MEASUREMENTS OF THE LONGITUDINAL IMPEDANCE OF A COAXIAL CAVITY COUPLED WITH A CIRCULAR PIPE THROUGH SLOTS

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Abstract

The results of the measurements of the longitudinal impedance of a coaxial cavity coupled with a circular pipe through four slots are shown. Because of the slots, placed in the same longitudinal position but at different azimuthal angles, the device is rotationally asymmetric. The measurements have been performed with the coaxial wire method, and the results compared with theoretical ones obtained by applying the modified Bethe's theory.

1 INTRODUCTION

Beam screens in a vacuum chamber are widely used, to protect pumps, or, as in LHC[1], superconducting magnets from synchrotron radiation. Slots and holes of screens allow pumps to create high vacuum in the beam pipe. The resulting coupling impedance has been studied by utilizing the modified Bethe's theory[2, 3]. In this paper we present the measurements of the longitudinal impedance of a coaxial cavity coupled with a circular pipe through four slots, and compare the results to the theory. The paper is structured as follows: first we briefly review the method of the coaxial wire which we used for the measurements, then, by applying the modified Bethe's theory, we obtain an analytical expression of the longitudinal coupling impedance of the device, and finally we show the results of the measurements in comparison to the theory.

2 COAXIAL WIRE METHOD

The longitudinal coupling impedance of an ultra-relativistic point charge q traveling along the z axis of a beam pipe is defined as[4]:

$$Z(\omega) = -\frac{1}{q}\int_{-\infty}^{\infty} E_s(r=0, z; \omega)\, e^{jk_0 z} dz \qquad (1)$$

where E_s is the Fourier transform of the electric field scattered by the discontinuities of the pipe and $k_0 = \omega/c$. The impedance of a generic component is measured by means of the coaxial wire method, through the transmission S-parameter of a coaxial line obtained inserting a thin central wire[5, 6, 7, 8]. Several formulae have been proposed to express the coupling impedance as function of the transmission S-parameter. A widely used expression, valid for a single lumped impedance, which is exact in the frame of the transmission line S-parameters, has been provided by Hahn and Pedersen [9]:

$$Z(\omega) = \frac{Z_0}{\pi}\log\left(\frac{b}{a}\right)\left(\frac{S_{2,1}^{REF} - S_{2,1}^{DUT}}{S_{2,1}^{DUT}}\right) \qquad (2)$$

$S_{2,1}^{DUT}$ being the transmission parameter of the device under test (DUT), $S_{2,1}^{REF}$ the parameter of a portion of unperturbed coaxial line of the same length, Z_0 the impedance of free space, b the radius of the pipe, and a that of the coaxial wire.

The presence of the wire on the axis of the device strongly affects the fields, therefore it has been for long time discussed to which extent this technique is able to measure the impedance defined in eq. (1). A detailed analysis of the validity of the coaxial wire method, for azimuthally symmetric geometry, has been provided by Gluckstern and Li[10], who showed that the difference between (1) and (2) is of the order of $\log^{-1}(b/a)$. The device under test, in our case, shows four slots on the inner tube, losing the symmetry. For such asymmetric structures, the validity of the method, and eq. (2), has been recently discussed in [11] where, through a perturbation method, the following expression of the impedance has been derived,

$$Z(\omega) = \frac{Z_0}{\pi}\log\left(\frac{b}{a}\right)\frac{S_{2,1}^{DUT} S_{2,1}^{*REF}}{\left|S_{2,1}^{DUT}\right|^2} \left(\frac{S_{2,1}^{REF} - S_{2,1}^{DUT}}{S_{2,1}^{REF}} - 2\left|S_{2,1}^{REF} - S_{2,1}^{DUT}\right|^2\right) \qquad (3)$$

which, for our measurements, gives the same impedance value of eq. (2).

3 ANALYTICAL CALCULATION

A method for calculating analytically the impedance of a coaxial cavity, based on a modified version of the classical Bethe's theory of diffraction, has been presented in[3]. The fundamental steps to extend this method to the case of multiple coupling apertures are reported in[12]. The longitudinal impedance of N identical apertures positioned all around the same beam pipe transverse section at $z = z_0$, seen by an ultra-relativistic charge q, can be expressed by

$$Z(\omega) = j\frac{\omega Z_0}{2\pi b q} N\left(\frac{1}{c}M_\varphi + P_r\right) \qquad (4)$$

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where M_φ and P_r are the aperture equivalent dipole moments, which depend on the aperture shape and dimension, their position relative to the coaxial cavity and on the cavity dimensions themselves. Assuming that only a TEM mode is resonating in the cavity and limiting the calculation to frequencies below the beam pipe cutoff, we may write:

$$\begin{aligned} P_r &= \varepsilon_0 \alpha_e \left(E_{0r} - N E_{cr}\right) \\ M_\varphi &= \alpha_{m\perp} \left(H_{0\varphi} - N H_{c\varphi}\right) \end{aligned} \tag{5}$$

In eqs. (5) we have indicated by the subscript 0 the charge wake field in the unperturbed beam pipe; the subscript c indicates the fields in the coaxial cavity and α_e and $\alpha_{m\perp}$ are the electric and transverse magnetic polarizabilities. All the fields in eqs. (5) are calculated at the center of the aperture, that is for $r = b$, so that we have:

$$E_{0r} = \frac{Z_0 q}{2\pi b} \quad \text{and} \quad H_{0\varphi} = \frac{q}{2\pi b} \tag{6}$$

The scattered fields E_{cr} and $H_{c\varphi}$ can be expressed through the cavity eigenfunctions e_{rn} and $h_{\varphi n}$ and the coupling coefficients c_{en} and c_{hn} obtained applying the reciprocity theorem[13]

$$E_{cr} = c_{en}\, e_{rn}|_{r=b} \quad \text{and} \quad H_{c\varphi} = c_{hn}\, h_{\varphi n}|_{r=b} \tag{7}$$

The coupling coefficients, in turn, depend on the equivalent dipole moments:

$$\begin{aligned} c_{en} &= \frac{-j\omega\mu_0 k_n h_{\varphi n} M_\varphi + \omega^2 \mu_0 \left[1 + (1-j)/Q_n\right] e_{rn} P_r}{k_n^2 - k_0^2 \left[1 + (1-j)/Q_n\right]} \\ c_{hn} &= \frac{j\omega k_n e_{rn} P_r + k_0^2 h_{\varphi n} M_\varphi}{k_n^2 - k_0^2 \left[1 + (1-j)/Q_n\right]} \end{aligned} \tag{8}$$

where $k_0 = 2\pi/\lambda$, $k_n = n\pi/L$. L is the cavity length, Q_n its quality factor for the $\mathrm{TEM_n}$ mode and e_{rn} and $h_{\varphi n}$ are given by

$$\begin{aligned} e_{rn} &= \frac{\sin(k_n z_0)}{b\sqrt{\pi L \ln(d/b)}} \\ h_{\varphi n} &= \frac{\cos(k_n z_0)}{b\sqrt{\pi L \ln(d/b)}} \end{aligned} \tag{9}$$

with d the cavity radius. Replacing eqs. (6), (7), (8) and (9) in eq. (5), we get a linear system for the equivalent dipole moments. Using a perturbation method which neglects high order terms in the polarizability factors we can write

$$P_r = \alpha_e \varepsilon_0 \left[E_{0r} - \frac{N}{\tilde{k}} \left(-j\omega\mu_0 k_n e_{rn} h_{\varphi n} \alpha_{m\perp} H_{0\varphi} + \right.\right.$$
$$\left.\left. + \omega^2 \mu_0 \tilde{q} e_{rn}^2 \alpha_e \varepsilon_0 E_{0r}\right)\right]$$

$$M_\varphi = \alpha_{m\perp} \left[H_{0\varphi} + \frac{N}{\tilde{k}} \left(j\omega k_n e_{rn} h_{\varphi n} \alpha_e \varepsilon_0 E_{0r} + \right.\right.$$
$$\left.\left. + k_0^2 h_{\varphi n}^2 \alpha_{m\perp} H_{0\varphi}\right)\right] \tag{10}$$

with $\tilde{q} = 1 + (1-j)/Q_n$ and $\tilde{k} = k_n^2 - k_0^2 \tilde{q}$. In our case, since the slots are centered with respect to the cavity, the eigenfunction $h_{\varphi n}$ is zero. From eqs. (4) and (10) we can easily calculate the longitudinal impedance.

4 LONGITUDINAL IMPEDANCE MEASUREMENTS

The device under test is shown in Fig. 1. A 70 cm copper

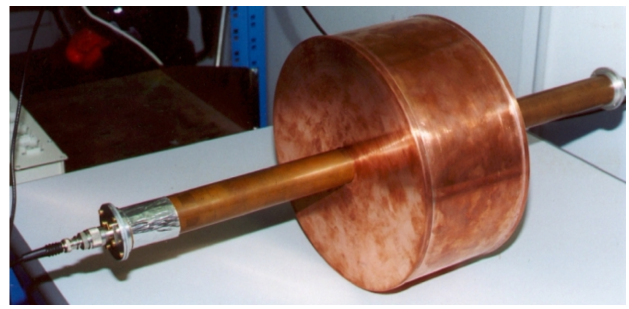

Figure 1: Device under measurement.

pipe of 2 cm internal radius and 1 mm thickness has 4 slots, 8 mm wide, positioned at its mid-length and azimuthally symmetric. A copper pill-box cavity, with radius 15 cm and length 14 cm, is placed coaxially around the pipe. The measuring wire has a diameter of 1.12 mm. The measurements have been performed using the Network Analyzer HP 8753E.

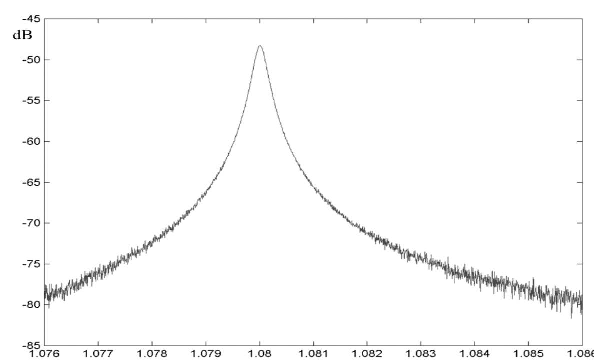

Figure 2: $S_{2,1}$ transmission parameter of the cavity.

In order to obtain the theoretical value of the impedance, the quality factor of the coaxial cavity without slots has to be known. The measurement of the $S_{2,1}$ transmission parameter as a function of frequency shown in Fig. 2 gives $Q = 2200$. It is important to underline that this measurement is very sensitive to the degree of oxidation of the copper, which can change rapidly. Thus the quality factor varies from day to day. Another critical point of the measurements is the electric contact between the pipe and the coaxial cavity.

In order to maximize the coupling impedance of the DUT, a first set of measurements has been performed with a length of the slots equal to that of the cavity, i. e. 14 cm. In this case the perturbation with respect to a reference pipe, even if small, has been clearly evidenced by the $S_{2,1}$ transmission parameter around a frequency of 1.046 GHz. From eq. (3) we get the real and imaginary part of the longitudinal coupling impedance shown in Figs. 3 and 4. The peak

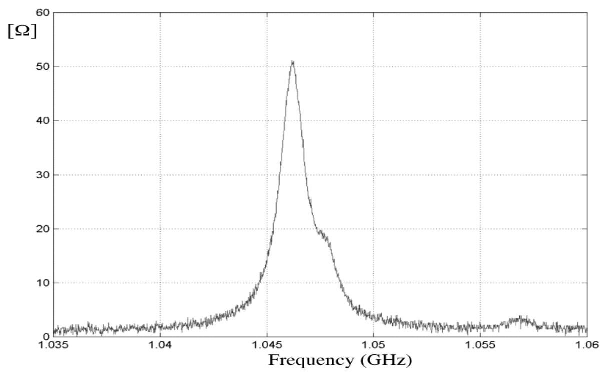

Figure 3: Long slots. Real part of the impedance.

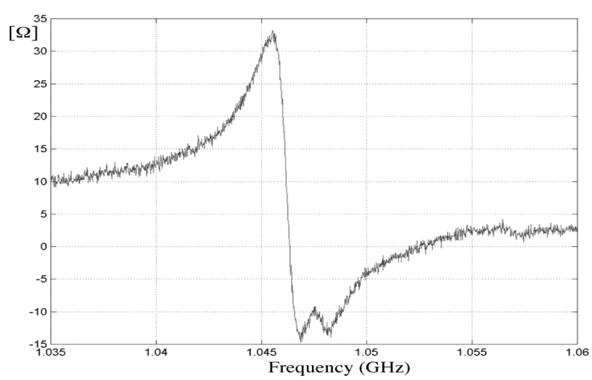

Figure 4: Long slots. Imaginary part of the impedance.

value of the real part is about $51\ \Omega$ at 1.046 GHz, quite close to the theoretical predictions, $53\ \Omega$ at a frequency of 1.071 GHz, obtained using eq. (4), with the same geometry and the measured Q value.

Actually the modified Bethe's theory is valid when the condition $\lambda/l\ <<\ 1$ is verified, with λ the cavity resonant mode wavelength and l the length of the slots. Since the former case is at the limit of validity of the theory $(\lambda/l = 0.5)$, we have performed another set of measurements with a reduced length of the slots $(\lambda/l = 0.25)$. The real and imaginary part of the measured impedance are shown in Figs. 5 and 6. The perturbation induced by the slots is very small, and the peak impedance is about $5\ \Omega$, a factor 10 less than the previous case. This result is consistent with what the theory predicts if we assume a Q reduced to $\simeq 1000$, which is likely due to the rapid oxidation of the copper cavity.

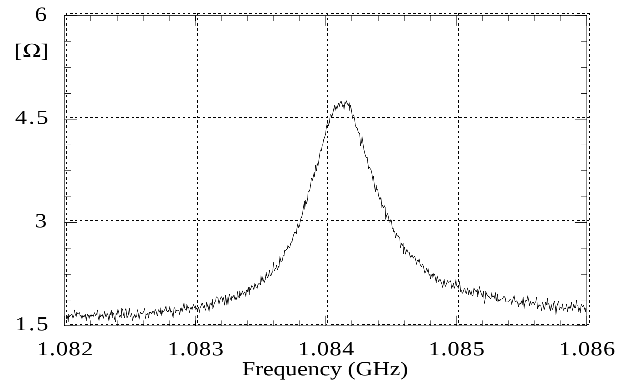

Figure 5: Short slots. Real part of the impedance.

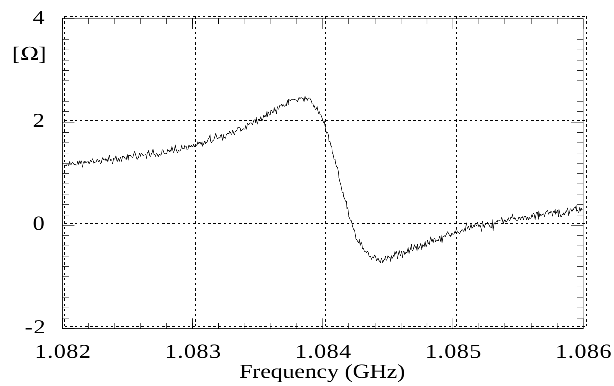

Figure 6: Short slots. Imaginary part of the impedance.

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