A NEW TYPE OF HIGH-RESOLUTION POSITION SENSOR FOR ULTRA-RELATIVISTIC BEAMS

G. Doucas, Particle & Nuclear Physics, Univ. of Oxford, Oxford OX1 3RH, UK; M. F. Kimmitt, Univ. of Essex, Colchester, UK; J. H. Brownell, S. R. Trotz and J. E. Walsh, Dartmouth College, NH 03755-3528, USA

Abstract

The radiation emitted through the interaction of an ultra-relativistic particle with a metallic grating (Smith-Purcell radiation) could be used as a very sensitive position sensor for a future Linear Collider or X-ray FEL, with position resolution of $1\mu m$, or better. This article discusses the physical principles underlying the operation of such a device and concludes with some of the practical issues in its implementation.

1 INTRODUCTION

The next generation of Linear Colliders and X-ray Free Electron Lasers is expected to place severe demands on the beam diagnostic systems of these machines. Both the TESLA and the NLC/JLC Colliders, for example, envisage beam dimensions at the final focus in the *nm* range. If the expected luminosity is to be achieved, high-resolution position sensors and fast feedback systems will have to be incorporated in these accelerators. In general, it is true to say that existing devices offer a resolution that is not better than a few microns. We present here an alternative approach to the problem that is based on the radiation emitted by the interaction of an ultra-relativistic particle with a periodic metallic structure, such as a grating, also known as Smith-Purcell (SP) radiation. This was first observed in 1953[1] and had remained over the years an interesting phenomenon, but without an obvious application. Recent work[2,3] has focused on its theoretical description and on its potential use as a tuneable source of radiation in the far infrared. We examine here the possibility of using SP radiation to detect the beam position with a resolution of one micrometre. We assume that the beam will have an energy of about 500GeV and that it must not be brought closer than 1*mm* to the grating surface.

2 PHYSICAL PRINCIPLES

2.1 Basic formulae

When an electron beam passes close to the surface of a grating, radiation is emitted because of the interaction of the beam with the periodic structure. The origin of this radiation can be understood in terms of currents induced on the surface of the grating by the passing electron and 'accelerated' by the periodic profile of the grating. Although alternative approaches have been suggested [4,5], the surface current picture is easier to follow and has received experimental support, but at energies of a few MeV only. This radiation has interesting features, which are outlined in this section. Its wavelength λ depends on the period l of the grating, the speed of the electron, expressed in terms of β, and also on the angle of observation θ relative to the beam direction, i.e.

$$\lambda = \frac{l}{n}(1/\beta - \cos\theta) \qquad (1)$$

where n is the order of the radiation. Equation (1) is well established experimentally and indicates one of the interesting features of SP radiation, namely the ability to select a wavelength by varying the angle of observation θ. It is also worth noting the possibility of using harmonics higher than 1. The calculation of the **spontaneous** power dP emitted in a solid angle $d\Omega$, in a direction θ and an azimuthal angle ϕ, by a beam current I passing at a distance x_0 above a grating of length Z is more complicated. Since the details of the calculation have been published [2,3] already, we simply state the formula:

$$\frac{dP}{d\Omega} = 2\pi e I \frac{Z}{l^2}\frac{\beta^3 n^2}{(1-\beta\cos\theta)^3}\exp\left[-\frac{2x_0}{\lambda_e}\right]R^2 \qquad (2)$$

The quantity R^2 in (2) is a complicated expression that depends on the profile of the grating and the wavelength of the radiation and which at high energies is essentially a δ-function around the angle of maximum emission. The quantity λ_e is called the 'evanescent wavelength' and is defined as:

$$\lambda_e = \lambda \frac{\beta\gamma}{2\pi\sqrt{1+\beta^2\gamma^2\sin^2\theta\sin^2\phi}}$$

Effective coupling between beam and grating means small x_0, that is close proximity of the beam centre to the grating, and a large evanescent wavelength λ_e. As a qualitative comment, we note that the radiation is sharply peaked in the forward direction but at an angle that can be adjusted by selection of the grating parameters. We treat here only gratings whose profile is of the echelle type, consisting of two facets one of which forms an angle α_1 with the beam direction while the other is perpendicular to it. It can be shown that for a given x_0 there is an optimum wavelength for maximum power given by:

$$\lambda_m = \frac{4\pi x_0}{3\beta\gamma} \qquad (3)$$

and a corresponding emission angle θ_m given by (1). However, for very large γ this leads, for any practicable value of x_0, to grating periods of the order of μm and wavelengths of nm . The blaze angle of the grating has a very sharp optimum value when $\alpha_1 = \theta_m / 2$.

3 SELECTION CRITERIA

3.1 Wavelength and emission angle

Instead, we select an emission angle that will allow the radiation to be taken out of the beam line without difficulty. We stipulate that 6^o is a reasonable value. From equation (1) we obtain a value of 0.0055 for the quantity $n\lambda / l$ and we are free to select the wavelength of the radiation, its order and the grating period. If we select a grating period $l = 3mm$, then the fundamental wavelength is $\lambda = 0.0055\, l = 16.5 \mu m$. The first five emission orders have a sharp angular distribution with a peak at 6^o .If one uses the 5^{th} order emission, then the wavelength of the radiation is $3.3 \mu m$. Is this a suitable wavelength to work with? The near-infrared region (1-20μm) offers a variety of excellent detectors. For the proposed position sensor an ideal detector would have the following characteristics: fast response, good detectivity (sensitivity), room temperature operation and ruggedness. The two detectors that come close to the ideal characteristics are indium arsenide (InAs) and germanium (Ge) photodiodes. InAs has a long wavelength limit of 3.6μm, with peak response at 3.2μm. The response time can be as short as 2ns but this requires the junction to be 'reverse biased'. Without bias InAs photovoltaic detectors have response times of ~10ns, which should be adequate. Ge photodiodes offer similar speed and much better detectivity than InAs but with a long wavelength limit of only 1.8μm, which is somewhat shorter than the expected optimum wavelength.

3.2 Position sensitivity

We demonstrate first that, within the parameters of our problem, it is not possible to detect changes in x_0 of the order of $1\mu m$. This quantity appears only in the exponential term and its influence on the emitted power can be calculated analytically. If we assume that the power arriving at the detector should change by about 10% for a change in x_0 of 1μm, then it is easy to show that $\lambda_e \cong 20 \mu m$, i.e. it has to be made small. This can only be achieved by bringing the beam very close to the grating surface (see equation 3). This, however, is contrary to the original stipulation of $x_0 = 1mm$. We conclude, therefore, that variations of the exponential term on their own, cannot achieve the micron resolution without having the beam close to the grating. We consider next the influence of any change in the emission angles. A vertical displacement dx , or a horizontal one dy , can be translated into changes of θ and ϕ as follows:

$$d\theta \cong \frac{dx}{r\cos\theta\cos\phi} \cong \frac{dx}{r} \quad \text{and} \quad d(\theta\phi) \cong \frac{dy}{r}$$

Both angles are assumed to be small. If the distance $r \cong 25$ mm and $dx = dy = 1\mu m$, then $d\theta \cong 40 \mu rad$ and $\theta d\phi + \phi d\theta = 40 \mu rad$. In principle, a horizontal displacement (i.e. across the surface of the grating) will change both ϕ and θ but if we assume that the latter change is much smaller than the change in ϕ and we set $\theta = 6^o$ and $\phi = 1^o$, we obtain $d\phi \cong 400 \mu rad \cong 0.023^o$. This is greater than the produced by a similar displacement in the vertical direction. In order to translate changes in emission angles, relative to a fixed detector, into changes of measured infrared power, it is necessary to consider the contour plots of the emission spectrum. One such plot for emission order 5 is shown in fig. 1.

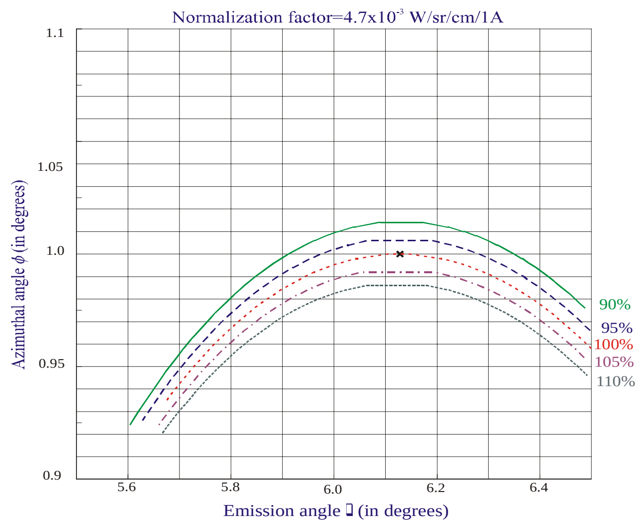

Fig.1 Equal power contours for order 5. γ=10^6, θ=6^o, ϕ=1^o, $x_0 = 1mm$

The point marked with a cross is the 'reference' point, i.e. the point at which the output is equal to the normalization factor. A change $d\phi = 0.02^o$, arising out of a $1\mu m$ displacement, causes a drop (or increase) in output of about 5% in the case of emission order 3 and 10% for order 5, reflecting the sharper nature of the emission pattern in the latter case. Since variations in output power at the level of 5-10% are detectable by the current near-infrared detectors, we conclude that it is possible, in principle, to detect horizontal displacements of the beam of the order of $1\mu m$ or, possibly, smaller.

4 PRACTICAL CONSIDERATIONS

A practical beam position sensor based on Smith-Purcell radiation would involve two detectors so that a difference can be taken in order to remove the influence of beam intensity fluctuations. The detectors would have to be calibrated individually and would be mounted symmetrically around the axis vertical to the grating. The magnitude of the subtended solid angle $\delta\Omega$ will depend on the selected angular acceptances $\delta\theta, \delta\phi$ and it could be as low as $10^{-5} sr$. The grating can easily be made 10cm long and the beam intensity in the pulse will probably be of the order of 50A. Combining these figures with the normalization factor of $4.9\times10^{-3}W/sr/cm/A$ from figure 2, we arrive at an absolute power of about $25\mu W$ at the detector. We note, again, that this power level is calculated solely on the basis of spontaneous emission and does not take into account any coherent enhancement of the radiation.

5 OTHER SOURCES OF RADIATION

The proposed beam-position monitor will operate in the near infrared and would thus be well clear of the main harmonics of the machine. However, the introduction of the grating itself in the beam pipe will produce some diffraction radiation, which will be peaked almost along the electron beam trajectory. Nevertheless, it is of some interest to compare the two sources of radiation in order to establish how much of the latter might find its way into the detectors and whether it would constitute an unacceptable level of 'background'. Formula (2) can be rewritten to give the intensity *W* produced by a single charge emitting at a wavelength λ and over the frequency range $d\omega$. This can be compared with the corresponding formula for the diffraction radiation from a single charge going through the centre of a circular hole of radius a in an infinite screen [6]:

$$\frac{d^2W}{d\omega d\Omega}=\frac{e^2}{2\pi^2 c}\frac{\theta^2}{(1-\beta^2+\theta^2)^2}J_0^2\left(\frac{2\pi a\theta}{\lambda}\right)$$

where J_0 is the Bessel function. If we denote with *r* the ratio of SP to diffraction, we obtain:

$$r\cong\frac{2\pi^2\theta^2\dfrac{l^2}{\lambda^2}\exp\left(-\dfrac{2x_0}{\lambda_e}\right)R^2}{J_0^2}$$

and substituting the appropriate numerical values in the above expression ($x_0=1mm$, $l=3mm$, $\lambda\cong3\mu m$, $a=1mm$, $\lambda_e\cong0.273$ and $R^2\cong10^3$) we find that $r\cong10^9$. This is a rough calculation done in order to estimate the order of magnitude of *r*, which is clearly >>1. This can be explained by the fact that we are observing radiation at an angle that is well outside the emission cone for such a high value of γ and at a rather long wavelength. We conclude, therefore, that whatever radiation is detected by the two detectors at $\theta=6^o$ and $\phi=1^o$ will be Smith-Purcell radiation.

6 SUMMARY AND CONCLUSIONS

The proposed method produces radiation from a very simple device and can be used in linear or circular machines. The radiation pattern from echelle gratings has certain features that make it an interesting candidate for the construction of beam position sensors with sub-micron resolution, especially designed for electron or positron beams with energies of hundreds of GeV:

a. The blaze angle of the grating 'lifts' the whole radiation pattern and separates it from the direction of the electron beam and other sources of radiation.
b. There is wavelength selectivity through its dependence on the angle of observation of the radiation and on the period of the grating.
c. Apart from the fundamental, significant power levels are available in the higher harmonics but the emitted power does decrease with harmonic number.
d. The angular distribution of the radiated power is sharp and it becomes sharper as the harmonic number is increased.
e. The required gratings are easy to fabricate and inexpensive.

Using two infrared detectors per grating and taking the difference between the two signals, we conclude that lateral displacements of the beam across the grating surface of $1\mu m$ will manifest themselves as a difference in signal level at the two detectors of between 10-20%, depending on the selected order of radiation. By operating the detector in the near infrared ($3.3\mu m$) one has the additional advantage of being far removed from the frequencies of the RF accelerating system or the bunch frequency. This does not imply, however, that this particular wavelength is the optimum and that the device might not work even better at shorter wavelengths, even in the visible.

We believe that the theory of the emission process is robust but most of the experiments up to now have been carried out at energies that are orders of magnitude lower [6-9]. What is clearly needed is experimental evidence at energies in the GeV region.

REFERENCES

1. S. J. Smith and E. M. Purcell, Phys. Rev. **92**, (1953), 1069
2. J. H. Brownell, J. E. Walsh and G. Doucas, Phys. Rev. E **57**, (1998), 1075
3. S.R. Trotz, J.H. Brownell, J.E. Walsh and G. Doucas, Phys. Rev. E **61**, (2000), 7057.
4. O. Haeberlé, P. Rullhusen, J-M. Salomé and N. Maene, Phys. Rev. E **49**, (1994), 3340
5. A. P. Potylitsyn, Phys. Rev. Lett. A **238**, (1998), 112
6. M. L. Ter-Mikaelian, 'High-Energy processes in condensed media', Wiley-Interscience (1972), Ch. 5
7. J. H. Brownell, G. Doucas, M.F. Kimmitt, J.H. Mulvey, M. Omori, and J.E. Walsh, J. Phys. D: Appl. Phys. **30**, (1997), 2478
8. Y. Shibata et al. Phys. Rev. E **57**, (1998), 1061.
9. G. Kube et al. paper presented at the International Workshop on Radiation Physics with Relativistic Electrons, June 1998, Tabarz, Germany (unpublished)