3D PIC SPACE CHARGE SOLVER OF PHOTOELECTRON CLOUD

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Abstract

The Particle-In-Cell (PIC) method is applied to the 3D space charge of the photoelectron cloud. Irregular mesh is applied in order to get good approximation in boundary shape treatment. The technique of the method, such as charge assignment, POISSON solver and interpolation of the field on the mesh nodes, etc., has been studied in detail. Numerical results are shown.

1 INTRODUCTION

A blow-up of the vertical beam size, which is caused by photoelectron cloud, is observed in KEKB positron ring (LER)[1]. A single bunch instability model is applied to study the blow-up[2] and various simulation program are developed to study the effects of the electron cloud. However, all simulations for photoelectron cloud used 2D space charge force[2-4], which means the electron cloud distribution was assumed to be uniform longitudinally. Our 3D program [5-6] shows that the real distribution is not uniform in a non-uniform field. A 3D PIC space charge program has been completed recently to study the space charge force of the electron cloud in LER.

Unlike the bunched beam, the electron cloud is distributed in the whole vacuum chamber, which is round shape in LER. Therefore, the uniform rectangle mesh is difficult to be appllied for the charge assignment as is done in the study of the bunched beam. An irregular 3D mesh is used in our 3D space charge program and there are some novel features in our method, which are different from the general particle simulation methods such as beam-beam interaction. The technique of the program is discussed in this paper.

2 PIC TECHNIQUE

The direct particle-particle method is easy for developing the program and has high accuracy. However, it has a very low efficiency. The mesh method seems to be applied by all particle simulation programs. The general particle in cell (PIC) method includes four principal steps:

(1) Assign charge to the mesh node.
(2) Solve the field equation on that mesh.
(3) Calculate the mesh-defined force field.
(4) Interpolate to find forces on the particles.

The vacuum chamber of LER is round shape with a radius of 50 mm. Photoelectrons are distributed within the chamber as shown in Figure 1 for one example. The regular mesh as applied in bunch beam case can't satisfy here. Therefore, an irregular mesh is applied for the photoelectron cloud as shown in Figure 2.

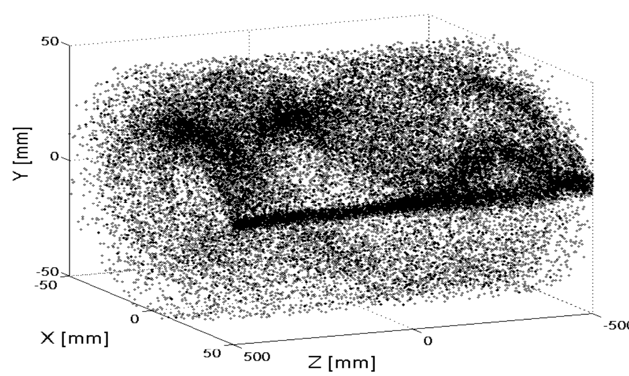

Figure 1: Example of photoelectron cloud distribution in the vacuum chamber

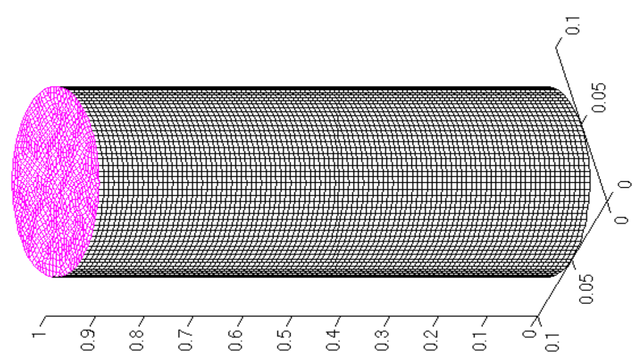

Figure 2: Mesh example of the vacuum chamber for photoelectron cloud

There are many charge assignment methods. The mesh in our method is an irregular mesh with brick elements. The charge Q_0 of a photoelectron is assigned to each node i of the element in which the photoelectron stays according to the shape function N_i

$$Q_i = N_i Q_0 . \quad (1)$$

Figure 3 shows the distribution of the macro-particle and charge at mesh node in one transverse section. The number of elements in this transverse section is 276, which is a small number. It already shows good representing of the real electron cloud distribution.

For the isoparametric element, the charge assignment scheme in Eq.(1) has all characters of charge assignment function such as

$$\sum_i N_i = 1 \quad (2)$$

$$\sum_i N_i \mathbf{r}_i = \mathbf{r} \quad (3)$$

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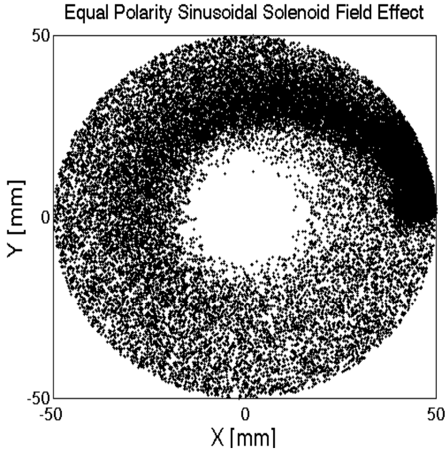
The property of the shape function in Eq.(2) keeps the charge conservation. It can be called Cloud-in-a-Cell (CIC) scheme. But it is different from the so-called CIC scheme applied in general particle simulation:

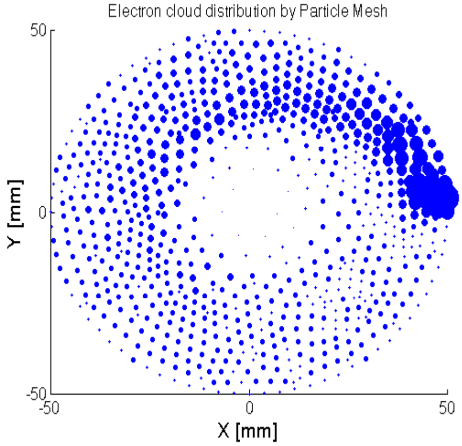
(1) The general CIC method applies a regular mesh. However, our scheme uses an irregular mesh, which makes this method can be successfully applied to the complex boundary problem such as the very flat beam case and ante-chamber.

(2) General CIC is for 2D, the charge assignment function has a clear meaning such as the cloud area. Our scheme is for 3D and the assignment has not a clear physical meaning for a high order element. There are many kinds of elements in the finite element methods. Among of them, the high order element can be applied to improve the accuracy of the method.

Therefore, our scheme has very serious advantages: general boundary and high accuracy. At the same time, it is somehow difficult to be applied comparing with the regular mesh CIC method.



(a)



(b)

Figure 3: Charge assignment
(a)Transverse distribution of the macro-particles.
(b)Transverse distribution of the mesh-defined charge by charge assignment.

The electron cloud (both the density and distribution) changes with time. The electron cloud can be extracted at some moment and we assume a quasi-static condition. The scalar potential satisfies (at each moment)

$$\Delta\phi = -\rho / \varepsilon_0 , \tag{4}$$

Eq.(4) can be solved by using the finite element method. We can get the finite element equation

$$\mathbf{A}\phi = \mathbf{B} . \tag{5}$$

Here the stiffness matrix **A** depends only on the mesh and **B** are the source term. The matrix **A** is extremely sparse and there are well-known methods for handing such linear problems, such as conjugate gradient method, profile or frontal technique. Fortunately, the vacuum chamber of LER is round shape. We can also find the Green function to get the potential. The potential ϕ at **R** is available with the Green function G(**R**, **R'**)

$$\phi(\mathbf{R}) = \int_0^L dz' \int_0^{2\pi} d\theta' \int_0^a r'dr' f(\mathbf{R}')G(\mathbf{R},\mathbf{R}') \tag{6}$$

$$\begin{aligned} G(\mathbf{R},\mathbf{R}') &= \frac{e}{L}\ln\frac{\rho^2 + r^2 r'^2/\rho^2 - 2rr'\cos(\theta-\theta')}{r^2 + r'^2 - 2rr'\cos(\theta-\theta')} \\ &+ \frac{4e}{L}\sum_{n=1}^{\infty}\cos nk(z-z')\Big\{K_0\big(nk\sqrt{r^2+r'^2-2rr'\cos(\theta-\theta')}\big) \\ &- \sum_{m=0}^{\infty}(2-\delta_{m0})\frac{K_m(nk\rho)}{I_m(nk\rho)} I_m(nkr)I_m(nkr')\cos m(\theta-\theta')\Big\} \end{aligned} \tag{7}$$

where L is the period length of the vacuum chamber, ρ is the pipe radius, **R'** is the source position, and **R** is the potential position, $k=2\pi/L$. The cylindrical coordinates with z-axis along the axis of the pipe, **R**=(r,θ,z), **R'**=(r',θ',z') are used.

After finding the potential, the force on each particle is interpolated by using the same shape function in order to keep the momentum conservation. Unlike the general PIC method, we calculate the force on particle directly using the potential at mesh node instead of the mesh-defined force field

$$\mathbf{E} = \sum_i \nabla N_i \cdot \phi_i \tag{8}$$

3 NUMERICAL EXAMPLE

We study the space charge effect of the photoelectron cloud in a solenoid field. When periodic solenoids are arranged with the same current direction in all the coils, the magnetic field can be approximately expressed as

$$B_z(x,y,z) = B_{z0} + B_0 \sin kz , \tag{9}$$

$$B_x(x,y,z) = -0.5B_0 kx\cos kz , \tag{10}$$

$$B_y(x,y,z) = -0.5B_0 ky\cos kz . \tag{11}$$

with B_{z0}=30G, B_0=20G, λ=1m for LER. The simulation parameters used in this paper are shown in Table 1.

The potential and field of the space charge at one transverse section (z= -0.5m) are shown in Figure 4. There are different potentials at different transverse sections (with different z). The space charge field is strong where the photoelectron density is large. Figure 5 shows the electron cloud distribution and the space charge force in the vertical-longitudinal plane. Figure 6 compares the effect of space charge force on the photoelectron line density. The difference in density is less than 5%. The space charge doesn't change the electron cloud distribution in the transverse plane so much, but it makes the longitudinal distribution more uniform due to the longitudinal space charge force. The secondary emission is not included in this study. However, it is expected to be not important unless the multipacting happens, because most of the photoelectrons have the energy less than 100 eV in the equilibrium state.

Variable	Symbol	Value
Ring circumference	C	3016.26 m
RF bucket length	s_{rf}	0.589 m
Bunch spacing	s_b	7.860 ns
Bunch population	N	3.3×10^{10}
Vertical betatron function	β_x	10 m
Horizontal betatron function	β_y	10 m
Horizontal emittance	ε_x	1.8×10^{-8} m
Vertical emittance	ε_y	3.6×10^{-10} m
Rms bunch length	σ_l	4 mm

Table 1: Parameters assumed for the electron cloud simulation

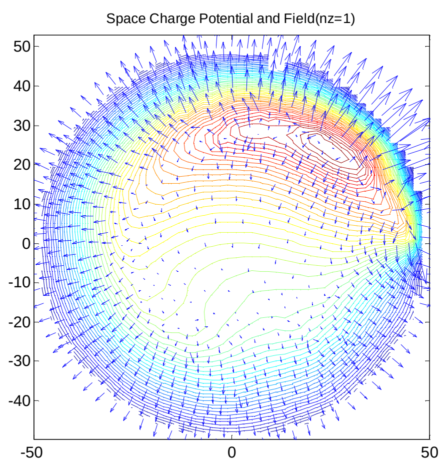
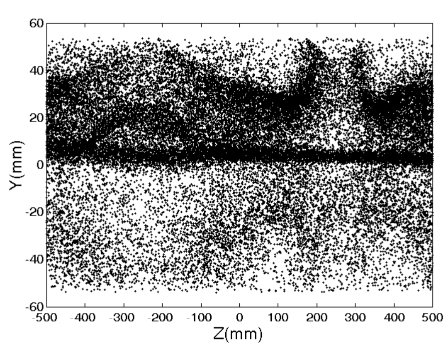
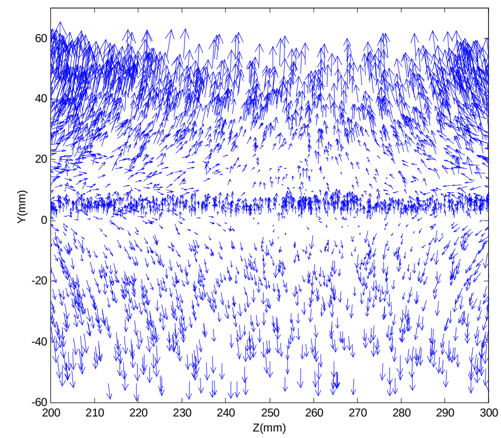


Figure 4: Potential and field of space charge at one transverse section



(a)



(b)

Figure 5: Electron cloud distribution (a) and the space charge force (b) in the vertical-longitudinal plane.

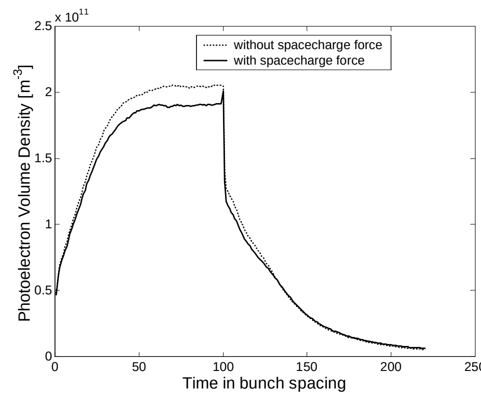


Figure 6: Effect of space charge force on the volume density

5 ACKNOWLEDGEMENT

Author L. Wang thanks Prof. Yokoya for introducing him to the particle simulation field.

6 REFERENCE

[1] H. Fukuma, et al., "Observation of vertical beam blow-up in KEKB low energy ring", EPAC2000, 2000.

[2] K. Ohmi and F. Zimmermann, "Head-tail instability caused by electron cloud in positron storage rings", Physics Review Letter (85):3821, 2000.

[3] F.Zimmermann, "Electron-Cloud Studies for the Low-Energy Ring of KEKB", CERN-SL-Note-004, 2000.

[4] O. Bruning, "Simulations for the Beam-Induced Electron Cloud in the LHC beam screen with Magnetic Field and Image Charges" LHC Project Report 158 (1997).

[5] L.F. Wang, H. Fukuma, K. Ohmi, "Simulation Study of Photoelectron Motion in Solenoid Field", KEK-2001-2, 2001.

[6] L. Wang, et al., "3D Simulation of Photoelectron Cloud", PAC2001, Chicago, Illinois, 2001