

A GROUP OF CODES FOR FINDING THE POLE PROFILE IN QUADRUPOLES WITH HIGH FIELD QUALITY

M. K. Dey, P. R. Sarma and R. K. Bhandari
Variable Energy Cyclotron Centre, Department of Atomic Energy
Block-AF, Bidhannagar, Kolkata-700 064, India

Abstract

A group of codes has been developed using a new method for designing the pole profile of a quadrupole magnet. In this method the error harmonics in the quadrupole magnetic field are optimized to get a pole-profile with constant width yielding a very high field quality. The code adopts an iterative procedure to optimize the error harmonics and visualize the pole-profile. It also calculates and plots the magnetic field quality. The method can be extended to design the pole profiles of other multipole magnets also.

1 INTRODUCTION

Quadrupole magnets are one of the most essential elements in any beam line transporting charged particle beams. In many applications, especially in high resolution beam lines and in synchrotrons, a highly pure quadrupole field is required in order to preserve the beam quality as the beam traverses through the quadrupole.

The field quality of a room-temperature iron magnet depends mainly on the profile of the pole. The ideal profile of a quadrupole is hyperbolic extending up to infinity. In reality, however, one has to settle for a finite pole width by truncating the pole which gives rise to field errors. There are various ways of reducing the field errors in a quadrupole. One can use a circular pole and optimize the radius to reduce the field error [1,2]. For obtaining a better field quality people use various other pole profiles [3,4]. Some people use hyperbolic profile terminated by shims at the coil window. Some others truncate the hyperbolic profile with straight lines at the outer edges. Poles consisting entirely of plane surfaces are also used [6,7].

In the design of the pole profile of a magnet, the general procedure is to choose a pole profile and calculate the field produced by that profile with the help of standard codes like POISSON, SUPERFISH etc. The errors or the deviations from the ideal field are then estimated. If the errors are not smaller than the chosen value, one changes the geometrical parameters of the pole to repeat the procedure.

Another way of designing the pole of a quadrupole was developed by Sarma and Bhandari [8] in which one first chooses the tolerable error in the field and then determines the profile which will keep the error below the tolerance. Thus in this method one is not burdened with the task of selecting the geometrical parameters a priori. This method gives a smaller pole width than that obtained with the other methods.

Based on the new method, now we have developed an algorithm which calculates the pole profile of a quadrupole of required field tolerance and optimizes the pole width also. This report discusses the iterative method of profile optimization and describes the codes which design the profile and ascertains the field quality by calculating the field by POISSON code for the required field level.

2 METHOD

The equipotential lines for a pure quadrupole field are given by

$$C_2 r^2 \cos(2\theta) = 1 \quad (1)$$

where C_2 is the coefficient in the harmonic expansion of the magnetic potential. The above equation gives the pole profile for magnets of low strength and electrostatic quadrupoles. In practical magnets the potential equation contains higher harmonics due to finite termination of the pole

$$C_2 r^2 \cos(2\theta) + C_6 r^6 \cos(6\theta) + C_{10} r^{10} \cos(10\theta) + \dots = 1$$

The C_6 - and C_{10} -terms are the major error terms and have the major contribution to the field error. So we try to find the profile by taking $C_6 = C_{10} = 0$. Thus the profile equation becomes

$$C_2 r^2 \cos(2\theta) + C_{14} r^{14} \cos(14\theta) + C_{18} r^{18} \cos(18\theta) + \dots = 1$$

In Cartesian coordinates this becomes,

$$C_2 \operatorname{Re}[(x + iy)^2] + C_{14} \operatorname{Re}[(x + iy)^{14}] + C_{18} \operatorname{Re}[(x + iy)^{18}] + \dots = 1 \quad (2)$$

For a given set of harmonic coefficients ($C_2, C_{14}, C_{18}, \dots$) the locus of the point $P(x, y)$ gives the pole profile. We terminate at C_{30} because the higher order terms have little influence on the field in the region of interest. As we wish to find the solution set $\{Y_i\}$ for a given set $\{x_i\}$, ($i = 1, N$), we write the above equation in terms of a polynomial in y as

$$f(y) = k_0 + k_2 y^2 + k_4 y^4 + k_6 y^6 + \dots + k_{28} y^{28} + k_{30} y^{30} = 1 \quad (3)$$

where k 's are function of x and $C_2, C_{14}, C_{18}, \dots, C_{30}$. Here f is an explicit function of y and an implicit function of x and C 's. The solution set Y_i , hence, can be written as a function of C 's as

$$Y_i = Y_i(C_2, C_{14}, C_{18}, C_{22}, C_{26}, C_{30}) \quad (4)$$

The harmonic coefficient C_{14} has now the dominant contribution to the error in quadrupole field (in absence of the abandoned coefficients C_6 and c_{10}). This is the term which mainly determines the pole width and, obviously, the field quality. In designing a quadrupole magnet for any application, one has to have a specific field quality and in our method this is set by fixing C_{14} . We then get the optimized profile by varying the other higher harmonics $C_{18}, C_{22}, C_{26}, C_{30}$. As equation (4) is nonlinear, we linearize it by differentiating.

$$\Delta Y_i = \frac{\partial Y_i}{\partial C_{18}} \Delta C_{18} + \frac{\partial Y_i}{\partial C_{22}} \Delta C_{22} + \dots \quad (5)$$

we get N equations like eqn.(5) for N points x_i . In the matrix form we write,

$$\Delta Y = A \Delta C \quad (6)$$

Eqn.(6) gives the variation in the profile for a given variation in the harmonic coefficients $C_{18}, C_{22}, C_{26}, C_{30}$. A is therefore a $(4 \times N)$ matrix. Conversely, if we vary a given profile we can find out the corresponding variation ΔC in harmonic coefficients.

$$\Delta C = A^{-1} \Delta Y$$

But the matrix A is not a square one. So ΔC is actually calculated from [9]

$$A^T \Delta Y = A^T A \Delta C$$

ΔC now comes out to be

$$\Delta C = (A^T A)^{-1} A^T \Delta Y \quad (7)$$

These variations ΔC are then taken into account in eqn.(3) and a new profile is generated.

We want a profile which has a small pole width and is constant for higher x -values so that energizing coils can be accommodated easily. The optimized values of the harmonics are found from eqn.(7). The calculation however is done iteratively.

3 DESCRIPTION OF CODE

The code is interactive and Windows based one comprising of four programs:

1. Pole profile optimization (**PROG1**): It finds out iteratively an optimized pole-profile with a given field quality (specified by the harmonic coefficient C_{14}) and minimum width.
2. Pole profile visualization (**PROG2**): It visualizes the pole profile at any stage of iteration so that progressive improvement in the profile can be tracked.
3. Generating the input file of POISSON (**PROG3**): This program generates the input file to the POISSON for the optimized profile so that the magnetic field data can be calculated for this profile which will be used in the next

program. This program also visualizes the full quadrupole profile.

4. Field quality determination (**PROG4**): The error analysis is done in this program and hence the field quality is calculated and shown.

3.1 PROG1

In **PROG1** we give the desired field quality (specifying C_{14}) and the set of points x_i ($i = 1, N$) where optimization is to be done. Generally we desire that for lower values of x the locus of $y(x)$ be nearly hyperbolic and for larger values of x it is constant. We choose x_i values on the pole stem as it is the pole width which is to be optimized by the program. In fact only the first point x_1 and N are to be specified. The other points are taken at regular intervals already specified within the program. Initially the higher error harmonics are set to zero, i.e., we put $C_{18} = 0, C_{22} = 0, C_{26} = 0, C_{30} = 0$. The pole profile (e.g. profile-1 in Fig.1) is then calculated by solving the equation

$$f(y; C, x) - 1 = 0 \quad (8)$$

One needs to solve this equation repeatedly and it is done by Gauss-Jordan method. The initial guess values for y_1 is taken from a table already provided in the program which is prepared by trials for some specific values of field quality and x_1 . **PROG1** calculates the initial profile y_i at the specified points x_i . Then a small variation in the profile Δy_i is considered, e.g., profile-2 in Fig.1) is the varied profile. The corresponding variation in the higher harmonics ($\Delta C_{18}, \Delta C_{22}, \Delta C_{26}, \Delta C_{30}$) are calculated by solving eqn.(7). A small fraction (r) of these error harmonics is fed back and the new profile is calculated again by solving eqn.(8) with specified C_{14} and $C_j = C_j + r \Delta C_j$, for $j = 18, 22, 26, 30$ and $r = 0.05$ to 0.25 generally. A further variation in this new profile is considered and the same process is done iteratively until the final optimized profile (shown as profile-5, 6 in Fig.1) with specified C_{14} and a constant optimum width is achieved. Once the optimization is complete pole profile becomes stable and no further variation is achieved (see profile-4).

3.2 PROG2

Throughout the above optimization process the intermediate profiles can be visualized by **PROG2**. It is an interactive program and uses Windows graphics. In fact, the whole optimization process is to be done interactively by running **PROG1** and **PROG2** simultaneously. Fig.1 shows the intermediate profiles after different iterations and the finally optimized profile.

3.3 PROG3

PROG3 makes use of the finally optimized profile and generates the input file to POISSON for the desired field value (B_0 , to be given as input). Then with the help of POISSON and SF7 the magnetic field is calculated for this optimized

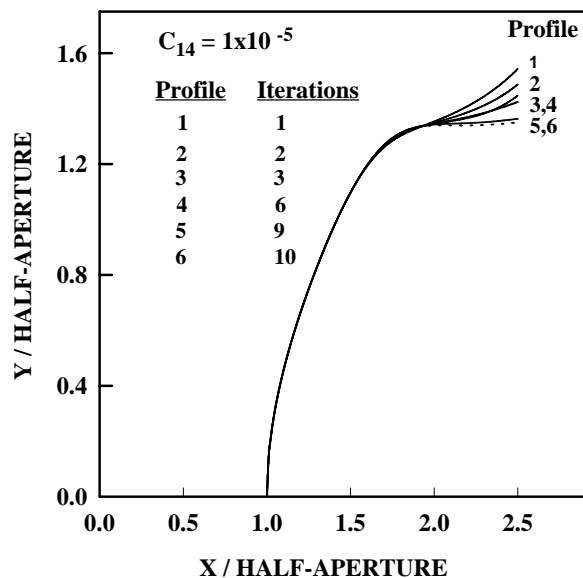


Figure 1: Improvement of field profile after various iterations

profile. It is to be noted that for higher fields the field error calculated by POISSON code are higher than that by our code. It is because at higher fields the pole profile no longer remains an equipotential. One can easily circumvent this problem by initially choosing a lower tolerance than desired.

3.4 PROG4

Error analysis is done with this program and the field quality is calculated. It fits the error harmonics with χ^2 fitting and calculates the deviation of the field data from the ideal quadrupole field and visualizes it on a graph (Fig.2).

4 DISCUSSION

The group of codes described above shows a way of designing quadrupoles of highly accurate field generally required in synchrotrons and storage rings where the beam passes through the magnets many times. We emphasize that the pole width obtained by the above method remains small compared the widths generally used. One can readily modify the codes for designing other types of magnets like sextupoles and octupoles.

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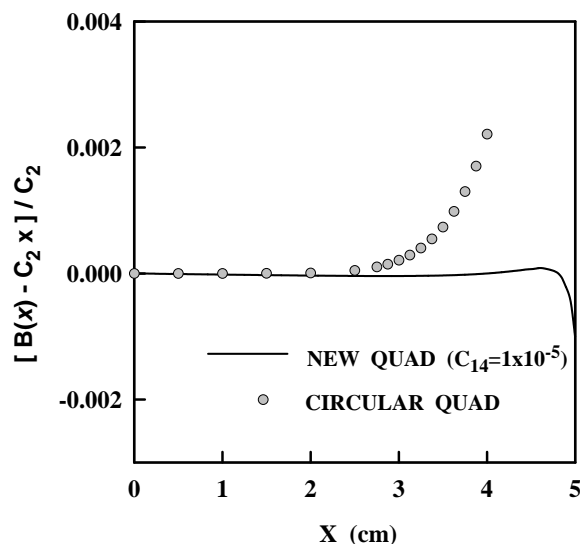


Figure 2: Comparison of the error field of the new quadrupole with that of a circular quadrupole. Half-aperture is 5cm.

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