper choice of RF phase shift $\Delta\varphi_{RF}$ between the cavities we can create a quasi-synchronous motion, and in total we have a stable motion in the whole accelerator. Such structures are used for the acceleration of particles with different mass-charge ratio in the low energy region [1] and in the high-energy region for proton linear accelerators [2]. These structures are cheaper, since they have the simpler cavity geometry. However, due to the bigger phase oscillation the particles are appeared in the nonlinear part of the separatrix, which one causes the growth of the longitudinal emittance. We developed the separatrix formalism for the optimized design of such linear accelerators.

1 METHOD OF ACCELERATION IN PHASE-STEPPED STRUCTURES

The separatrix is stationary in case of the structure phase velocity is constant. In such structure the particles with small energy deviation from synchronous energy oscillate around the synchronous level inside separatrix (see fig.1). They have the closed phase trajectories and after integer number of oscillations the energy remains to be constant. The particles with bigger absolute energy deviation will drift along the phase trajectories to left or right directions depending on the ratio between the particles and the structure phase velocities. For such particles the initial and final energies ration is determined by many parameters. So, such structures with many longitudinal phase oscillations are unsuitable for acceleration. However, in the shorter structures the mechanism of acceleration can be realized as well. Instead of the length increasing of the acceleration period proportional to velocity like in the DTL-structure, we introduce the additional phase shift between RF-field oscillations in the neighbour cavities. This phase shift is analogous to replacement of the separatrix as a whole in the direction of phase axis. Thus, the particles make short sliding in each cavity in the right direction for $\beta < \beta_{str}$, then they are coming inside separatrix and after when $\beta > \beta_{str}$ we manage them to slide to the left direction (see fig.1).
Thus, there is the fundamental difference between the acceleration method in the DTL-structure where the particles move inside the separatrix during the whole of acceleration process, and the acceleration method in the RF phase-stepped structures, where the particles, starting its motion from the unstable region, go through the stable region and then again come to the unstable region.

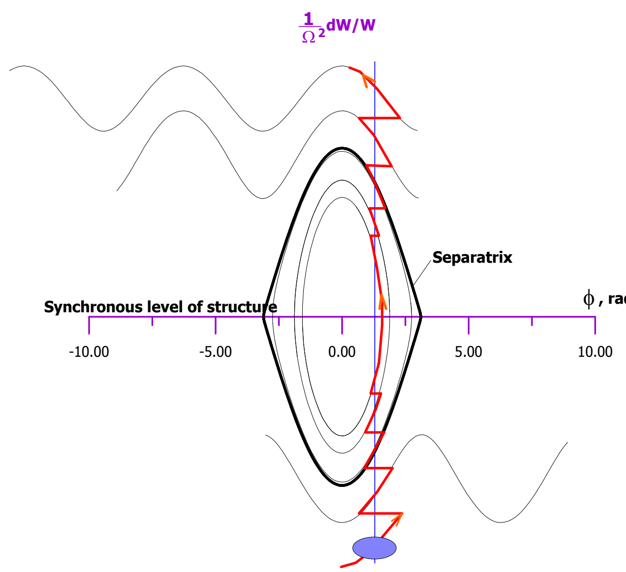


Figure 1: The acceleration scheme.

Now in accordance with these special features we have to answer for the next most important questions.

- How many families of cavities with the same constant structure velocity are necessary?
- How many cavities does the family contain?
- How many accelerating cells have to be in one cavity?

Obviously, the required final energy and the optimum number of cavity in each family determine the number of families. In the same time the number of cavities in one family depends on the beam sliding relatively of travelling RF wave. The last one is entirely determined by $(\beta - \beta_{str})/\beta$ value and number of accelerating periods in each cavity. There is the limit for maximum difference between particle velocity and structure phase velocity, when the structure gives the positive gain of energy. Moreover, the real limit differs from the case when the potential accelerating exists, since after some value $(\beta - \beta_{str})/\beta$ the efficiency of such acceleration will be unacceptably low. This fact defines the number of required cavities in one family and as result the number of families themselves. The number of accelerating cells directly affects on the particle sliding value and therefore on maximum possible value of $(\beta - \beta_{str})/\beta$.

It is easy to get the energy gain expression for the n-periods cavity $\Delta W = \frac{\beta_{str}\lambda}{2} eE_1 T\cos\varphi$, where E_1 - the 1st harmonic of accelerating field and the coefficient T:

$$T = \frac{\sin\left[\pi\left(\frac{1}{\beta_{str}} - \frac{1}{\beta_{part}}\right)n\beta_{str}\right]}{\pi\left(\frac{1}{\beta_{str}} - \frac{1}{\beta_{part}}\right)n\beta_{str}},$$

where n is a number of accelerating cells in one cavity. It describes the influence of nonsynchronism to the acceleration efficiency.

As it was to be expected, the energy gain is the maximal in case of coincidence β_{part} with β_{str} that corresponds to the precise synchronism. Behaviour of the transient time is very important for choosing of the cells, cavities and families number.

2 PARAMETRIC RESONANCE

In case of drift space between cavities we have the longitudinal motion perturbation. For the low energy region it is one of the most important phenomena. In super-conducting accelerators, the cavities are placed in

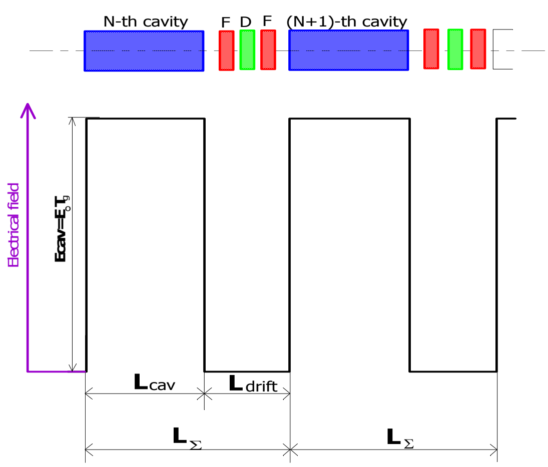


Figure 2: Field distribution in one period of acceleration.

cryo-module and additional drift space is needed for the installation of the focusing, diagnostic, vacuum, etc. systems. The real distribution of the RF-field along the accelerator axis is shown on figure 2.

Obviously, the field distribution modulation should lead to the modulation of the longitudinal frequency, and as result to the parametric resonance. The equation of the particle motion in the linear approximation is $\frac{d^2\psi}{d\tau^2} + \Omega^2(1 + e_1\cos\nu\tau)\psi = 0$, where, e_1 is the amplitude of the first harmonic of perturbation. The value $\Omega^2 = \frac{eE_0 T_g T_{cav}\lambda\sin\varphi_s}{2\pi\beta\gamma^3 m_0 c^2}\frac{L_{cav}}{L_\Sigma}$ is the longitudinal frequency and ν is the perturbation frequency. The parametric resonance will be exited if the perturbation frequency is close to the doubled longitudinal frequency and it is within limits: $2\Omega\left(1 - \frac{e_1}{4}\right) < \nu < 2\Omega\left(1 + \frac{e_1}{4}\right)$. In the linear case the amplitude of oscillation would increase with exponent law. In reality the amplitude is stabilized on certain level.

3 APPLICATION FOR THE PHASE-STEPPED STRUCTURE

In high-energy region ($W_{kin} > 200\,\mathrm{MeV}$) the super-conducting phase-stepped structure is very convenient, since the velocity gain per one period is quite small and increasing of the acceleration period is negligible.

Super-conducting cavities are adopted to use in ESS high-energy linac to accelerate protons from 185 MeV to 1345 MeV. In the energy range from 185 MeV to 450 MeV cavities have the structure velocity equal to 0.68 and from 450 MeV up to the final energy 1345 MeV it is β_{str}=0.86. The first part consists of 45 cavities and the second one has 92 cavities. Each cavity has five accelerating cells for both families.

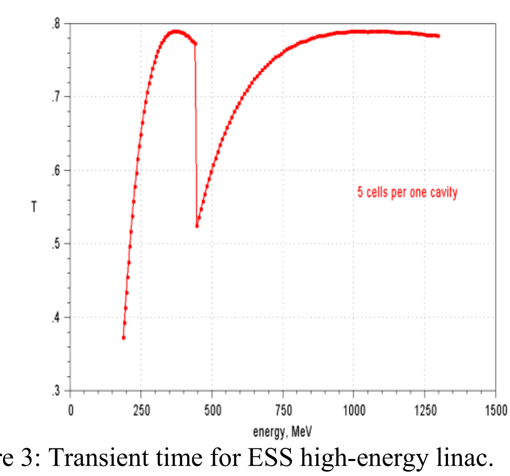


Figure 3: Transient time for ESS high-energy linac.

The transient time behaviour for all region 185-1345 MeV is illustrated by the figure 3. Numerical estimation allows making a conclusion that particles are involved in the parametric resonance just from beginning up to 250 MeV. The influence of resonance on the effective separatrix is presented on figure 4.

Another example of phase-stepped structure application is COSY-injector linac. It has to deliver two kinds of particles, protons and deuterium, which differ by mass-charge ratio by factor two. The only phase-stepped structure enables to accelerate particles with such a big difference in charge to mass ratio.

Initial energy for the protons and deuterium are 5 MeV and 2.5 MeV correspondently. Desirable final energy for both beams is 50 MeV.

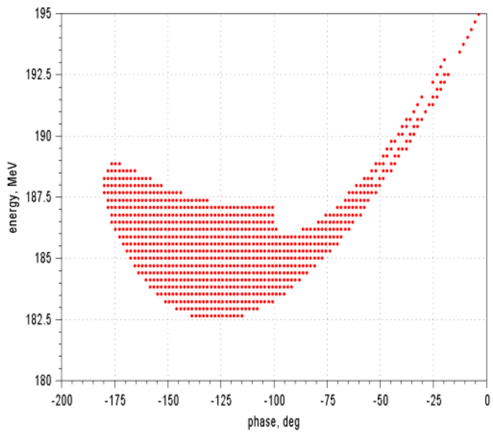


Figure 4: Effective separatrix of ESS high-energy linac.

From our estimation the optimum number of families is 2. The first family operates at 160 MHz and it has 20 cavities. The second one operates at 320 MHz and contains 24 cavities. The cavity is based the half-wave structure and it has two accelerating cells. Thus, we have the frequency jump.

The transient time factor for both beams is shown on figure 5.

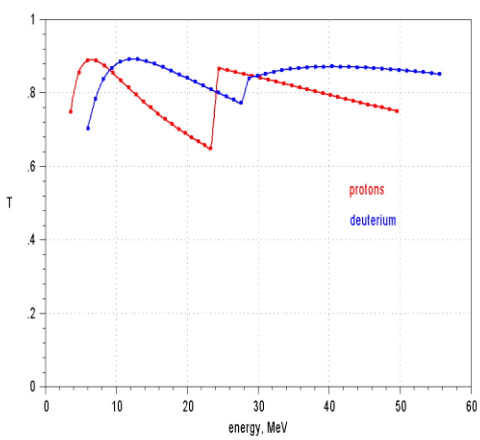
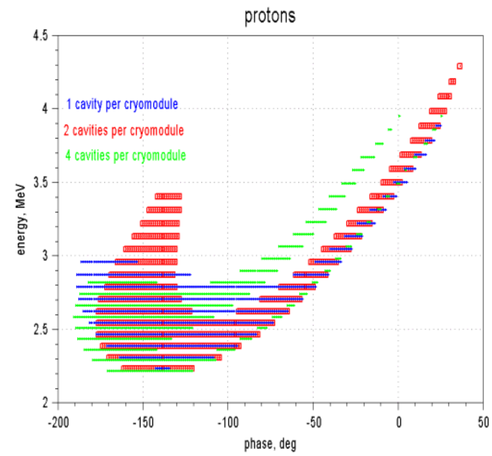
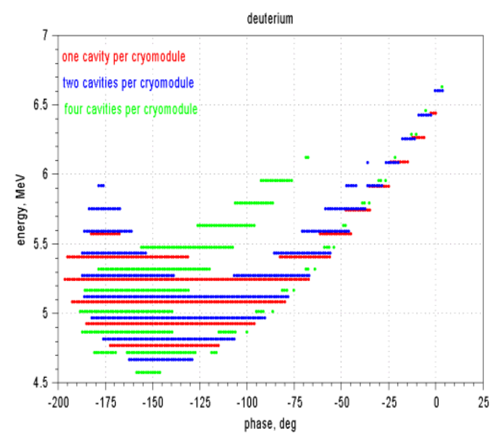


Figure 5: Transient time factor for COSY-injector linac.

In parametric resonance the longitudinal and perturbation frequencies are defined by the ratio L_{cav} / L_{Σ}. Therefore changing the cavities number in one cryo-module we can change the resonant conditions. Calculations show that the option “4 cavities in one cryo-module” is the most sensitive to the parametric resonance. The particles will be involved in the resonance from the beginning and up to 10 MeV for protons and up to 13 MeV for deuterium. The effective separatrises of the 1st part of COSY-injector linac versus of the cavities number in one cryo-module is shown on figure 6.



a). Protons



b). Deuterium particles.

Figure 6: Effective separatrix of COSY-injector linac.

4 CONCLUSION

In this paper we considered the acceleration from separatrix formalism point of view. We optimised the accelerating regime in phase-stepped structure by RF phasing cavities. The efficiency of acceleration depends on how far the particles from the synchronism with the travelling wave. Also the phenomenon of the parametric resonance was considered. It arises due to the drift space between cavities.

5 REFERENCES

[1] R. Toelle et. al., A Superconducting Injector LINAC for COSY, this proceeding.

[2] A next generation Neutron Source for Europe, ESS preprint, ESS-96-53-M, 1996.