

CHARACTERISTICS AND POSSIBLE CURES OF THE HEAD-TAIL INSTABILITY OF COLLIDING BUNCHES

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Abstract

A theoretical, numerical and experimental study is presented of the coherent synchro-betatron modes due to the coherent beam-beam interaction. Possible coherent beam-beam instability of the head-tail type caused by the combined effect of the impedance elements in the machine and linearized coherent beam-beam interaction is discussed, as well as the ways to stabilize the beam-beam system, including optimal choice of the chromaticity. Special case of linac-ring collision scheme is also described.

1 INTRODUCTION

Coherent beam-beam modes have been studied theoretically and experimentally for a long time [1]-[5]. But recently it has been shown [6] that because of the finite length of the colliding bunches they can act upon each other as media passing information from the leading particles to the trailing ones. This interaction couples the coherent synchro-betatron beam-beam modes. The paper presents experimental evidence of these modes at the VEPP-2M collider. Under certain conditions the mode coupling is able to make the system unstable thus giving birth to the coherent beam-beam instability of the head-tail type.

In this paper we shall use a theoretical model of the effect based on the circulant matrix formalism and compare it with the results of experimental observations and numerical simulations.

2 THEORETICAL MODEL

Detailed theory of the circulant matrix approach to description of the synchro-betatron motion is given in [7]. Here we shall focus only on expansion of the method on the case of two colliding bunches.

We use the so-called "hollow beam" model. It assumes that all particles of the beam have equal synchrotron amplitudes and are evenly spread over the synchrotron phase forming a ring in the synchrotron phase space. The ring is divided into N mesh elements, each characterized with its dipole moment and synchrotron phase. The betatron motion will be described in terms of the normalized betatron variables.

The synchro-betatron oscillations of N elements forming a bunch is defined with the turn matrix

$$M = C \otimes B$$

(\otimes stands for the outer product),

with B being the betatron motion matrix

$$B = \begin{pmatrix} \cos \mu & \sin \mu \\ -\sin \mu & \cos \mu \end{pmatrix}.$$

C is the circulant matrix with elements [7]

$$C_{ij} = \frac{\sin N\varphi_{ij}}{N \sin \varphi_{ij}}, \varphi_{ij} = \frac{1}{2}(\mu_s - (N - i + j) \frac{2\pi}{N})$$

Here μ and μ_s are the betatron and the synchrotron phase advances. The eigenvectors and eigenfrequencies of M exactly correspond to first $-m..m$ harmonics ($2m + 1 = N$).

Expansion of the model to the case of two noninteracting bunches is straightforward by using the matrix

$$M_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes M$$

The linearized beam-beam interaction is described by matrix M_{bb} consisting of consequent short kicks and drifts between interactions of macroparticles. For example let us consider interaction of two bunches each consisting of 3 elements. Fig. 1 shows position of particles before the first interaction. But the M_2 matrix makes transformation

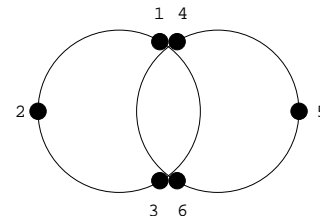


Figure 1: Position of macroparticles in the synchrotron phase space.

from IP to IP. So the first action is longitudinal unfolding of the bunch. The next step is interaction between particles 1,3,4 and 6 which is linear in relative distance (for instance, $dp_1 = -4\pi\xi(x_1 - x_6) - 4\pi\xi(x_1 - x_4)$ where x and p are the particles coordinate and momentum and ξ is the beam-beam space charge parameter). Further follows the free drift and interaction of particles 2,4,6 and 1,3,5; next drift and interaction of the "tail" particles no. 2, 5 and return to the IP. Generalization of the algorithm to the case of $N > 3$ is evident and can be left for the reader.

Complete matrix M_t is the product of the turn matrix and the beam-beam matrix $M_t = M_2 M_{bb}$. Its eigenfrequencies and eigenvectors can be obtained numerically using a computer algebra system. Complexity of the method consists in necessity of manual construction of the beam-beam matrix.

3 NUMERICAL SIMULATION

Assumptions used in the numerical simulation code are the same as in the matrix model - we use linearized beam-beam kick and hollow beam, while the latter restriction was maintained only for comparison needs and in extended simulation the beam was filled. Description of synchrotron and betatron oscillations in numerical model is much simpler and can be implemented for arbitrary number of particles per bunch.

In the tracking code each macroparticle is described with its longitudinal coordinate s and momentum deviation δp and transverse coordinate and momentum x and p . Between interactions at IP particles execute free betatron and synchrotron oscillations with frequencies ν and ν_s , with permutation of their longitudinal motion.

The main difficulty consists in ensuring correct beam-beam interaction sequence. Before the collision act we must sort the particles in each bunch by longitudinal coordinate, and the transverse coordinates are transformed from the reference IP $s = 0$ to the actual interaction point

$$s_{ij} = \frac{s_{1i} - s_{2j}}{2}$$

indices 1,i and 2,j label macroparticles i,j in the two bunches.

The transformation of coordinates is given by

$$x_{1i,2j} = x_{1i,2j} \pm p_{1i,2j} \cdot s_{i,j}$$

Next the beam-beam kick can be calculated thus giving change in the particles momenta.

$$dp_{1i,2j} = \pm \frac{4\pi\xi}{N} \cdot (x_{2j} - x_{1i})$$

Inverse recalculation of the coordinates to $s = 0$ with the new momenta followed by betatron and synchrotron transformation on the arc closes the full turn cycle.

Effect of the machine impedance can be taken into consideration by adding the collective kick to each bunch. We investigated the case of constant wake. The kick of the i -th particle in the bunch is

$$dp_i = \sum_{j=1}^{i-1} Q \cdot x_j$$

The center of mass position of the bunch is stored turn by turn. By applying the Fourier transform to this data we obtain the transverse mode spectrum.

4 OBSERVATION OF THE COHERENT BEAM-BEAM MODES

Experimental investigation of the coherent oscillations of colliding bunches has been performed at the VEPP-2M electron-positron collider [8]. Vertical oscillations of the electron bunch were excited with a short pulse applied to

the injection plates. Coherent oscillations of the bunches were observed on SR beam sensors and vertical coordinates were sampled turn by turn by fast ADC for 8K turns. Fourier transform of the collected data gives the coherent modes spectrum. Luminosity information from two particle detectors was used to determine the ξ value and its dependence on the bunch current. Experimental data obtained in this measurements allows to evaluate the ratio between the coherent π and σ modes tuneshift $\Delta\nu$ and ξ . According to our data this factor comes to approximately 1.1.

5 RESULTS

Results of calculation within the linearized beam-beam interaction model are presented in fig 2. It is evident that no mode merge occurs in this framework for all values of colliding currents. The only one possible case of instability is when a tune of one of the modes reaches zero or 1/2 values. Figure 3 shows comparison of the measured spec-

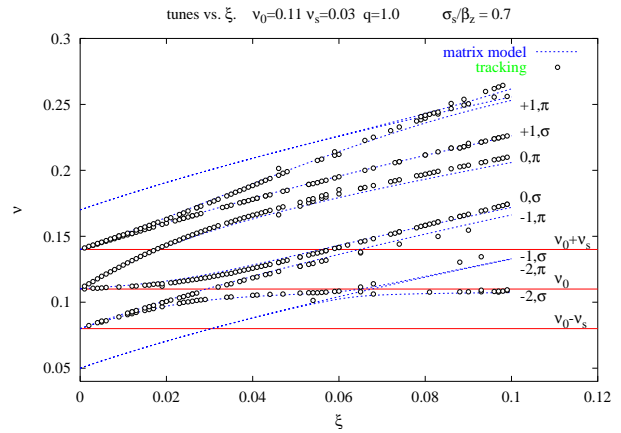


Figure 2: Comparison of matrix model with tracking.

trum with the calculated one. The VEPP-2M machine has a very small value of the transverse impedance and therefore a small value of the single bunch coherent tuneshift.

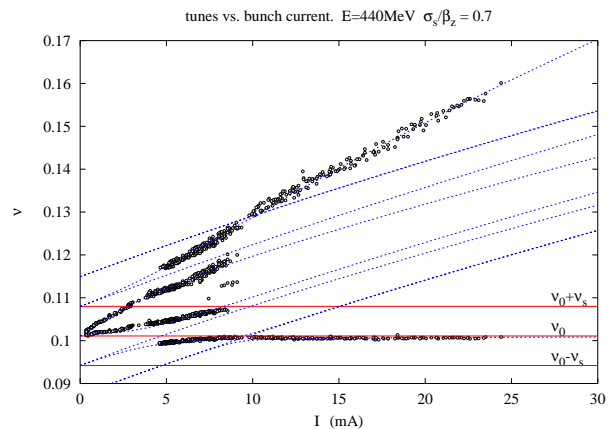


Figure 3: Measured spectrum and matrix model.

So the measurements have very good agreement with the calculations neglecting the machine impedance.

This agreement gives us a possibility to apply the technique when effect of the machine impedance is not negligible. In this case the situation changes drastically: we have the head-tail instability of colliding bunches [6]. The system is unstable without threshold and values of increments are linear in the transverse impedance and quadratic in beam current for small ξ [6] (fig. 4). Change of the beta-

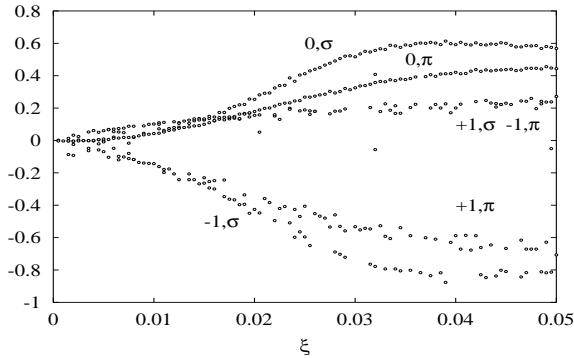


Figure 4: Increments $\times 10^4$ vs. ξ for combined action of beam-beam and machine impedance (tracking).

tron tune chromaticity χ can redistribute increments in this system, but zero values for all modes can not be achieved simultaneously.

6 LINAC-RING COLLISIONS

Special attention has been paid to the case of interaction of a beam circulating in a storage ring with a beam coming from linac. In the tracking algorithm this means that one of the two bunches has zero x and p before each turn. For the matrix model the linac-ring scheme can be implemented using matrix

$$M_{lc} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes M$$

instead of M_2

In the linac-ring collision scheme the head-tail instability results from the betatron phase advance over the beam-beam interaction length, and its effect can be exactly compensated by the chromaticity. For zero chromaticity some modes are unstable (fig. 5) but it is possible to stabilize all of them applying some positive χ value (fig. 6). This value does not depend on the beam-beam parameter ξ so this is a good cure of the instability.

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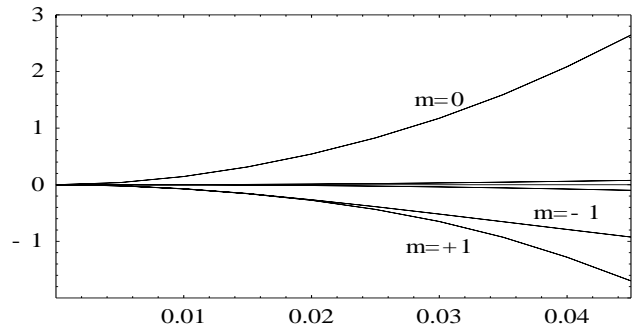


Figure 5: Increments $\times 10^3$ vs. ξ for linac-ring scheme.

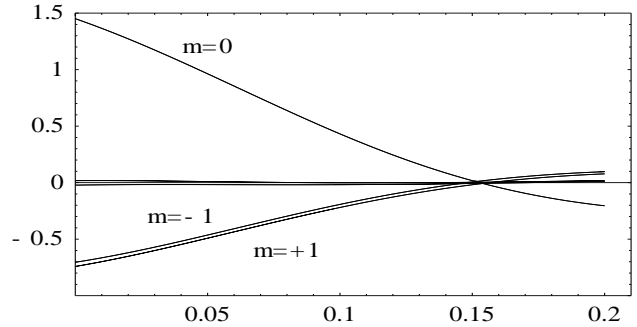


Figure 6: Increments $\times 10^4$ vs. the chromaticity χ for constant $\xi = 0.01$.

8 REFERENCES

- [1] A. Piwinski, "Observation of the beam-beam effects in PETRA", IEEE Trans. Nucl. Sci., vol. NS-26, No.3, (1979) p.4268
- [2] H. Koiso et al., "Measurement of the coherent beam-beam tune shift in the TRISTAN accumulation ring", Particle Accelerators, 1990, vol. 27, p.83
- [3] K. Hirata, "Coherent betatron oscillation modes due to beam-beam interaction", Nucl. Instr. Meth. A 269 (1988), p.7
- [4] A.W. Chao and R.D.Ruth, "Coherent beam-beam instability in colliding-beam storage rings", Particle Accelerators, 1985, vol. 16, p.201
- [5] K. Yokoya and H. Koiso, "Tune shift of coherent beam-beam oscillations", Particle Accelerators, 1990, vol. 27, p.181
- [6] E.A. Perevedentsev, "Simplified theory of the head-tail instability of colliding bunches", in Proc. 1999 Particle Accelerator Conf., New York (1999), vol. 3, p.1521; E.A. Perevedentsev, "Possible coherent beam-beam instability of the head-tail type", in Proc. Int. Workshop on Electron-Positron Factories, Sept. 21-24 1999, Tsukuba, KEK Proceedings 99-24, Feb. 2000
- [7] V.V. Danilov, E.A. Perevedentsev, "Feedback system for elimination of the transverse mode coupling instability", Nucl. Instr. Meth. A 391 (1997), p.77
- [8] P.M. Ivanov et al., "Luminosity and the beam-beam effects on the electron-positron storage ring VEPP-2M with superconducting wiggler magnet", in Proc. Third Advanced ICFA Beam Dyn. Workshp., May 29 - June 3, Novosibirsk, p.26