HEAD TAIL INSTABILITY OBSERVATION AND STUDIES AT THE PROTON SYNCHROTRON BOOSTER

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Abstract

Since many years the Proton Synchrotron Booster (PSB) high intensity beams have shown head tail instabilities in all of the four rings at around 100 ms after the injection. In this paper we present the latest observations together with the evaluation of the instability rise time and its dependence on the bunch intensity. The acquired head tail modes and the growth rates are compared with HEADTAIL numerical simulations, which together with the Sacherer theory points at the resistive wall impedance as a possible source of the instability.

INTRODUCTION

The PSB is made of four stacked similar rings and we reported the last observations of the horizontal instabilities in Ring4 which are observed for the 50 MeV-1.4 GeV cycle (NORMGPS).

We compared the observations of the horizontal bunch profiles with the Sacherer theory. We have also calculated the bunch frequency spectrum for a parabolic bunch. In fact the longitudinal bunch profile seems to be best fitted by a parabola instead of a Gaussian.

In Ring4 from the experimental data we calculated the growth rates of the instability which develops 100 ms after the beam injection: while increasing the bunch intensity we observed that, despite the fact that the bunch length stays the same, the number of nodes decreases. The Sacherer theory foresees higher number of modes respect to those observed.

THE INSTABILITIES AND THE PARAMETERS

In this section we report the typical pattern of the losses that can be observed at the PSB while increasing the bunch intensity without the transverse damper. All the following measurements have been taken using only the 1^{st} harmonic cavity (C02). Some of the machine parameters are reported in Table 1.

Table 1: PSB Parameters for Ring4 at Time 378 ms During The Cycle

Q_x/Q_y	H/V tune	4.22/4.4
R	Machine radius [m]	25
α	Momentum compaction factor	$6.1 \cdot 10^{-2}$
ξ_x/ξ_y	Chromaticity	-0.95/-2.1

The two instabilities would develop either 100 ms or 200 ms after the injection into the PSB, which approximately correspond to 378 ms and 478 ms in the PSB magnetic cycle. We have only carried out the analysis in the Ring4, after observing the same unstable behavior in all four rings.

The losses are easily triggered by increasing the intensity Fig. 1. In fact it is clear that the instability has a strong dependence on the bunch current. Both the instabilities occur for a current higher that $\approx 2.5 \cdot 10^{12}$ ppb and sometimes they even appear both during the same cycle.

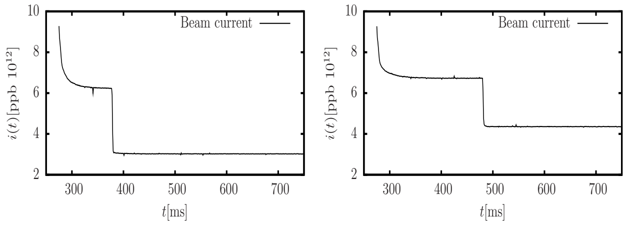


Figure 1: Typical pattern of the losses observed at 378 ms (left) and 478 ms (right) the injection. The kinetic energies are ≃131 MeV and ≃330 MeV respectively.

The pattern observed from the horizontal pick-up (both instabilities start from the horizontal plane [1]) is displayed in Fig. 2.

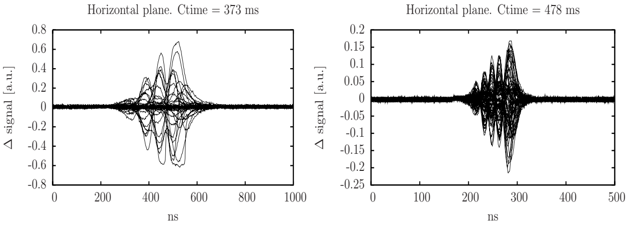


Figure 2: Typical pattern from the pick-up signal. Both the pictures refer to the horizontal plane.

From now on, we will only focus on the first instability (378 ms). In the theory of the head tail instability the chromatic frequency shift should indicate which mode will be driven unstable with the shortest rise time by the resistive wall wake field. For this purpose we have also measured the horizontal and vertical chromaticity. The curves are reported in Fig. 3.

We have also observed that the bunch length stays almost constant respect to the bunch population (Fig. 4).

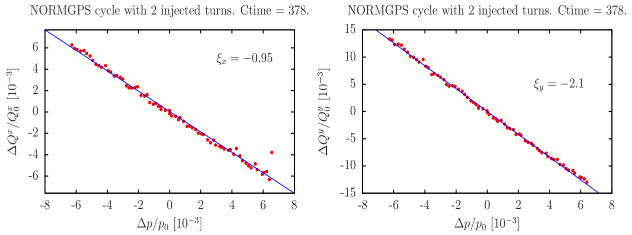


Figure 3: Measured horizontal (left) and vertical (right) chromaticities.

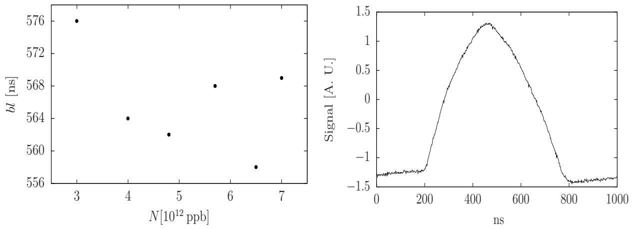


Figure 4: Longitudinal bunch length measurements at 378 ms. Left: the acquired bunch lengths as a function of the bunch intensity. Right: an example of the bunch profile acquired for a bunch intensity of $3 \cdot 10^{12}$ protons.

The longitudinal bunch profile seems to be best fitted by a parabola instead of a Gaussian curve. Since the longitudinal bunch shape plays a fundamental role in the head tail theory we calculated the bunch spectrum for a parabolic bunch.

THE PARABOLIC BUNCH

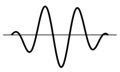
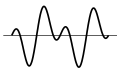
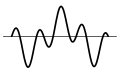
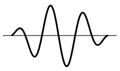
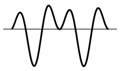
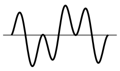
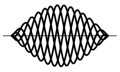
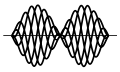
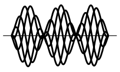
In this section we calculate the bunch spectrum for a parabolic distribution in the longitudinal plane. We want to describe the modes with a parabolic longitudinal shape of the bunch instead of a sin/cos shape. As suggested in [2] the pick-up signal has the form

$$f(t;j;n) = -4\frac{(n+1)^2}{bl^2}t^2 + 4(n+1)\frac{2j+1}{bl}t + 4j^2 - 4j(j+1) \qquad (1)$$

where bl[s] is the bunch length. Eq. (1) describes the bunch shape for the mode n in the portion of the bunch from $blj/(n+1)$ and $bl(j+1)/(n+1)$. So the full pick-up signal is given at the k^{th} turn by

$$F(t;n;k) = \sum_{j=0}^{n}(-1)^j\left[\theta(t - bl\frac{j}{n+1}) - \theta(t - bl\frac{j+1}{n+1})\right]\cdot f(t;j;n)e^{i(\omega_\xi + 2\pi k Q_x)} \qquad (2)$$

Table 2: Head tail modes for $\xi = -1$, for a parabolic distribution Eq.(2). We used the parameters of the PSB.

	$n = 0$	$n = 1$	$n = 2$
$k = 0$			
$k = 4$			
$k = 0,\cdots,4$			

where Q_x stands for the tune and $\omega_\xi = \xi Q_x \beta c/R\eta$. The power spectrum is a function of $h(\omega)$ where

$$h(\omega) = \int_{\mathbb{R}} dt e^{i\omega t}\sum_{j=0}^{n}(-1)^j\Big[\theta(t - bl\frac{j}{n+1}) - \theta(t - bl\frac{j+1}{n+1})\Big] f(t;j;n)\cdot e^{i(\omega_\xi + 2\pi k Q_x)}. \qquad (3)$$

Letting $A = bl\dfrac{j}{n+1}, B = bl\dfrac{j+1}{n+1}, \alpha = -4\dfrac{(n+1)^2}{bl^2}, \beta = 4(n+1)\dfrac{2j+1}{bl}, \gamma = 4j^2 - 4j(2j+1)$, and introducing the functions

$$\begin{cases} \mathcal{I}_1(\omega;j;n) = \alpha\left[Be^{i\omega B}\left(\dfrac{B^2}{i\omega} - 2\dfrac{B}{(i\omega)^2} + \dfrac{2}{(i\omega)^3}\right) - \right. \\ \qquad \left. -Ae^{i\omega A}\left(\dfrac{A^2}{i\omega} - 2\dfrac{A}{(i\omega)^2} + \dfrac{2}{(i\omega)^3}\right)\right] \\ \mathcal{I}_2(\omega;j;n) = \dfrac{\beta}{(i\omega)^2}\left[e^{i\omega B}(i\omega B - 1) - e^{i\omega A}(i\omega A - 1)\right] \\ \mathcal{I}_3(\omega;j;n) = \dfrac{\gamma}{i\omega}\left[e^{i\omega B} - e^{i\omega A}\right] \end{cases} \qquad (4)$$

$h(\omega)$ of Eq. (3) can be written as

$$h(\omega;n) = \sum_{j=0}^{n}(-1)^j\left[\mathcal{I}_1(\omega;j;n) + \mathcal{I}_2(\omega;j;n) + \mathcal{I}_3(\omega;j;n)\right] \qquad (5)$$

The instability occurs if, by the beam spectrum-impedance spectrum interaction, the imaginary part of the coherent frequency shift $\Delta\omega_n$ is positive:

$$\mathcal{I}m(\Delta\omega_n) > 0. \qquad (6)$$

The bunch spectrum is given by $g_n(\omega) = |h(\omega;n)|^2$. In Fig. 5 we show the bunch spectrum for the PSB parameters Table 1.

For a bunched beam the coherent tune shift for the mode n, involves a sum over the bunch spectrum multiplied by the impedance

$$\Delta\omega_n = -\frac{i}{n+1}\frac{e^2 N_b}{4\pi Q_x bl\gamma m_0 c}\frac{\sum_p Z_\perp(\omega_p)g_n(\omega_p - \omega_\xi)}{\sum_p g_n(\omega_p - \omega_\xi)}. \qquad (7)$$

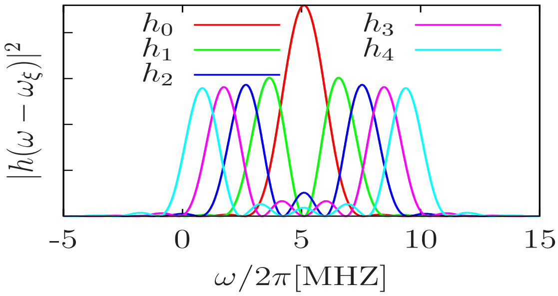


Figure 5: Bunch profile for the PSB bunch with a parabolic distribution. We used the data Table 1 and the beam parameters at time 378 ms.

We choose $Z(\omega) = (\text{sgn}\omega - i)\frac{R}{b^3}\sqrt{\frac{2\rho}{\epsilon_0|\omega|}}$, $\omega_p = (p + Q_x)\omega_0 + n\omega_s$, ω_s is the synchrotron frequency and we assumed $\rho = 10^{-6}\Omega$m and $b = 3.5$cm. For the instability growth rates $\tau_n = \mathcal{I}m(\Delta\omega_n)^{-1}$ we obtain the results in Fig. 6, while keeping the bunch intensity at $N_b = 5 \cdot 10^{12}$ ppb. As expected from the Fig. 5 the first unstable mode is the $n = 6$ one using a resistive wall impedance. Using different types of impedance will give a higher mode and produce an even larger discrepancy with the experimental observations.

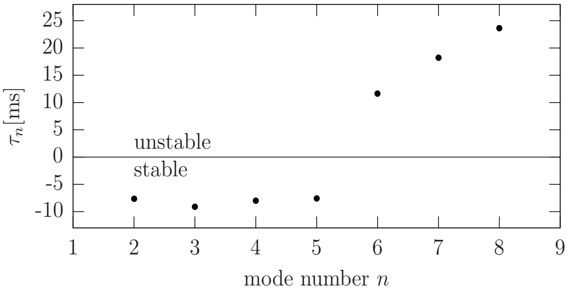


Figure 6: Growth rate of the PSB instability as a function of the mode number n.

MEASUREMENTS OF GROWTH RATES VS. BUNCH CURRENT

In this section we show the measurement of the growth rates as a function of the bunch intensity. As shown in Fig. 7 we observe a clear mode "3" (left) and a mode "2" (right) oscillation while increasing the bunch intensity. In fact from Fig. 8 we see that the higher the intensity, the shorter is the rise time, and in addition the mode number n is changing from $n = 3$ to $n = 2$. For the first instability at 378 ms we took a set of 5 pick-up data acquisitions for each current and we analyzed the results fitting the beam pick-up envelope with an exponential curve.

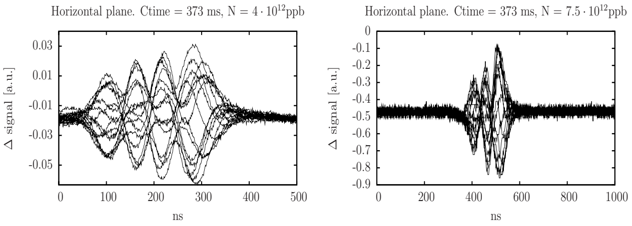


Figure 7: Pick-up signal at 378 ms.

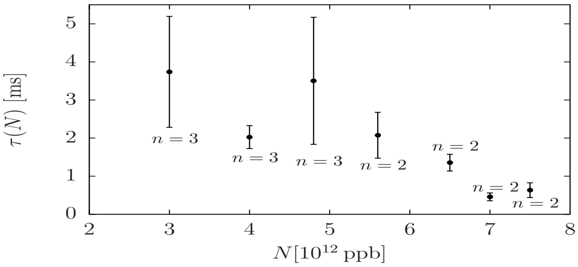


Figure 8: Growth time of the first (100 ms after the injection) PSB instability as a function of the beam intensity.

The observations and the theory together with HEAD-TAIL code [3] simulations show discrepancies concerning the numbers of nodes: while from one side numerical simulations agree with the theory, the data exhibit a lower number of nodes. Using a resistive wall impedance (which drives the smaller n mode unstable) the theory and the simulations show that the first unstable mode is $n = 6$.

CONCLUSIONS

We observed clear signs of head tail instability in the PSB and experimentally obtained the growth rates as a function of the bunch intensity. We calculated the modes for a parabolic bunch and found a discrepancy between theory/numerical simulations and experimental observations. This might be explained taking into account space-charge effects: as reported in recent literature [4] space charge forces might play a role in head tail instabilities when the ratio between space charge incoherent tune shift and the synchrotron tune is big $\Delta Q_{\text{s.c.}}/Q_s \gg 1$: in the PSB case under discussion we have $\Delta Q_{\text{s.c.}}/Q_s \approx 50$. Further numerical and experimental studies are ongoing.

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[3] G. Rumolo, F. Zimmermann, *Practical user guide for HEAD-TAIL*, SL-Note-2002-036-AP (2002).

[4] A. Burov, *Head-tail modes for strong space charge*, Phys. Rev. ST Accel. Beams **12**, 044202 (2009).