COD RESPONSE TO MIS-ALIGNMENT MODE OF SYNCHROTRON QUADRUPOLES

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Abstract

The effect of the misalignment of quadrupoles on the closed orbit can be treated as each contribution of the eigenmode of the eigenvalue problem for the precise magnet alignment of the synchrotron. As each eigenmode corresponds to its proper quadrupole displacement mode, the net misalignment is given by the linear combination of all eigenvectors. However, an individual mode contribution to the closed orbit distortion is different. The COD response to each misalignment mode is analyzed for the different survey meshes and the relations between the eigenmode and the misalignment pattern are given.

1 INTRODUCTION

To ensure the least COD (closed orbit distortion) to the beam operation of the synchrotron, magnets must be aligned precisely on the designed orbit. Generally the synchrotron is installed in the tunnel built for just a size allowing the machine space and the narrow passage for both the component installation and maintenance along the synchrotron ring. Therefore, a survey network shall be confined in the small space where the triangulation works take place using the overlapped flat triangles. The surveyed data are analyzed numerically by the least squares method (LSM) to make the alignment error minimum for all magnets. Even if the survey is performed to an accuracy of 0.01 mm, the LSM gives larger positional error of the magnet than expected.

After an installation of magnets is made in a few mm accuracy depending on the ground survey in the accelerator tunnel, the precise alignment of 0.1 mm accuracy follows using special tools developed for the accelerator survey or special purposes such as distinvar, distometer, offset measuring device and/or the laser tracker of which cat's eye reflector is tracked by the light of an incorporated laser interferometer measuring both the angle and the distance. By using either tools or the laser tracker, the raw survey data are obtained and processed mathematically to deduce the suitable data for the magnet re-alignments.

Several survey meshes are introduced at the different laboratories depending on the ring structure and the geometry of the magnet lattice. However, the survey mesh shall be selected what kind of survey data is useful to determine the magnet position accurately. This study is intended to find which mis-alignment mode affects seriously to the beam orbit. Since the quad mis-alignment gives the direct deflection to the beam, only the quad mis-alignment is considered here.

The present-day large synchrotron is composed of the same magnet lattice except for the experimental insertions. This kind of synchrotron is classified here as an asymmetric ring of which shape is different from the circle, in contrast with the symmetric ring which holds the circular shape.

2 SURVEY DATA ANALYSIS

If N quads having an individual positioning target point make a whole ring, at least two survey variables S_i and P_i $(i = 1, 2, ..., N)$, short chord and perpendicular, respectively, are necessary to each quad. These variables are expressed as a function of the radial and azimuthal coordinates of target points, R_i and Θ_i.

$$S_i = F(R_i, \Theta_i) \text{ and } P_i = G(R_i, \Theta_i), \qquad (1)$$

where the subscript $\boldsymbol{i}$ of R and Θ is considered for all quads. For an example, if S_i is a distance between the adjacent quads, $S_i = F(R_i, R_{i+1}, \Theta_i)$. Differentiating the above relations to the first order,

$$s_i \equiv \Delta S_i = \sum_i \frac{\partial F}{\partial R_i} r_i + \sum_i \frac{\partial F}{\partial \Theta_i}(\theta_{i+1} - \theta_i) \qquad (2)$$

and

$$p_i \equiv \Delta P_i = \sum_i \frac{\partial G}{\partial R_i} r_i + \sum_i \frac{\partial G}{\partial \Theta_i}(\theta_{i+1} - \theta_i), \qquad (3)$$

where (s_i and p_i) are the deviations from the design values which are determined from the precise survey and (r_i and θ_i) the quad displacements to the radial and azimuthal directions, respectively. Applying relations (2) and (3) to all quads, the following equation is obtained,

$$\begin{pmatrix} p_1 \\ : \\ p_N \\ s_1 \\ : \\ s_N \end{pmatrix} = \begin{pmatrix} a_{11} & \cdot\cdot & a_{1N} & c_{11} & \cdot\cdot & c_{1N} \\ : & & : & : & & : \\ a_{N1} & \cdot\cdot & a_{NN} & c_{N1} & \cdot\cdot & c_{NN} \\ b_{11} & \cdot\cdot & b_{1N} & d_{11} & \cdot\cdot & d_{1N} \\ : & & : & : & & : \\ b_{N1} & \cdot\cdot & b_{NN} & d_{N1} & \cdot\cdot & d_{NN} \end{pmatrix} \begin{pmatrix} r_1 \\ : \\ r_N \\ \theta_1 \\ : \\ \theta_N \end{pmatrix}, \qquad (4)$$

where the 2Nx2N matrix depends on the ring geometry. If 3 variables (S_i, P_i and Q_i) are considered, the matrix becomes 3Nx2N and the left column vector has 3N elements [1]. In any case the matrix equation (4) is expressed simply as

$$\mathbf{p} = (A)\mathbf{r} \qquad (5)$$

and solved by LSM as

$$\mathbf{h} \equiv (A^*)\mathbf{p} = (A^* A)\mathbf{r}, \qquad (6)$$

where A^* is the transposed matrix of A and $A^* A$ the 2Nx2N matrix. From this relation the precise magnet alignment problem can be treated as an eigenvalue problem. Replacing the matrix A^*A with H, the eigenvalue λ_i is given by the following equation,

$$(H)\mathbf{v}_i = \lambda_i \mathbf{v}_i, \qquad (7)$$

where $\mathbf{v}_i$ is the eigenvector belonging to the eigenvalue λ_i. The magnet displacements r_i and θ_i consist of a linear combination of the eigenvectors as follows,

$$\mathbf{r} = \sum_k c_k \mathbf{v}_k, \qquad (8)$$

where

$$c_k = (\mathbf{v}_k^T \mathbf{h}) / \lambda_k, \qquad (9)$$

and the j-th component of the displacement vector $\mathbf{r}$ is

$$r_j = \sum_k \frac{(\mathbf{v}_k^T \mathbf{h})}{\lambda_k} v_{kj}, \qquad (10)$$

where v_{kj} is the j-th component of the eigenvector $\mathbf{v}_k$ [2].

3 SURVEY MESHES

Three meshes which are treated here are depend on the survey variables,

(Case#1) short chord and perpendicular,

(Case#2) long chord and perpendicular, and

(Case#3) short chord and 2 perpendiculars,

as shown in Fig.1.

The matrix equations for 3 cases are derived for the ideal symmetric ring and the asymmetric ring TRISTAN (converted already to KEK-B factory) assuming that both rings have the same diameter and the same number of quads (392 quads). The eigenvectors for the largest and smallest eigenvalues are shown for the asymmetric and symmetric 3 cases in Fig.2, where each eigenvector is plotted to the order of the quad arrangements and a left (right) half corresponds to the radial (azimuthal) displacements. There are distinct difference between the asymmetric and symmetric cases and the characteristic displacement patterns appear for the modes with large eigenvalues.

If the lattice symmetry is lost as in the case of a colliding machine, the radial and azimuthal displacements can be treated separately for larger eigenvalues but both displacements are combined each other for smaller ones. Whereas in the symmetric quad arrangement, the case#1 shows the strong coupling between the radial and azimuthal displacements for larger eigenvalues.

The mode with the smallest eigenvalue gives one sinusoidal variations along the synchrotron ring to both radial and azimuthal directions. If the eigenvalue increases, the number of oscillations increases.

The relation (8) or (10) means that the contribution of eigenvectors which are less significant to the beam motion may be omitted from the solution. In general, the smaller the eigenvalue the larger it contributes to the magnet displacement as given by (10) and it is desirable to neglect the eigenvectors having little contribution to the beam. Considering the eigenvector elimination for the modes with small eigenvalues, the significant amount of the displacement is left uncorrected. This kind of displacement often appears in the solution as pseudo-components which have relatively large coefficients of (9) for smaller eigenvalues.

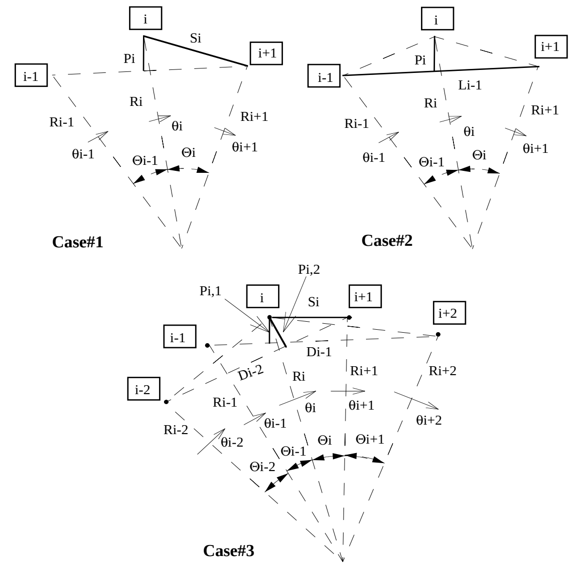

Figure 1: Survey variables for 3 cases.

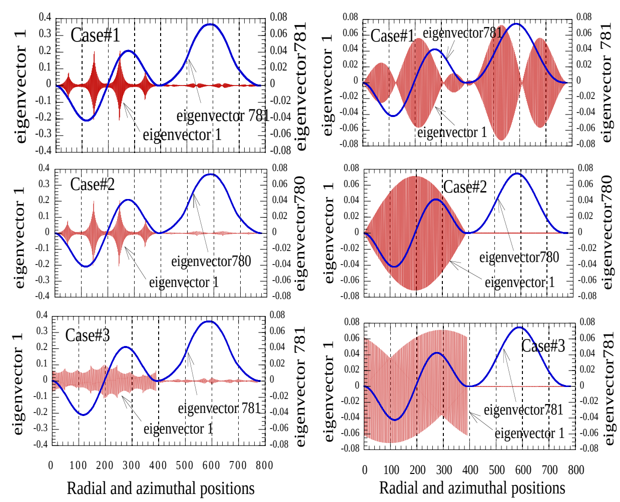

Figure 2: Eigenvectors with the largest (numbered as 1) and smallest (numbered as 781 or 780) eigenvalues for the asymmetric (left) and symmetric (right) cases. The left and right half of the eigenvector in each figure correspond to the radial and azimuthal displacements, respectively, to the order of quad arrangements. Case#1: upper, Case#2: middle, and Case#3: lower.

4 MIS-ALIGNMENT MODE AND COD

The quad mis-alignment effect on COD can be analyzed for each eigenmode that is converted directly to the mis-alignment of quads to estimate its effect by the beam simulation code in the linear motion regime [3]. As deviations of the perpendicular and short chord or long chord length are given by the corresponding eigenvector or mis-alignment mode, the radial and azimuthal positional errors are calculated. Assuming deviations of survey variables thus obtained, the corresponding eigenvector is regarded as the mis-alignment mode. Each eigenmode can be treated as a quad displacement mode separately in the beam simulation. The rms radial displacement error is adjusted to 0.1 mm for the fair evaluation of the individual eigenmode. The resultant rms COD is given in Fig.3-1, -2, -3 for all eigenmodes of every asymmetric and symmetric cases when the horizontal and vertical tunes are 37.2 and 39.1, respectively. Eigenmodes to be omitted from the magnet re-alignment can be selected by judging which mode has less contribution to COD.

There are some differences in contributions to rms COD between the survey meshes. As modes with small eigenvalues has relatively large displacement amplitudes, it is desirable to eliminate from the re-alignment procedure to avoid the unnecessary or not always necessary corrections. This analysis provides with a measure which mode can safely be omitted.

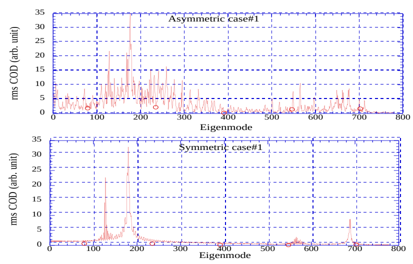

Figure 3-1: The rms COD for the 0.1 mm rms radial eigenmode offset of quads for the asymmetric (top) and symmetric (bottom) case#1.

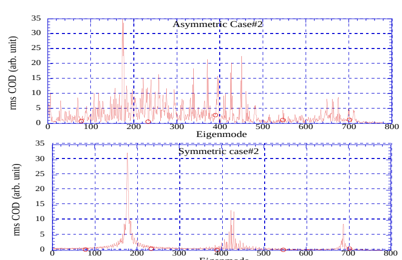

Figure 3-2: The rms COD for the 0.1 mm rms radial eigenmode offset of quads for the asymmetric (top) and symmetric (bottom) case#2.

The COD response is modified by the change of the tune (betatron oscillations per revolution) as shown in Fig.4, where two examples are compared for the asymmetric case#1. There is a linear relation between the eigenmode and the number of the mis-alignment periods of the whole ring as shown in Fig.5 for asymmetric and symmetric case#1. Two kinds of symbols (circles and triangles) correspond to eigenmodes to which the peaked COD responses are obtained at different horizontal tunes.

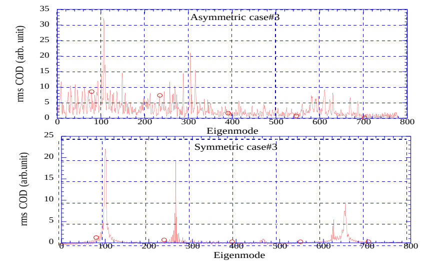

Figure 3-3: The rms COD for the 0.1 mm rms radial eigenmode offset of quads for the asymmetric (top) and symmetric (bottom) case#3.

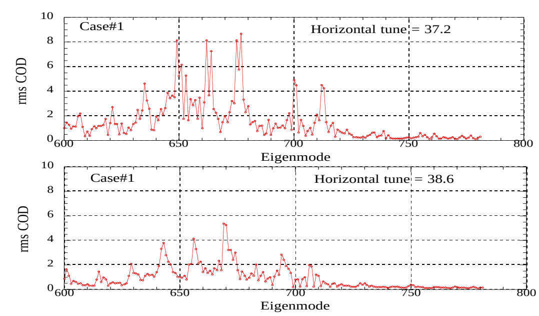

Figure 4: The COD response changes by the tune variation.

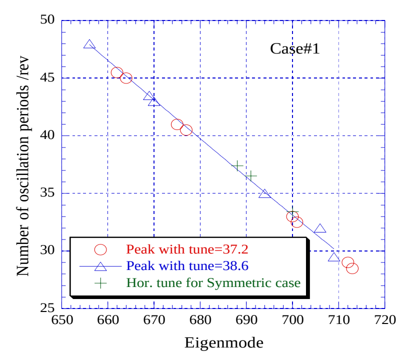

Figure 5: Peaks of the COD response shift to the different eigenmodes by changing the tune. Two symbols (circles and triangles) correspond to eigenmodes giving the peak COD response to two kinds of hor. tunes. For reference, tunes are shown with crosses for the symmetric case#1.

REFERENCES

[1] K. Endo and K. Mishima, "Survey Meshes for Synchrotron Magnet Alignment," Proc. EPAC2000, Vienna, pp.2385-7.

[2] J.H. Wilkinson, "The Algebraic Eigenvalue Problem," Oxford Univ. Press, 1965.

[3] A.S. King, M.J. Lee and W.W. Lee, "MAGIC, A Computer Code for Design Studies of Insertions and Storage Ring," SLAC-183, 1975. It was modified greatly to apply for the COD calculation due to the quad displacements.