THEORETICAL ANALYSIS OF METAMATERIAL INSERTIONS FOR RESISTIVE-WALL BEAM-COUPLING IMPEDANCE REDUCTION

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Abstract

Resistive-wall impedance usually constitutes a significant percentage of the total beam-coupling impedance budget of an accelerator. Reduction techniques often entail high electrical-conductivity coatings. This paper investigates the use of negative-permittivity or negative-permeability materials for sensibly reducing or theoretically nearly cancelling resistive-wall impedance. The proposed approach is developed by means of an equivalent transmission-line model. The effectiveness of such materials is discussed both for negative-permittivity and for negative-permeability cases, which actually show different impacts and can be then target of proper engineering. This first-stage study opens the possibility of considering metamaterials for impedance mitigation or for proper experimental setups.

INTRODUCTION

The resistive-wall term of beam-coupling impedance is essentially due to the finite electrical conductivity of the beam pipe walls. Many countermeasures are often used to mitigate this, as conductive coating or ceramic inserts [1]. In particular, the choice of coating materials is crucial.

Metamaterials or, more in detail, composite materials with negative values of either relative permittivity or relative permeability have been intensively studied in the last decades [2]-[3], in the framework of RF cloaking [2], [4], as well as in waveguides [2]. Some publications also addressed the application of metamaterials in particle accelerators [5], but no contribution is known concerning metamaterials insertions for impedance mitigation.

This paper theoretically evaluates the effect of metamaterial insertions for the reduction of resistive-wall beam-coupling impedance in accelerator rings.

PROPOSAL

The maximum acceptable impedance of a particle accelerator (i.e. the "impedance budget") is following a decreasing trend for modern machines (e.g. HL-LHC). On the other hand, many current and future machines present very short bunches, leading to a much wider frequency range of interest.

The theoretical study presented in this paper aims at describing the effect of a negative-permittivity (ENG) or a negative-permeability (MNG) material on the resistive-wall impedance on a wide frequency range. The effect has been studied thanks to an equivalent transmission-line model of the beam pipe with different coating layers (TL-Wall [6]), and consequently benchmarked with a full 2D field model, widely used for wall impedance investigations at CERN (ReWall [7]).

THEORETICAL ANALYSIS

The Validation Codes

TL-Wall is a MATLAB-based code which calculates the resistive-wall impedance of a 2D multilayer cylindrical beam pipe modelling it as an equivalent radial transmission line, starting at the interface between the pipe and the first layer, and ending with an infinite last layer [6] (Figure 1). The condition of validity of this approach is that the surface impedance (computed transporting the layer impedances to the first interface) is constant with the incident wave direction [6]. This condition is satisfied when the permittivity and permeability of the first layer are such that [6]

$$|\varepsilon_1\mu_1| \gg \varepsilon_0\mu_0 \qquad (1)$$

where ε_0 and μ_0 denote vacuum permittivity and permeability. It has to be noted that (1) can still be applied for $\varepsilon_1 < 0$ or $\mu_1 < 0$. As discussed in [6], for ultra-relativistic beams the transmission-line approach gives satisfactory results.

The longitudinal impedance is calculated through the equivalent impedance ζ_m of the layers, transported at the beam pipe-1st layer interface. The transverse impedance is computed from the longitudinal one [6].

ReWall is a code developed with Mathematica. It solves the Maxwell's equations of a point beam travelling at any speed in the 2D domain of an infinite-length cylindrical beam pipe, assigning field matching boundary conditions along the radial coordinate. It is possible to consider several (up to 5) layers as boundaries. It is also possible to use frequency-dependent expressions of permittivity and permeability. From the field solutions, it calculates the wall impedance [7].

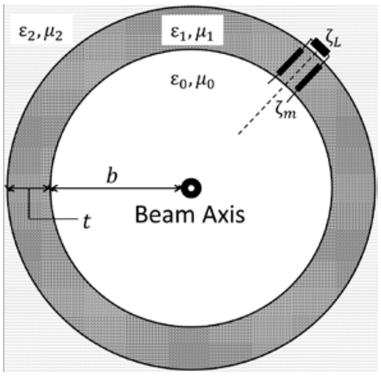

Figure 1: Transverse geometry of the problem. The metamaterial layer is put between the vacuum and the beam pipe (which extends to infinity).

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Effect of Metamaterial Insertions

Figure 1 shows the geometry of the electromagnetic problem. The metamaterial insertion covers the internal side of the beam pipe for a thickness *t*. The thickness of the beam pipe wall, modelled as a conductive medium with $\varepsilon_2 = \varepsilon_0$ and $\mu_2 = \mu_0$, is assumed to be infinite. This approximation holds for small skin depths (compared to the pipe wall thickness).

The results obtained with TL-Wall are here presented for an ultra-relativistic beam. Figure 2 and Figure 3 show the calculated longitudinal and transverse (in this case only dipolar) impedances when an insertion layer of t = 10 mm with a negative permittivity or a negative permeability is alternatively considered. The parameters used for the calculation are listed in Table 1. For this first theoretical analysis, the constitutive parameters have been assumed frequency-independent.

In the equivalent transmission-line model, the insertion of a layer entails an additional line sector, to be modelled with its characteristic impedance and wave number. In the case of a metamaterial, the characteristic impedance is imaginary. In fact, taking the correct determinations of the square roots [3], the ENG and the MNG material lines are described by inductive and capacitive characteristic impedance, respectively. This means that, as observed for metamaterials in general [2], [3], the electromagnetic wave in these sectors is evanescent. Therefore, the impedance transportation along such a line leads to an intrinsic alteration of the resistive-wall impedance.

As a matter of fact, Fig. 2 shows that both ENG and MNG layers determine a major decrease of the real part of longitudinal impedance above a characteristic frequency (in this case, about 300 MHz for ENG and 5 GHz for MNG insertions), which in principle depends both on constitutive parameters and on layer thickness. Nevertheless, the impact is significant, since the real part decreases by several orders of magnitude (e.g. more than 10, for ENG insertions). The influence on the imaginary part is instead opposite: whereas the MNG insertion translates the imaginary part from inductive to capacitive (the negative part is not displayed in logarithmic scale), the ENG layer increases it up to a constant above the previously-defined characteristic frequency.

Table 1: Parameters of the Electromagnetic Problem

Parameter	Value
b [mm]	31.5
t [mm]	10.0
Length of the line [m]	1
$\varepsilon_{r,1} = \varepsilon' + j\varepsilon''$ $\mu_{r,1} = \mu' + j\mu''$ (for ENG insertions)	$\varepsilon_{r,1} = -200 + j \cdot 10^{-12}$ $\mu_{r,1} = 1$
$\varepsilon_{r,1} = \varepsilon' + j\varepsilon''$ $\mu_{r,1} = \mu' + j\mu''$ (for MNG insertions)	$\varepsilon_{r,1} = 1$ $\mu_{r,1} = -0.5 + j \cdot 10^{-12}$
σ_{el}(conductivity of beam pipe wall) [S/m]	10^7

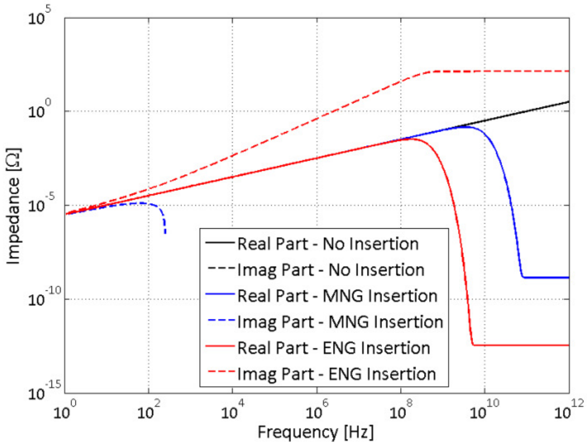

Figure 2: Effect of metamaterial insertions on real and imaginary part of longitudinal impedance.

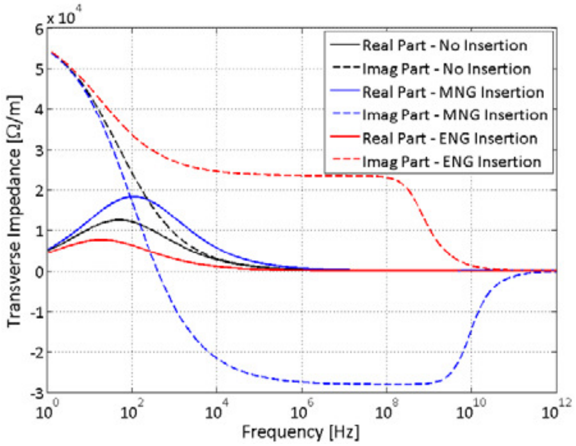

Figure 3: Effect of metamaterial insertions on real and imaginary part of transverse impedance.

On the transverse plane, the effect is also evident. The MNG insertion increases the real part and decreases the imaginary one, down to capacitive (i.e. negative) values.

Instead, the ENG layer significantly decreases the real part and increases the imaginary one. In both cases though, the imaginary parts are approaching zero above their characteristic frequencies.

Other than for this first-approach case, the impact of the metamaterial layer has been clearly observed even with much smaller values of the thickness *t*. Overall, the observed results demonstrate a remarkable influence on the resistive-wall beam-coupling impedance, which can lead to the individuation of theoretical design rules for impedance mitigation, exploiting the different degrees of freedom which emerged from this analysis: the type of material (ENG or MNG), its values of constitutive parameters, its thickness and its length. As a consequence, a proper engineering of such insertions can be performed, with the aim of substantially reducing the resistive-wall impedance of a beam line.

As an example, for the considered 1-m-long cylindrical pipe, Fig. 4 shows the results when a simple linear rule has been adopted to distribute ENG and MNG layers along the length, in order to integrate the effect of both.

The results show that the longitudinal impedance is substantially decreased, whilst in the transverse plane, a slight increase of the real part is observed. The imaginary part is also decreased, down to negative values (but smaller, in absolute value, than the ones obtainable with 100 % of MNG material, as testified in Figure 3). This kind of arrangement would, for example, suit a need for lowering the longitudinal real part, with some margin on the transverse plane. On the other hand, the introduction of capacitive impedance on this plane may also be useful to minimize the total transverse impedance.

This example of metamaterial insertion engineering has taken into account just one degree of freedom. A wider and more complex optimization can be thought *ad-hoc* considering all design parameters.

Re-Wall Benchmarking

In order to assess the reliability of the results obtained with TL-wall, the same metamaterial layers have been submitted to Re-Wall. Figure 5 shows a comparison in the longitudinal plane for the ENG case described in Table 1, zooming in the 100 Hz-4 GHz range for better resolution. The two codes agree very well. The same results have been observed for transverse plane and for MNG layers.

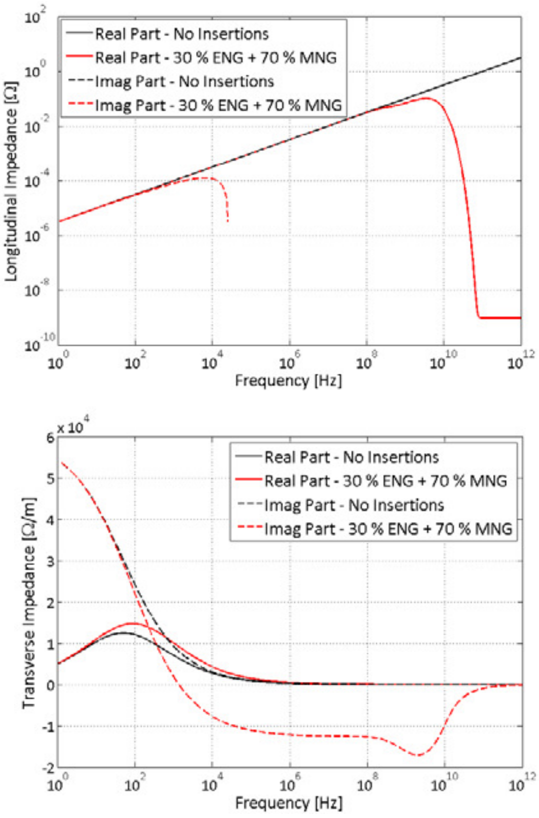

Figure 4: Longitudinal and transverse impedances of a cylindrical beam pipe as of Table 1, loaded with 30 % ENG and 70 % MNG layers along its length.

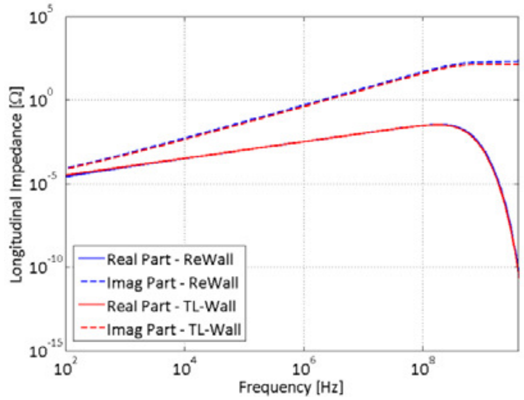

Figure 5: Benchmarking of metamaterial analysis with ReWall in the 100 Hz-4 GHz frequency range.

CONCLUSIONS AND OUTLOOK

The effect of metamaterial insertions on beam-coupling impedance is studied theoretically by means of a transmission-line model and benchmarked with a 2D full-field code. Metamaterials showed to impact the resistive-wall impedance both in the longitudinal and transverse planes. In the case considered, the real longitudinal impedance is significantly decreased. An example of engineering is also given.

More complex studies, with frequency-dependent negative permittivity and permeability are foreseen, as well as an experimental study with an actual metamaterial insertion.

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