

COIL SHAPE OPTIMIZATION IN SUPERCONDUCTING DIPOLE MAGNETS

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Abstract

The field in coil-dominated superconducting magnets can be maximized by optimizing the coil shape. In this process the coil size and hence the cost of the magnet can be reduced. We have used the variational calculus and the technique of random search optimization to investigate the optimum coil shape and have shown that the optimized shape differs markedly from the conventional $\cos\theta$ shape. A block coil dipole magnet giving a field of 8.4 T has been designed as a demonstration.

INTRODUCTION

Superconducting magnets have become an essential component in high energy accelerators like LHC, RHIC, HERA etc. [1-3]. Many large detectors also use such magnets [4-5]. In these magnets one has to produce a large magnetic field using coils with a current density below the allowed value. At the same time the quantity of superconducting cable is also not unlimited. In this situation one has to optimize the coil shape so that the required field (with the requisite field quality) is produced by using minimum amount of cable.

Superconducting dipole magnets used in accelerators are variations of the so-called $\cos\theta$ magnet. A current sheet having an azimuthal shape which varies like $\cos\theta$ on a cylindrical surface gives a perfectly uniform dipole field [6]. But in such $\cos\theta$ magnets, though the field is perfect, the coil size is large. In practical magnets one can tolerate a finite field deviation of about 10^{-4} for $\Delta B/B$. Therefore one can deviate from the $\cos\theta$ shape in order to reduce the coil size. This also means that for a given coil size and allowed current density one can generate higher magnetic field by such optimization.

In this work we have investigated the shapes of the inner and outer edges of the coil by using the calculus of variation and the technique of random search optimization. Our investigation has shown that the coil width should increase marginally as one goes away from the median plane, whereas in the $\cos\theta$ design the width is maximum in the median plane.

FIELD DUE TO COIL

The vertical component $B(x_0, z_0)$ of the field at a point (x_0, z_0) due to N turns in a coil carrying a current I is

$$B(x_0, y_0) = 2NI \left[f(x_0, y_0) + f(-x_0, y_0) + f(x_0, -y_0) + f(-x_0, -y_0) \right] \quad (1)$$

where

$$f(x_0, z_0) = \int_{-z_{\max}}^{z_{\max}} \left[\frac{1}{(a(z)^2 + z^2)^{5/2}} - \frac{1}{(b(z)^2 + z^2)^{5/2}} \right] dz \quad (2)$$

Here z_{\max} is half the height of the coil, $a(z)$ and $b(z)$ are the shapes of the inner and outer edges (Fig.1).

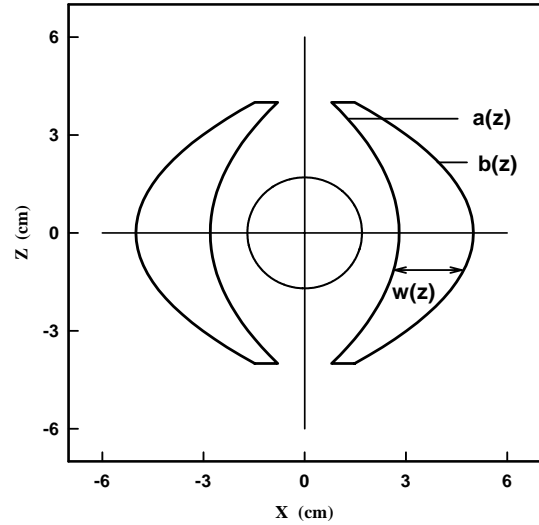


Figure 1. Geometry and shape of the coil.

OPTIMIZATION OF COIL SHAPE

We have attempted to optimize the shape of the coil by the technique of random search. However, we have first used the method of calculus of variation to increase the magnetic field for a given coil size (irrespective of the field quality).

Optimization by calculus of variation

The field $B(0,0)$ at the centre of the magnet is

$$B(0,0) = 8NI \int_{-z_{\max}}^{z_{\max}} \left[\frac{1}{(a(z)^2 + z^2)^{5/2}} - \frac{1}{(b(z)^2 + z^2)^{5/2}} \right] dz = \int_{-z_{\max}}^{z_{\max}} [F(a, z) - F(b, z)] dz \quad (3)$$

$F(a, z)$ and $F(b, z)$ are the two parts of the integrand. The Euler-Lagrange equation for maximizing $B(0,0)$ is

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$$\frac{\partial F}{\partial a(z)} = 0 = \frac{\partial F}{\partial b(z)} \quad (4)$$

This gives

$$F[a(z)] = \text{constant} \quad (5)$$

From eq.(3) and eq.(5) one gets

$$a(z)^2 + z^2 = r^2 \quad (6)$$

Therefore the inner edge of the coil comes out to be circular. Similarly the outer edge also should be circular if the field is to be maximized. This maximization is at the cost of deteriorating the field quality. A $\cos\theta$ coil shape gives a very uniform field but it is not maximum achievable with the given quantity of cable. If one wants to increase the magnetic field but at the same time wishes to keep the field quality under control, the coil shape is expected to be in between the circular coil shape and the $\cos\theta$ shape.

Optimization by random search method

In the variational method we have not put any restriction on the field uniformity $\Delta B/B$. In the random search method [7-9] we optimize the coil shape for generating the required field with the minimum $\Delta B/B$ possible and using a limited quantity of superconducting cable. The technique of random search optimization is very easy to use and can be applied to a variety of linear as well as non-linear problems.

Instead of finding the shapes $a(z)$ and $b(z)$ analytically, in the random search method we find these shapes in terms of some parameters. We first select a probable shape for $a(z)$ and $b(z)$. We have taken the following polynomial-like shapes.

$$\begin{aligned} a(z) &= a_1 \left[1 - a_2 z^2 - a_3 z^4 \right]^5 \\ b(z) &= b_1 \left[1 - b_2 z^2 - b_3 z^4 \right]^5 \end{aligned} \quad (7)$$

The parameters $a_1, a_2, a_3, b_1, b_2, b_3$ determine the coil shape. In the search process we first select a random set of these parameters (within some ranges of probable values), calculate $a(z)$ and $b(z)$ and then calculate the field using eq.(1) at a number of points within the region of interest (i.e., the region through which the beam passes and where the field deviation should be small). In our computation we choose the points along the horizontal and vertical axes and on the circular boundary of the good field region, as shown in Fig.2. At each chosen point the field deviation is calculated.

$$\Delta B_i = |B(x_i, y_i) - B(0,0)| \quad (8)$$

where $B(0,0)$ is the required field, $i=1, \dots, n$ and n is the number of points. The maximum value of all these ΔB_i 's is the overall field deviation ΔB for the set of parameters chosen at random. For each set of parameters we calculate the area of the cross-section of the coil. If the area is larger than the allowed area, we reject the parameter set.

This process is repeated a large number of times. If ΔB for a trial set becomes smaller than that of a previous set, the trial set becomes the new set of parameters. After a number of successful trials we shift the parameter space by observing whether the value of a parameter has increased or decreased over the previous trials. After every cycle of such trials we gradually decrease the range of parameters so that the search becomes faster.

RESULTS AND DISCUSSIONS

For demonstrating our method we have optimized the coil shape of a dipole magnet producing a field of 8.4 T. This is the same as that of the LHC main dipoles at CERN. The good field region has a radius of 17 mm and the aperture radius is 28 mm. The overall current density (over copper and superconductor together) has been taken to be 460 A/mm². This is equivalent to 11850 ampere of current in the LHC superconducting cables.

We have assumed that a field of 7 T is generated by the superconducting coil and the rest 1.4 T is generated by the iron yoke surrounding the coil. This is a benefit we get from the yoke whose main purpose is to contain the field. Fig. 2 shows the shape of the coil optimized for a coil cross-section of 11.5 cm² (the same as that of the LHC dipoles) in a quadrant.

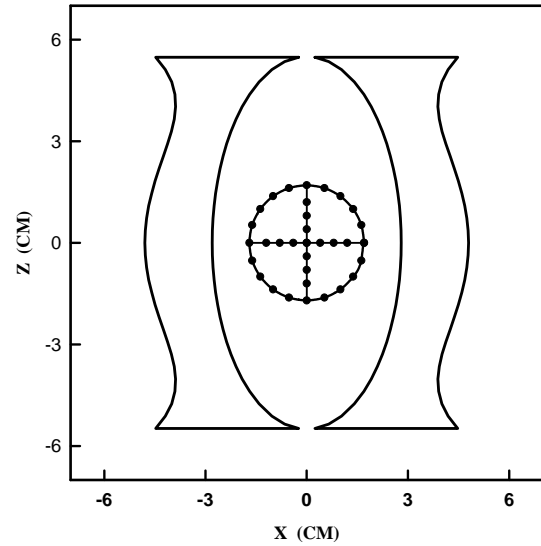


Figure 2. Coil optimized by random search method.

It is seen that the width $w(z)$ of the coil is larger at the end of the coils in comparison with that at the median plane. The coil shapes for two different coil areas have been compared in Fig. 3. The coil shape becomes smoother as the coil area increases. As is seen from Fig. 3 the aperture area inside the coil is elliptical rather than the circular aperture generally adopted. It has been observed that field quality improves as the coil area is increased.

It is difficult to fabricate a coil whose width varies continuously as shown in Fig. 2. One has to adopt a

simpler coil design. The coil fabrication becomes easy if the coil consists of rectangular blocks.

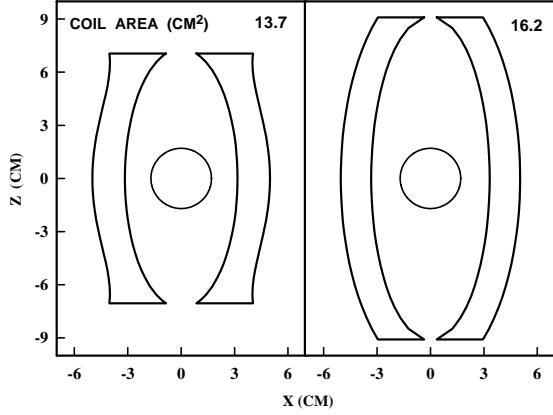


Figure 3. Optimized shapes for two different coil sizes.

The block-coil [10,11] designed at Texas A&M University is an example of such design. Here, instead of the polynomial shape, we adopt a rectangular shape. The calculation of field becomes easier in this case as no numerical integration is necessary. The optimization also becomes much faster as a result. The field B_q due to one quadrant of a rectangular coil block is

$$B_q(x_0, z_0) = 2NI \left[(d - z_0) \ln \frac{(d-z_0)^2 + (b-x_0)^2}{(d-z_0)^2 + (a-x_0)^2} - (c - z_0) \ln \frac{(c-z_0)^2 + (b-x_0)^2}{(c-z_0)^2 + (a-x_0)^2} + 2(b - x_0) \tan^{-1} \frac{(d-c)(b-x_0)}{(b-x_0)^2 + (d-z_0)(c-z_0)} - 2(a - x_0) \tan^{-1} \frac{(d-c)(a-x_0)}{(a-x_0)^2 + (d-z_0)(c-z_0)} \right] \quad (9)$$

where a and b are the distances of the vertical planes of the rectangular cross-section of the coil from the z -axis, and c and d are the distances of the horizontal planes of the coil from the x -axis. The total field due to all the four quadrants is

$$B(x_0, z_0) = B_q(x_0, z_0) + B_q(x_0, -z_0) + B_q(-x_0, z_0) + B_q(-x_0, -z_0) \quad (10)$$

We have used 5 coil blocks (on one side) in our design. The LHC dipole uses 12 coil blocks and the Texas A&M design has 10 blocks. Fig. 4 shows the optimized design.

Table 1: Geometrical parameters of the coil blocks

Parameter	Block A	Block B	Block C
a (mm)	28.0	24.8	9.13
b (mm)	42.9	55.5	56.9
c (mm)	-26.2	26.4	39.1

d (mm)	26.2	38.8	47.0
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Table 1 gives the geometrical parameters of the coil. The maximum field deviation for this design is 4.10^{-5} . One can use more number of blocks and have more number of optimizable parameters so that one gets even lower field deviation. But for very low field deviations the mechanical tolerances becomes impossible to achieve.

Recently common coil dipole magnets [12] have been proposed for use in dual beam accelerators. Such magnets also can be designed by our technique. Superconducting quadrupole and sextupole magnets can also be optimally designed by the method described here.

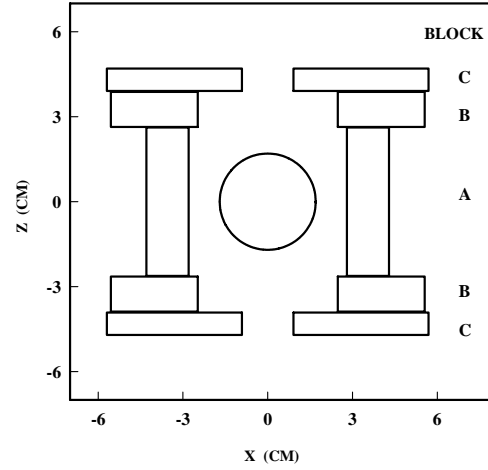


Figure 4. Block-coil design for a field of 8.4 T

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