# FIELD TOLERANCES FOR THE TRIPLET QUADRUPOLES OF THE LHC HIGH LUMINOSITY LATTICE \* <sup>†</sup>

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#### Abstract

It has been proposed to implement the so-called Achromatic Telescopic Squeezing (ATS) scheme [1] in the LHC high luminosity (HL) lattice to reduce beta functions at the Interaction Points (IP) up to a factor of 8. As a result, the nominal 4.5 km peak beta functions reached in the Inner Triplets (IT) at collision will be increased by the same factor. This, therefore, justifies the installation of new, larger aperture, superconducting IT quadrupoles. The higher beta functions will enhance the effects of the triplet quadrupole field errors leading to smaller beam dynamic aperture (DA). To maintain the acceptable DA, the effects of the triplet field errors must be re-evaluated, thus specifying new tolerances. Such a study has been performed for the so-called "4444" collision option of the HL-LHC layout version SLHCV3.01, where the IP beta functions are reduced by a factor of 4 in both planes with respect to a pre-squeezed value of 60 cm at two collision points. The dynamic aperture calculations were performed using Six-Track. The impact on the triplet field quality is presented.

# **INTRODUCTION**

The LHC high luminosity (HL) lattice is designed for up to a factor of 8 lower beta function at two Interaction Points (IP1 and IP5), as compared to the nominal lattice. Consequently, beta functions in the Inner Triplet (IT) quadrupoles adjacent to these IPs will increase by the same factor, resulting in a larger beam size. This, therefore, justifies installation of new, larger aperture, superconducting (SC) triplet quadrupoles, where the coil diameter  $d_c$  is increased from the nominal 70 mm to, e.g., 120-150 mm, and the operating gradient is lower (120 T/m in SLHCV3.01). The higher IT beta functions will result in larger aberrations caused by the triplet chromatic and non-linear field errors. These must be corrected and/or reduced in magnitude in order to maintain a sufficient dynamic aperture (DA). The triplet linear and non-linear chromatic effects will be compensated by implementing the Achromatic Telescopic Squeezing (ATS) optics [1]. On the other hand, the triplet non-linear field errors will be corrected only to low order [2]. Therefore, the magnitude of the field errors must be re-evaluated, leading to new field quality specification for the triplet. This study was performed for the "4444" collision option of the HL-LHC lattice V3.01, where the

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beta functions at the IP1 and IP5 are  $\beta^* = 15 \text{ cm}$  in both planes. The goal was to obtain the field error tolerances for the new SC triplet quadrupoles with a reference coil aperture of  $d_c = 120 \text{ mm}$ , yielding a minimum DA of 12-13  $\sigma$  in 60 random sets of machine errors, where  $\sigma$  is the rms beam size. The adopted strategy is similar to the one used in the "Phase-1" triplet study [3].

The core of this study consisted of dynamic aperture calculations for various IT field errors using long-term tracking in SixTrack [4]. The conditions used in the DA simulations were:  $10^5$  turns, 11 angles, 30 particle pairs per amplitude step  $(2\sigma)$ , 60 error seeds for final tracking, 20 seeds for multipole sensitivity scans, 7 TeV beam energy with initial energy offset  $\Delta p/p = 2.7 \times 10^{-4}$ , and normalized beam emittance of  $3.75 \,\mu$ m·rad. The arc errors and their correction were included, as well as the low order IT multipole field correction [2]. The latter utilized non-linear field correctors implemented on the outer side of each triplet to compensate the effects of  $a_3, a_4, b_3, b_4, b_6$  multipole terms (see definition below). Finally, no field errors were considered for the D1, D2 separation dipoles and Q4 quadrupoles.

## **MULTIPOLE FIELD SCALING**

Magnetic field in a quadrupole can be expanded as [5]

$$B_y + iB_x = 10^{-4} B_2 \sum_{n=2}^{\infty} (b_n + ia_n) \left(\frac{x + iy}{r_0}\right)^{n-1},$$
(1)

where  $a_n$ ,  $b_n$  are skew and normal multipole coefficients in units of  $10^{-4}$  at a reference radius  $r_0$ , and  $B_2$  is the main quadrupole field at  $r_0$ . In LHC studies [6], the  $a_n$  and  $b_n$ are split in the "uncertainty" and "random" components related to systematic and random type errors, and their values represent Gaussian sigmas of the error distributions.

It is logical to start the search for the new specifications with the measured field quality for the existing IT quadrupole listed in Table 1 [7], where  $r_0 = 17$  mm, coil diameter  $d_c = 70$  mm, and  $a_{nu}$ ,  $b_{nu}$  and  $a_{nr}$ ,  $b_{nr}$  are the uncertainty and random terms. These field error tolerances provide acceptable dynamic aperture for the existing lattice with the IT peak beta function  $\beta_m = 4.5$  km. As a next step, one can re-scale the Table 1 values to the new proposed reference radius of 50 mm and coil diameter of 120 mm according to [5]

$$a_n, b_n \propto r_0^{n-2} d_c^{1-n}.$$
 (2)

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This yields coefficients in Table 2 named "target3", where the expected values for n=12-14 were added from the "Phase-1" study [3]. These field tolerances should be achievable in a real magnet, however the corresponding dynamic aperture is insufficient as seen in Fig. 1, where the

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Table 1: Measured multipoles at  $r_0 = 17$  mm for the nominal MQXB quadrupole with coil diameter  $d_c = 70$  mm.

n	$a_{nu}$	$a_{nr}$	$b_{nu}$	$b_{nr}$
3	0.5235	0.6354	0.4135	0.7873
4	0.4432	0.3883	0.1552	0.1563
5	0.0874	0.1423	0.1142	0.2171
6	0.2306	0.2637	0.2089	0.3088
7	0.0254	0.0411	0.0311	0.0374
8	0.0140	0.0280	0.0060	0.0096
9	0.0127	0.0078	0.0085	0.0116
10	0.0094	0.0179	0.0303	0.0086
11	0.0046	0.0028	0.0084	0.0106

3.0) Table 2: Error table "target3" obtained by scaling the mea-- cc Creative Commons Attribution 3.0 (CC BY sured field to  $r_0 = 50 \text{ mm}$  and coil diameter  $d_c = 120 \text{ mm}$ .

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n	$a_{nu}$	$a_{nr}$	$b_{nu}$	$b_{nr}$
3	0.5239	0.6359	0.4139	0.7879
4	0.7611	0.6667	0.2664	0.2683
5	0.2574	0.4191	0.3365	0.6396
6	1.1655	1.3328	1.0603	1.5608
7	0.2203	0.3564	0.2701	0.3244
8	0.2087	0.4162	0.0889	0.1423
9	0.3238	0.2003	0.2165	0.2971
10	0.4137	0.7838	1.3256	0.3755
11	0.3457	0.2116	0.6340	0.7965
12	0.1863	0.1863	0.1863	0.1863
13	0.1164	0.1164	0.2328	0.1164
14	0.4366	0.1455	0.5821	0.1455

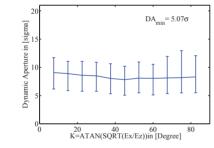


Figure 1: Dynamic aperture for the error table "target3".

line is the average aperture for 11 angles, and the error bars show the span between the minimum and maximum DA in 60 random seeds. This small aperture is the consequence of high triplet beta function  $\beta_m$  in the new lattice enhancing the field error effects. As a next step, it is therefore reasonable to require that the non-linear kicks from the triplet field errors remain about the same with  $\beta_m$  [8]. Therefore, the "target3" coefficients should be scaled as

$$a_n, b_n \propto \beta_m^{-\frac{n}{2}}.$$
 (3)

Scaling from the nominal  $\beta_m = 4.5 \,\mathrm{km}$  to 21.5 km in the new lattice yields coefficients in Table 3 named "target31". The corresponding minimum DA exceeds the goal of 12-13  $\sigma$  as shown in Fig. 2, however such tight field tolerances may be difficult to achieve in a real magnet. Therefore, the final step of this study is to relax the "target31" tolerances while satisfying the DA goal.

Table 3: Error table "target31" obtained from table "target3" by scaling the triplet  $\beta_m$  from 4.5 km to 21.5 km.

n	$a_{nu}$	$a_{nr}$	$b_{nu}$	$b_{nr}$
3	0.0502	0.0609	0.0396	0.0755
4	0.0333	0.0292	0.0117	0.0118
5	0.0052	0.0084	0.0067	0.0128
6	0.0107	0.0122	0.0097	0.0143
7	0.00092	0.00150	0.00113	0.00137
8	0.00042	0.00080	0.00015	0.00027
9	0.00029	0.00019	0.00019	0.00024
10	0.00018	0.00030	0.00054	0.00018
11	0.00007	0.00007	0.00015	0.00015
12	$1.6 \ 10^{-5}$	$1.6 \ 10^{-5}$	$1.6 \ 10^{-5}$	$1.6 \ 10^{-5}$
13	$0.4 \ 10^{-5}$	$0.4 \ 10^{-5}$	$0.9 \ 10^{-5}$	$0.4 \ 10^{-5}$
14	$0.8 \ 10^{-5}$	$0.3 \ 10^{-5}$	$1.0 \ 10^{-5}$	$0.3 \ 10^{-5}$

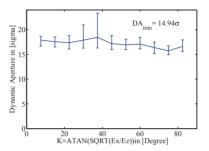


Figure 2: Dynamic aperture for the error table "target31". This result is close to DA without IT field errors [9].

### MULTIPOLE SCAN

Assuming that the multipole coefficients are independent, various solutions for the field tolerances are possible. Below we present the result based on DA sensitivity to individual multipoles. In this method, the coefficients are scanned one-by-one between their values in "target31" and "target3" tables while all other coefficients are kept at "target31" values. This involves a large number of DA calculations, but allows to determine the sensitivity to each coefficient. The resultant minimum DA versus relative change of  $a_{nr}$  and  $b_{nr}$  values is shown in Fig. 3,4. Similar dependences are obtained for the uncertainty coefficients, but their impact on the DA is typically smaller. One can observe that the aperture is not sensitive to the n=3,4 multipoles since their effect is compensated by the IT correctors. On the other hand, the  $b_{6r}$  term still causes the DA reduction even with the included corrector. The other uncorrected terms exhibit the DA reduction even at high multipole order. These results suggest that additional IT correctors, e.g., for  $a_5$ ,  $b_5$  and  $a_6$ , may be needed (they have already been included in the latest HL-LHC version).

Based on this data, one can find a set of tolerances, where the effects of individual multipoles on dynamic aperture are about equal. To do so, one can make a DA cut, such as shown in Fig. 3,4, to determine the coefficient values for the same minimum DA in the individual scans. Once all the coefficients are applied, the combined effect will produce a lower minimum aperture than in the individual scans. Therefore, a few iterations of the DA cut are needed

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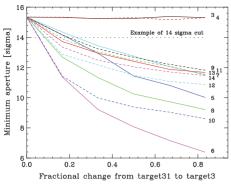


Figure 3: Individual scan of  $a_{nr}$  coefficients.

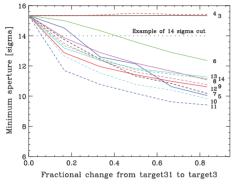


Figure 4: Individual scan of  $b_{nr}$  coefficients.

to reach the desired 12-13  $\sigma$  aperture.

In this method, the corrected and least sensitive coefficients can be relaxed and set to the "target3" values. However, the sensitive coefficients will have much tighter tolerances. Hence, in order to help relaxing the tightest tolerances, we intentionally set the least sensitive coefficients to a mid-value between the "target3" and "target31" settings (instead of more relaxed values from the DA cut). This provided a little more room for other coefficients to grow. The resultant tolerances are listed in Table 4, named "target39", and the corresponding dynamic aperture is shown in Fig. 5, where the minimum DA is  $12.3 \sigma$ , thus meeting the requirements. Alternatively, a similar DA can be obtained when the corrected and non-sensitive coefficients are set to "target3" values at the expense of somewhat tighter high order tolerances.

Table 4: Error table "target39" at  $r_0 = 50$  mm.

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n	$a_{nu}$	$a_{nr}$	$b_{nu}$	$b_{nr}$
3	0.2870	0.3484	0.2268	0.4317
4	0.3972	0.3480	0.1391	0.1400
5	0.1313	0.0295	0.0902	0.0711
6	0.4146	0.0270	0.5350	0.3892
7	0.1106	0.0256	0.0103	0.0122
8	0.1045	0.0082	0.0277	0.0056
9	0.1582	0.0104	0.0091	0.0084
10	0.0944	0.0092	0.0187	0.0067
11	0.0776	0.0070	0.0082	0.0100
12	0.0554	0.0035	0.0046	0.0036
13	0.0368	0.0044	0.0040	0.0027
14	0.0189	0.0036	0.0061	0.0036

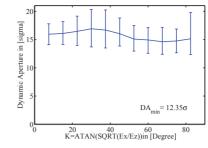


Figure 5: Dynamic aperture for the error table "target39".

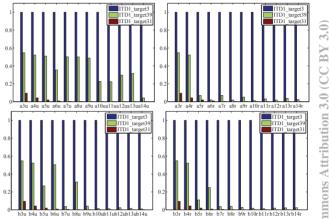


Figure 6: Relative values of  $a_{nu}$  (top left),  $a_{nr}$  (top right),  $b_{nu}$  (bottom left), and  $b_{nr}$  (bottom right) coefficients in the error tables "target3" (dark blue), "target31" (brown) and "target39" (green).

A direct comparison of the "target3", "target31" and "target39" coefficients is shown in Fig. 6. It clearly shows that the IT field correctors relax the corresponding tolerances, and that the random errors require tighter tolerances.

## CONCLUSION

This study showed that scaling with the triplet quadrupole reference radius, coil aperture, and peak beta function provides a reasonable first estimate of the required field quality for the new IT quadrupoles in the LHC high luminosity lattice. Further investigation of the individual multipole field sensitivities led to a solution with relaxed IT field error tolerances. Additional fine adjustment may be needed in order to reach the final specifications.

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