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A Compatible Approach to Temporal Description Logics

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1 Introduction

Temporal description logics (TDLs) have been studied by many researchers (see e.g., [1, 10] for surveys and [4, 2, 15] for recent or important results). These TDLs are, however, not compatible in the following sense: these are not embeddable into the standard (non-temporal) description logics (DLs), and hence the existing algorithms for testing satisfiability in the standard DLs are not available for these TDLs. Such a compatibility issue is important for obtaining reusable and practical algorithms for temporal reasoning in ontologies.

In this paper, two compatible TDLs, $\mathcal{XALC}$ and $\mathcal{BALC}_l$, are introduced by combining and modifying the description logic $\mathcal{ALC}$ [14] and *Prior's tomorrow tense logic* [12, 13]. $\mathcal{XALC}$ has the next-time operator, and $\mathcal{BALC}_l$ has some restricted versions of the next-time, any-time and some-time operators, in which the time domain is bounded by a positive integer l. *Semantical embedding theorems* of $\mathcal{XALC}$ and $\mathcal{BALC}_l$ into $\mathcal{ALC}$ are shown. By using these embedding theorems, the concept satisfiability problems for $\mathcal{XALC}$ and $\mathcal{BALC}_l$ are shown to be decidable. The complexities of the decision procedures for $\mathcal{XALC}$ and $\mathcal{BALC}_l$ are also shown to be the same complexity as that for $\mathcal{ALC}$. Next, tableau calculi, $\mathcal{TXALC}$ (for $\mathcal{XALC}$) and $\mathcal{TBALC}_l$ (for $\mathcal{BALC}_l$), are introduced, and *syntactical embedding theorems* of these calculi into a tableau calculus, $\mathcal{TALC}$ (for $\mathcal{ALC}$), are proved. The completeness theorems for $\mathcal{TXALC}$ and $\mathcal{TBALC}_l$ are proved by combining both the semantical and syntactical embedding theorems.

Prior's tomorrow tense logic, which is a base logic of $\mathcal{XALC}$ and $\mathcal{BALC}_l$, is regarded as the next-time fragment of *linear-time temporal logic* (LTL) [11], and hence $\mathcal{XALC}$ and $\mathcal{BALC}_l$ may also be familiar with many users of the existing LTL-based TDLs. The bounded temporal operators in $\mathcal{BALC}_l$ are, indeed, regarded as restricted versions of the corresponding LTL-operators. Although the standard temporal operators of LTL have an infinite (unbounded) time domain, i.e., the set ω of natural numbers, the bounded operators which are presented in this paper have a *bounded time domain* which is restricted by a fixed positive integer l, i.e., the set $\omega_l := \{x \in \omega \mid x \leq l\}$.

To restrict the time domain of temporal operators is not a new idea. Such an idea has been discussed [5–9]. It is known that to restrict the time domain is a technique to obtain a decidable or efficient fragment of first-order LTL [8].

Restricting the time domain implies not only some purely theoretical merits, but also some practical merits for describing temporal databases and planning specifications [6, 7], and for implementing an efficient model checking algorithm called *bounded model checking* [5]. Such practical merits are due to the fact that there are problems in computer science and artificial intelligence where only a finite fragment of the time sequence is of interest [6].

Finally in this section, other characters of $\mathcal{XALC}$ and $\mathcal{BALC}_l$ are summarized as follows: (1) the temporal operators in $\mathcal{XALC}$ and $\mathcal{BALC}_l$ are only applied to concepts and ABox assertions, (2) $\mathcal{XALC}$ and $\mathcal{BALC}_l$ are based on the assumptions of *rigid roles* and *rigid individual names*, i.e., the interpretations of atomic roles and individual names are not changed over time, and (3) $\mathcal{XALC}$ and $\mathcal{BALC}_l$ are based on the *constant domain* assumption, i.e., only one time domain is used in the logics.

2 Temporal Description Logic with Next-Time, $\mathcal{XALC}$

2.1 $\mathcal{ALC}$

The $\mathcal{ALC}$-language is constructed from atomic concepts, atomic roles, $\sqcap$ (intersection), $\sqcup$ (union), $\neg$ (classical negation or complement), $\forall R$ (universal concept quantification) and $\exists R$ (existential concept quantification). We use the letters A and A_i for atomic concepts, the letter R for atomic roles, and the letters C and D for concepts.

Definition 1 Concepts C *are defined by the following grammar:*

$$C ::= A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C$$

Definition 2 *An* interpretation $\mathcal{I}$ *is a pair* $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ *where*

1. $\Delta^{\mathcal{I}}$ *is a non-empty set,*
2. $\cdot^{\mathcal{I}}$ *is an interpretation function which assigns to every atomic concept A a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and to every atomic role R a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.*

The interpretation function is extended to concepts by the following inductive definitions:

1. $(\neg C)^{\mathcal{I}} := \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$,
2. $(C \sqcap D)^{\mathcal{I}} := C^{\mathcal{I}} \cap D^{\mathcal{I}}$,
3. $(C \sqcup D)^{\mathcal{I}} := C^{\mathcal{I}} \cup D^{\mathcal{I}}$,
4. $(\forall R.C)^{\mathcal{I}} := \{a \in \Delta^{\mathcal{I}} \mid \forall b\ [(a,b) \in R^{\mathcal{I}} \Rightarrow b \in C^{\mathcal{I}}]\}$,
5. $(\exists R.C)^{\mathcal{I}} := \{a \in \Delta^{\mathcal{I}} \mid \exists b\ [(a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}]\}$.

An interpretation $\mathcal{I}$ is a model of a concept C (denoted as $\mathcal{I} \models C$) if $C^{\mathcal{I}} \neq \emptyset$. A concept C is said to be satisfiable *in $\mathcal{ALC}$ if there exists an interpretation $\mathcal{I}$ such that $\mathcal{I} \models C$.*

The syntax of $\mathcal{ALC}$ is extended by a non-empty set N_I of individual names. We denote individual names by o, o_1, o_2, x, y and z.

Definition 3 *An* ABox *is a finite set of expressions of the form:* $C(o)$ *or* $R(o_1, o_2)$ *where* o, o_1 *and* o_2 *are in* N_I*,* C *is a concept, and* R *is an atomic role. An expression* $C(o)$ *or* $R(o_1, o_2)$ *is called an* ABox statement*. An interpretation* $\mathcal{I}$ *in Definition 2 is extended to apply also to individual names* o *such that* $o^{\mathcal{I}} \in \Delta^{\mathcal{I}}$*. Such an interpretation is a model of an ABox* $\mathcal{A}$ *if for every* $C(o) \in \mathcal{A}$*,* $o^{\mathcal{I}} \in C^{\mathcal{I}}$ *and for every* $R(o_1, o_2) \in \mathcal{A}$*,* $(o_1^{\mathcal{I}}, o_2^{\mathcal{I}}) \in R^{\mathcal{I}}$*. An ABox* $\mathcal{A}$ *is called* satisfiable *in* $\mathcal{ALC}$ *if it has a model.*

We adopt the following *unique name assumption*: for any $o_1, o_2 \in N_I$, if $o_1 \neq o_2$, then $o_1^{\mathcal{I}} \neq o_2^{\mathcal{I}}$.

Definition 4 *A* TBox *is a finite set of expressions of the form:* $C \sqsubseteq D$*. The elements of a TBox are called* TBox statments*. An interpretation* $\mathcal{I} := \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ *is called a* model *of* $C \sqsubseteq D$ *if* $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$*. An interpretation* $\mathcal{I}$ *is said to be a model of a TBox* $\mathcal{T}$ *if* $\mathcal{I}$ *is a model of every element of* $\mathcal{T}$*. A TBox* $\mathcal{T}$ *is called* satisfiable *in* $\mathcal{ALC}$ *if it has a model.*

Definition 5 *A knowledge base* Σ *is a pair* $(\mathcal{T}, \mathcal{A})$ *where* $\mathcal{T}$ *is a TBox and* $\mathcal{A}$ *is an ABox. An interpretation* $\mathcal{I}$ *is a model of* Σ *if* $\mathcal{I}$ *is a model of both* $\mathcal{T}$ *and* $\mathcal{A}$*. A knowledge base* Σ *is called* satisfiable *in* $\mathcal{ALC}$ *if it has a model.*

Since the satisfiability for an ABox, a TBox or a knowledge base can be reduced to the satisfiability for a concept [3], we focus on the concept satisfiability in the following discussion.

2.2 $\mathcal{XALC}$

Similar notions and terminologies for $\mathcal{ALC}$ are also used for $\mathcal{XALC}$. The symbol ω is used to represent the set of natural numbers. The $\mathcal{XALC}$-language is constructed from the $\mathcal{ALC}$-language by adding X (next-time operator). An expression $\mathrm{X}^n C$ is inductively defined by $\mathrm{X}^0 C := C$ and $\mathrm{X}^{n+1} C := \mathrm{X}\mathrm{X}^n C$.

Definition 6 Concepts C *are defined by the following grammar:*

$$C ::= A \mid \neg C \mid \mathrm{X}C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C$$

Definition 7 *A* temporal interpretation $\mathcal{TI}$ *is a structure* $\langle \Delta^{\mathcal{TI}}, \{\cdot^{\mathcal{I}^i}\}_{i \in \omega} \rangle$ *where*

1. $\Delta^{\mathcal{TI}}$ *is a non-empty set,*
2. *each* $\cdot^{\mathcal{I}^i}$ $(i \in \omega)$ *is an interpretation function which assigns to every atomic concept* A *a set* $A^{\mathcal{I}^i} \subseteq \Delta^{\mathcal{TI}}$ *and to every atomic role* R *a binary relation* $R^{\mathcal{I}^i} \subseteq \Delta^{\mathcal{TI}} \times \Delta^{\mathcal{TI}}$*,*
3. *for any atomic role* R *and any* $i, j \in \omega$*,* $R^{\mathcal{I}^i} = R^{\mathcal{I}^j}$*.*

The interpretation function is extended to concepts by the following inductive definitions:

1. $(\mathrm{X}C)^{\mathcal{I}^i} := C^{\mathcal{I}^{i+1}}$,

2. $(\neg C)^{\mathcal{I}^i} := \Delta^{\mathcal{TI}} \setminus C^{\mathcal{I}^i}$,
3. $(C \sqcap D)^{\mathcal{I}^i} := C^{\mathcal{I}^i} \cap D^{\mathcal{I}^i}$,
4. $(C \sqcup D)^{\mathcal{I}^i} := C^{\mathcal{I}^i} \cup D^{\mathcal{I}^i}$,
5. $(\forall R.C)^{\mathcal{I}^i} := \{a \in \Delta^{\mathcal{TI}} \mid \forall b\ [(a,b) \in R^{\mathcal{I}^i} \Rightarrow b \in C^{\mathcal{I}^i}]\}$,
6. $(\exists R.C)^{\mathcal{I}^i} := \{a \in \Delta^{\mathcal{TI}} \mid \exists b\ [(a,b) \in R^{\mathcal{I}^i} \wedge b \in C^{\mathcal{I}^i}]\}$.

For any $i \in \omega$, an expression $\mathcal{I}^i \models C$ is defined as $C^{\mathcal{I}^i} \neq \emptyset$. A temporal interpretation $\mathcal{TI} := \langle \Delta^{\mathcal{TI}}, \{\cdot^{\mathcal{I}^i}\}_{i \in \omega} \rangle$ is a model of a concept C (denoted as $\mathcal{TI} \models C$) if $\mathcal{I}^0 \models C$. A concept C is said to be satisfiable *in $\mathcal{XALC}$ if there exists a temporal interpretation $\mathcal{TI}$ such that $\mathcal{TI} \models C$.*

Definition 8 *A temporal interpretation $\mathcal{TI}$ in Definition 7 is extended to apply also to individual names o such that for any $i, j \in \omega$, $o^{\mathcal{I}^i} \in \Delta^{\mathcal{TI}}$ and $o^{\mathcal{I}^i} = o^{\mathcal{I}^j}$. Such a temporal interpretation is a model of an ABox $\mathcal{A}$ if for every $C(o) \in \mathcal{A}$, $o^{\mathcal{I}^0} \in C^{\mathcal{I}^0}$ and for every $R(o_1, o_2) \in \mathcal{A}$, $(o_1^{\mathcal{I}^0}, o_2^{\mathcal{I}^0}) \in R^{\mathcal{I}^0}$. Such a temporal interpretation is called a* model *of $C \sqsubseteq D$ if $C^{\mathcal{I}^0} \subseteq D^{\mathcal{I}^0}$. The satisfiability of ABox, a TBox or a knowledge base in $\mathcal{XALC}$ is defined in the same way as in $\mathcal{ALC}$.*

Remark that $\mathcal{XALC}$ is an extension of $\mathcal{ALC}$ since $\cdot^{\mathcal{I}^0}$ includes $\cdot^{\mathcal{I}}$. Remark also that $\mathcal{XALC}$ adopts the *constant domain assumption*, i.e., it has the single common domain $\Delta^{\mathcal{TI}}$, and the *rigid role and name assumption*, i.e., it satisfies the conditions: for any atomic role R, any individual name o and any $i, j \in \omega$, we have $R^{\mathcal{I}^i} = R^{\mathcal{I}^j}$ and $o^{\mathcal{I}^i} = o^{\mathcal{I}^j}$.

3 Semantical Embedding and Decidability

Definition 9 *Let N_C be a non-empty set of atomic concepts and N_C^i be the set $\{A^i \mid A \in N_C\}$ of atomic concepts where $A^0 = A$, i.e., $N_C^0 = N_C$.* [1] *Let N_R be a non-empty set of atomic roles and N_I be a non-empty set of individual names. The language $\mathcal{L}^x$ of $\mathcal{XALC}$ is defined using N_C, N_R, N_I, X, $\neg, \sqcap, \sqcup$, $\forall R$ and $\exists R$. The language $\mathcal{L}$ of $\mathcal{ALC}$ is obtained from $\mathcal{L}^x$ by adding $\bigcup_{i \in \omega} N_C^i$ and deleting* X.

A mapping f from $\mathcal{L}^x$ to $\mathcal{L}$ is defined inductively by

1. *for any $R \in N_R$ and any $o \in N_I$, $f(R) := R$ and $f(o) := o$,*
2. *for any $A \in N_C$, $f(\mathrm{X}^i A) := A^i \in N_C^i$, esp. $f(A) := A$,*
3. *For any $A(o) \in N_C$, $f(\mathrm{X}^i A(o)) := A^i(f(o)) \in N_C^i$, esp. $f(A(o)) := A(f(o))$,*
4. *$f(\mathrm{X}^i \neg C) := \neg f(\mathrm{X}^i C)$,*
5. *$f(\mathrm{X}^i (C \sharp D)) := f(\mathrm{X}^i C) \sharp f(\mathrm{X}^i D)$ where $\sharp \in \{\sqcap, \sqcup\}$,*
6. *$f(\mathrm{X}^i \forall R.C) := \forall f(R).f(\mathrm{X}^i C)$,*
7. *$f(\mathrm{X}^i \exists R.C) := \exists f(R).f(\mathrm{X}^i C)$.*

Lemma 10 *Let f be the mapping defined in Definition 9. For any temporal interpretation $\mathcal{TI} := \langle \Delta^{\mathcal{TI}}, \{\cdot^{\mathcal{I}^i}\}_{i \in \omega} \rangle$ of $\mathcal{XALC}$, we can construct an interpretation $\mathcal{I} := \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ of $\mathcal{ALC}$ such that for any concept C in $\mathcal{L}^x$ and any $i \in \omega$,*

[1] A can include individual names, i.e., A can be $A(o)$ for any $o \in N_I$.

$C^{\mathcal{I}^i} = f(\mathrm{X}^i C)^{\mathcal{I}}$.

Proof. Let N_C be a non-empty set of atomic concepts and N_C^i be the set $\{A^i \mid A \in N_C\}$ of atomic concepts where $A^0 = A$. Let N_R and N_I be sets of atomic roles and individual names, respectively.

Suppose that $\mathcal{TI}$ is a temporal interpretation $\langle \Delta^{\mathcal{TI}}, \{\cdot^{\mathcal{I}^i}\}_{i\in\omega}\rangle$ where

1. $\Delta^{\mathcal{TI}}$ is a non-empty set,
2. each $\cdot^{\mathcal{I}^i}$ $(i \in \omega)$ is an interpretation function which assigns to every atomic concept $A \in N_C$ a set $A^{\mathcal{I}^i} \subseteq \Delta^{\mathcal{TI}}$, to every atomic role $R \in N_R$ a binary relation $R^{\mathcal{I}^i} \subseteq \Delta^{\mathcal{TI}} \times \Delta^{\mathcal{TI}}$ and to every individual name $o \in N_I$ an element $o^{\mathcal{I}^i} \in \Delta^{\mathcal{TI}}$,
3. for any $R \in N_R$, any $o \in N_I$ and any $i, j \in \omega$, $R^{\mathcal{I}^i} = R^{\mathcal{I}^j}$ and $o^{\mathcal{I}^i} = o^{\mathcal{I}^j}$.

Suppose that $\mathcal{I}$ is an interpretation $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}\rangle$ where

1. $\Delta^{\mathcal{I}}$ is a non-empty set such that $\Delta^{\mathcal{I}} = \Delta^{\mathcal{TI}}$,
2. $\cdot^{\mathcal{I}}$ is an interpretation function which assigns to every atomic concept $A \in \bigcup_{i\in\omega} N_C^i$ a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, to every atomic role $R \in N_R$ a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and to every individual name $o \in N_I$ an element $o^{\mathcal{I}} \in \Delta^{\mathcal{I}}$,
3. for any $R \in N_R$, any $o \in N_I$ and any $i \in \omega$, $R^{\mathcal{I}} = R^{\mathcal{I}^i}$ and $o^{\mathcal{I}} = o^{\mathcal{I}^i}$.

Suppose moreover that $\mathcal{TI}$ and $\mathcal{I}$ satisfy the following condition: for any $A \in N_C$, any $oinN_I$ and any $i \in \omega$,

$A^{\mathcal{I}^i} = (A^i)^{\mathcal{I}}$ and $(A(o))^{\mathcal{I}^i} = (A^i(o))^{\mathcal{I}}$.

The lemma is then proved by induction on the complexity of C. The base step is obvious. We show some cases in the induction step below.

Case $C \equiv \neg D$: We obtain: $a \in (\neg D)^{\mathcal{I}^i}$ iff $a \in \Delta^{\mathcal{TI}} \setminus D^{\mathcal{I}^i}$ iff $a \in \Delta^{\mathcal{I}} \setminus D^{\mathcal{I}^i}$ (by the condition $\Delta^{\mathcal{TI}} = \Delta^{\mathcal{I}}$) iff $a \in \Delta^{\mathcal{I}} \setminus f(\mathrm{X}^i D)^{\mathcal{I}}$ (by induction hypothesis) iff $a \in (\neg f(\mathrm{X}^i D))^{\mathcal{I}}$ iff $a \in f(\mathrm{X}^i \neg D)^{\mathcal{I}}$ (by the definition of f).

Case $C \equiv \mathrm{X}D$: We obtain: $a \in (\mathrm{X}D)^{\mathcal{I}^i}$ iff $a \in D^{\mathcal{I}^{i+1}}$ iff $a \in f(\mathrm{X}^{i+1} D)^{\mathcal{I}}$ (by induction hypothesis) iff $a \in f(\mathrm{X}^i \mathrm{X} D)^{\mathcal{I}}$.

Case $C \equiv \forall R.D$: We obtain:

$d \in (\forall R.D)^{\mathcal{I}^i}$
iff $d \in \{a \in \Delta^{\mathcal{TI}} \mid \forall b\, [(a,b) \in R^{\mathcal{I}^i} \Rightarrow b \in D^{\mathcal{I}^i}]\}$
iff $d \in \{a \in \Delta^{\mathcal{I}} \mid \forall b\, [(a,b) \in R^{\mathcal{I}} \Rightarrow b \in D^{\mathcal{I}^i}]\}$ (by the conditions $\Delta^{\mathcal{TI}} = \Delta^{\mathcal{I}}$ and $R^{\mathcal{I}^i} = R^{\mathcal{I}}$)
iff $d \in \{a \in \Delta^{\mathcal{I}} \mid \forall b\, [(a,b) \in R^{\mathcal{I}} \Rightarrow b \in f(\mathrm{X}^i D)^{\mathcal{I}}]\}$ (by induction hypothesis)
iff $d \in (\forall R. f(\mathrm{X}^i D))^{\mathcal{I}}$
iff $d \in (\forall f(R). f(\mathrm{X}^i D))^{\mathcal{I}}$ (by the definition of f)
iff $d \in f(\mathrm{X}^i \forall R.D)^{\mathcal{I}}$ (by the definition of f).

∎

Lemma 11 *Let f be the mapping defined in Definition 9. For any temporal interpretation $\mathcal{TI} := \langle \Delta^{\mathcal{TI}}, \{\cdot^{\mathcal{I}^i}\}_{i\in\omega}\rangle$ of $\mathcal{XALC}$, we can construct an interpretation $\mathcal{I} := \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}\rangle$ of $\mathcal{ALC}$ such that for any concept C in $\mathcal{L}^x$ and any $i \in \omega$,*

$\mathcal{I}^i \models C$ *iff* $\mathcal{I} \models f(\mathrm{X}^i C)$.

Proof. We obtain: $\mathcal{I}^i \models C$ iff $C^{\mathcal{I}^i} \neq \emptyset$ iff $f(\mathrm{X}^i C)^{\mathcal{I}} \neq \emptyset$ (by Lemma 10) iff $\mathcal{I} \models f(\mathrm{X}^i C)$. ∎

Lemma 12 *Let f be the mapping defined in Definition 9. For any interpretation $\mathcal{I} := \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ of $\mathcal{ALC}$, we can construct a temporal interpretation $\mathcal{TI} := \langle \Delta^{\mathcal{TI}}, \{\cdot^{\mathcal{I}^i}\}_{i\in\omega} \rangle$ of $\mathcal{XALC}$ such that for any concept C in $\mathcal{L}^x$ and any $i \in \omega$,*

$\mathcal{I} \models f(\mathrm{X}^i C)$ *iff* $\mathcal{I}^i \models C$.

Proof. Similar to the proof of Lemma 11. ∎

Theorem 13 (Semantical embedding) *Let f be the mapping defined in Definition 9. For any concept C,*

C is satisfiable in $\mathcal{XALC}$ iff $f(C)$ is satisfiable in $\mathcal{ALC}$.

Proof. By Lemmas 11 and 12. ∎

Theorem 14 (Decidability) *The concept satisfiability problem for $\mathcal{XALC}$ is decidable.*

Proof. By decidability of the satisfiability problem for $\mathcal{ALC}$, for each concept C of $\mathcal{XALC}$, it is possible to decide if $f(C)$ is satisfiable in $\mathcal{ALC}$. Then, by Theorem 13, the satisfiability problem for $\mathcal{XALC}$ is decidable. ∎

The satisfiability problems of a TBox, an ABox and a knowledge base for $\mathcal{XALC}$ are also shown to be decidable.

Since f is a polynomial-time reduction, the complexities of the satisfiability problems of a TBox, an ABox and a knowledge base for $\mathcal{XALC}$ can be reduced to those for $\mathcal{ALC}$, i.e., the complexities of the problems for $\mathcal{XALC}$ are the same as those for $\mathcal{ALC}$. For example, the satisfiability problems of an acyclic TBox and a general TBox for $\mathcal{XALC}$ are PSPACE-complete and EXPTIME-complete, respectively. For the concept satisfiability problem for $\mathcal{XALC}$,

the existing tableau algorithms for $\mathcal{ALC}$ are applicable by using the translation f with Theorem 13.

4 Syntactical Embedding and Completeness

From a purely theoretical or logical point of view, a sound and complete axiomatization is required for the underlying semantics. In this section, we thus give such a tableau calculus $\mathcal{TXALC}$ for $\mathcal{XALC}$.

Definition 15 *A concept is called a* negation normal form *(NNF) if the classical negation connective $\neg$ occurs only in front of atomic concepts.*

Let $C(x)$ be a concept in NNF. In order to test satisfiability of $C(x)$, the tableau algorithm starts with the ABox $\mathcal{A} = \{C(x)\}$, and applies the inference rules of a tableau calculus to the ABox until no more rules apply.

Definition 16 ($\mathcal{TALC}$) *Let $\mathcal{A}$ be an ABox that consists only of NNF-concepts. The inference rules for the tableau calculus $\mathcal{TALC}$ for $\mathcal{ALC}$ are of the form:*

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{C_1(x), C_2(x)\}} \ (\sqcap)$$

where $(C_1 \sqcap C_2)(x) \in \mathcal{A}$, $C_1(x) \notin \mathcal{A}$ or $C_2(x) \notin \mathcal{A}$,

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{C_1(x)\} \quad | \quad \mathcal{A} \cup \{C_2(x)\}} \ (\sqcup)$$

where $(C_1 \sqcup C_2)(x) \in \mathcal{A}$ and $[C_1(x) \notin \mathcal{A}$ and $C_2(x) \notin \mathcal{A}]$,

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{C(y)\}} \ (\forall R)$$

where $(\forall R.C)(x) \in \mathcal{A}$, $R(x, y) \in \mathcal{A}$ and $C(y) \notin \mathcal{A}$,

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{C(y), R(x, y)\}} \ (\exists R)$$

where $(\exists R.C)(x) \in \mathcal{A}$, there is no individual name z such that $C(z) \in \mathcal{A}$ and $R(x, z) \in \mathcal{A}$, and y is an individual name not occurring in $\mathcal{A}$.

Definition 17 *Let $\mathcal{A}$ be an ABox that consists only of NNF-concepts. Then, $\mathcal{A}$ is called* complete *if there is no more rules apply to $\mathcal{A}$. $\mathcal{A}$ is called* clash *if $\{A(x), \neg A(x)\} \subseteq \mathcal{A}$ for some atomic concept $A(x)$. A tree produced by a tableau calculus from $\mathcal{A}$ is called* complete *if all the nodes in the tree are complete. A branch of a tree produced by a tableau calculus from $\mathcal{A}$ is called* clash-free *if all its nodes are not clash.*

The following theorem is known.

Theorem 18 (Completeness) *For any $\mathcal{ALC}$-concept C in NNF, $\mathcal{TALC}$ produces a complete tree with a clash-free branch from the Abox $\{C\}$ iff C is satisfiable in $\mathcal{ALC}$.*

The way of obtaining NNFs for $\mathcal{XALC}$-concepts is almost the same as that for $\mathcal{ALC}$-concepts, except that we also use the law: $\neg \mathrm{X}C \leftrightarrow \mathrm{X}\neg C$, which is justified by the fact: $(\neg \mathrm{X}C)^{\mathcal{I}^i} = (\mathrm{X}\neg C)^{\mathcal{I}^i}$ for any $i \in \omega$.

Definition 19 ($\mathcal{TXALC}$) *Let $\mathcal{A}$ be an ABox that consists only of NNF-concepts. The inference rules for the tableau calculus $\mathcal{TXALC}$ for $\mathcal{XALC}$ are of the form:*

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{\mathrm{X}^i C_1(x), \mathrm{X}^i C_2(x)\}} \ (\mathrm{X}\sqcap)$$

where $\mathrm{X}^i(C_1 \sqcap C_2)(x) \in \mathcal{A}$*,* $\mathrm{X}^i C_1(x) \notin \mathcal{A}$ *or* $\mathrm{X}^i C_2(x) \notin \mathcal{A}$*,*

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{\mathrm{X}^i C_1(x)\} \quad | \quad \mathcal{A} \cup \{\mathrm{X}^i C_2(x)\}} \ (\mathrm{X}\sqcup)$$

where $\mathrm{X}^i(C_1 \sqcup C_2)(x) \in \mathcal{A}$ *and* $[\mathrm{X}^i C_1(x) \notin \mathcal{A}$ *and* $\mathrm{X}^i C_2(x) \notin \mathcal{A}]$*,*

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{\mathrm{X}^i C(y)\}} \ (\mathrm{X}\forall R)$$

where $(\mathrm{X}^i \forall R.C)(x) \in \mathcal{A}$*,* $R(x,y) \in \mathcal{A}$ *and* $\mathrm{X}^i C(y) \notin \mathcal{A}$*,*

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{\mathrm{X}^i C(y), R(x,y)\}} \ (\mathrm{X}\exists R)$$

where $(\mathrm{X}^i \exists R.C)(x) \in \mathcal{A}$*, there is no individual name* z *such that* $\mathrm{X}^i C(z) \in \mathcal{A}$ *and* $R(x,z) \in \mathcal{A}$*, and* y *is an individual name not occurring in* $\mathcal{A}$*.*

An expression $f(\mathcal{A})$ denotes the set $\{f(\alpha) \mid \alpha \in \mathcal{A}\}$.

Theorem 20 (Syntactical embedding) *Let* $\mathcal{A}$ *be an ABox that consists only of NNF-concepts in* $\mathcal{L}^x$*, and* f *be the mapping defined in Definition 9. Then:*

> $\mathcal{TXALC}$ *produces a complete tree with a clash-free branch from* $\mathcal{A}$ *iff*
> $\mathcal{TALC}$ *produces a complete tree with a clash-free branch from* $f(\mathcal{A})$

Proof. ($\Longrightarrow$): By induction on the complete trees T with a clash-free branch from $\mathcal{A}$ in $\mathcal{TXALC}$. ($\Longleftarrow$): By induction on the complete trees T' with a clash-free branch from $f(\mathcal{A})$ in $\mathcal{TALC}$. ∎

Theorem 21 (Completeness) *For any* $\mathcal{XALC}$*-concept* C *in NNF,* $\mathcal{TXALC}$ *produces a complete tree with a clash-free branch from the Abox* $\{C\}$ *iff* C *is satisfiable in* $\mathcal{XALC}$*.*

Proof. Let C be a $\mathcal{XALC}$-concept in NNF. Then, we obtain:

$\mathcal{TXALC}$ produces a complete tree with a clash-free branch from $\{C\}$
iff $\mathcal{TALC}$ produces a complete tree with a clash-free branch from $\{f(C)\}$ (by Theorem 20)
iff $f(C)$ is satisfiable in $\mathcal{ALC}$ (by Theorem 18)
iff C is satisfiable in $\mathcal{XALC}$ (by Theorem 13).

∎

5 Temporal Description Logic with Bounded-Time, $\mathcal{BALC}_l$

5.1 $\mathcal{BALC}_l$

Similar notions and terminologies for $\mathcal{XALC}$ are also used for $\mathcal{BALC}_l$. The symbol $\geq$ or $\leq$ is used to represent a linear order on ω. In the following discussion, l is fixed as a certain positive integer. The $\mathcal{BALC}_l$-language is constructed from the $\mathcal{XALC}$-language by adding G (any-time operator) and F (some-time operator). Remark that the temporal operators X, G and F used in $\mathcal{BALC}_l$ are interpreted as some l-bounded versions of the original operators.

Definition 22 Concepts C *are defined by the following grammar:*

$$C ::= A \mid \neg C \mid \mathrm{X}C \mid \mathrm{G}C \mid \mathrm{F}C \mid C \sqcap C \mid C \sqcup C \mid \forall R.C \mid \exists R.C$$

Definition 23 *A* bounded-time interpretation $\mathcal{BI}$ *is the same as a temporal structure, i.e., it is obtained from a temporal structure* $\langle \Delta^{\mathcal{TI}}, \{\cdot^{\mathcal{I}^i}\}_{i\in\omega}\rangle$ *by replacing the notation* $\Delta^{\mathcal{TI}}$ *with the notation* $\Delta^{\mathcal{BT}}$*. The interpretation function is extended to concepts by induction on concepts. The definitions of the interpretation function is obtained from the conditions in Definitions 7 and 8 by replacing* $\Delta^{\mathcal{TI}}$ *with* $\Delta^{\mathcal{BT}}$*, deleting the condition 1 in Definition 7, and adding the following conditions:*

1. *for any* $i \leq l-1$, $(\mathrm{X}C)^{\mathcal{I}^i} := C^{\mathcal{I}^{i+1}}$,
2. *for any* $i \geq l$, $(\mathrm{X}C)^{\mathcal{I}^i} := C^{\mathcal{I}^l}$,
3. *for any* $m \in \omega$, $(\mathrm{X}C)^{\mathcal{I}^{l+m}} := C^{\mathcal{I}^l}$,
4. $(\mathrm{G}C)^{\mathcal{I}^i} := C^{\mathcal{I}^i} \cap C^{\mathcal{I}^{i+1}} \cap \cdots \cap C^{\mathcal{I}^{i+l}}$,
5. $(\mathrm{F}C)^{\mathcal{I}^i} := C^{\mathcal{I}^i} \cup C^{\mathcal{I}^{i+1}} \cup \cdots \cup C^{\mathcal{I}^{i+l}}$.

The notions of satisfiability etc. are defined in the same way as in $\mathcal{XALC}$*.*

Remark that the new conditions for the interpretation function in Definition 23 are intended to have the following axiom schemes:

1. for any $m \in \omega$, $\mathrm{X}^{l+m}C \leftrightarrow \mathrm{X}^l C$,
2. $\mathrm{G}C \leftrightarrow C \sqcap \mathrm{X}C \sqcap \cdots \sqcap \mathrm{X}^l C$,
3. $\mathrm{F}C \leftrightarrow C \sqcup \mathrm{X}C \sqcup \cdots \sqcup \mathrm{X}^l C$,
4. $\neg\mathrm{G}C \leftrightarrow \mathrm{F}\neg C$,
5. $\neg\mathrm{F}C \leftrightarrow \mathrm{G}\neg C$.

Remark also that the new conditions in Definition 23 are the l-bounded time versions of the following standard non-restricted conditions:

1. $(\mathrm{X}C)^{\mathcal{I}^i} := C^{\mathcal{I}^{i+1}}$,
2. $(\mathrm{G}C)^{\mathcal{I}^i} := \bigcap\{C^{\mathcal{I}^j} \mid i \leq j \in \omega\}$,
3. $(\mathrm{F}C)^{\mathcal{I}^i} := \bigcup\{C^{\mathcal{I}^j} \mid i \leq j \in \omega\}$.

These non-restricted conditions imply a standard LTL-based temporal description logic.

5.2 Semantical Embedding and Decidability

Definition 24 *The language $\mathcal{L}^b$ of $\mathcal{BALC}_l$ is obtained from the language $\mathcal{L}^x$ in Definition 9 by adding* G *and* F*. The language $\mathcal{L}$ of $\mathcal{ALC}$ is defined as the same language in Definition 9. A mapping f from $\mathcal{L}^b$ to $\mathcal{L}$ is obtained from the mapping defined in Definition 9 by adding the following conditions:*

1. *for any $m \geq l$, $f(\mathrm{X}^m\mathrm{X}C) := f(\mathrm{X}^l C)$,*
2. *$f(\mathrm{X}^i\mathrm{G}C) := f(\mathrm{X}^i C) \sqcap f(\mathrm{X}^{i+1}C) \sqcap \cdots \sqcap f(\mathrm{X}^{i+l}C)$,*
3. *$f(\mathrm{X}^i\mathrm{F}C) := f(\mathrm{X}^i C) \sqcup f(\mathrm{X}^{i+1}C) \sqcup \cdots \sqcup f(\mathrm{X}^{i+l}C)$.*

Lemma 25 *Let f be the mapping defined in Definition 24. For any bounded-time interpretation $\mathcal{BI} := \langle \Delta^{\mathcal{BI}}, \{\cdot^{\mathcal{I}^i}\}_{i\in\omega}\rangle$ of $\mathcal{BALC}_l$, we can construct an interpretation $\mathcal{I} := \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}\rangle$ of $\mathcal{ALC}$ such that for any concept C in $\mathcal{L}^b$ and any $i \in \omega$,*

$$C^{\mathcal{I}^i} = f(\mathrm{X}^i C)^{\mathcal{I}}.$$

Proof. Similar to the proof of Lemma 10 by replacing $\Delta^{\mathcal{TI}}$ with $\Delta^{\mathcal{BI}}$. ∎

We then obtain the key lemmas which correspond to Lemmas 11 and 12, and hence obtain the following theorems.

Theorem 26 (Semantical embedding) *Let f be the mapping defined in Definition 24. For any concept C,*

C is satisfiable in $\mathcal{BALC}_l$ iff $f(C)$ is satisfiable in $\mathcal{ALC}$.

Theorem 27 (Decidability) *The concept satisfiability problem for $\mathcal{BALC}_l$ is decidable.*

The complexity of the decision procedure for concept satisfiability in $\mathcal{BALC}_l$ is the same as that in $\mathcal{ALC}$.

5.3 Syntactical Embedding and Completeness

The way of obtaining NNFs for $\mathcal{BALC}_l$-concepts is almost the same as that for $\mathcal{XALC}$-concepts, except that we also use the laws: $\neg\mathrm{G}C \leftrightarrow \mathrm{F}\neg C$ and $\neg\mathrm{F}C \leftrightarrow \mathrm{G}\neg C$.

Definition 28 ($\mathcal{TBALC}_l$) *Let $\mathcal{A}$ be an ABox that consists only of NNF-concepts.*

The inference rules for the tableau calculus $\mathcal{TBALC}_l$ for $\mathcal{BALC}_l$ are of the form:

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{\mathrm{X}^l C(x)\}}\ (\mathrm{X})$$

where $\mathrm{X}^{l+m}C(x) \in \mathcal{A}$ for any $m \in \omega$,

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{\mathrm{X}^i C(x), \mathrm{X}^{i+1}C(x), ..., \mathrm{X}^{i+l}C(x)\}}\ (\mathrm{G})$$

where $\mathrm{X}^i\mathrm{G}C(x) \in \mathcal{A}$ *and* $\mathrm{X}^{i+j}C(x) \notin \mathcal{A}$ *for some* $j \in \omega_l$,

$$\frac{\mathcal{A}}{\mathcal{A} \cup \{\mathrm{X}^i C(x)\} \quad | \quad \mathcal{A} \cup \{\mathrm{X}^{i+1} C(x)\} \quad | \quad \cdots \quad | \quad \mathcal{A} \cup \{\mathrm{X}^{i+l} C(x)\}} \ (\mathrm{F})$$

where $\mathrm{X}^i\mathrm{F}C(x) \in \mathcal{A}$ *and* $[\mathrm{X}^iC(x) \notin \mathcal{A}$, $\mathrm{X}^{i+1}C(x) \notin \mathcal{A}$, ... , *and* $\mathrm{X}^{i+l}C(x) \notin \mathcal{A}]$.

Theorem 29 (Syntactical embedding) *Let $\mathcal{A}$ be an ABox that consists only of NNF-concepts in $\mathcal{L}^b$, and f be the mapping defined in Definition 24. Then:*

$\mathcal{TBALC}_l$ produces a complete tree with a clash-free branch from $\mathcal{A}$ iff $\mathcal{TALC}$ produces a complete tree with a clash-free branch from $f(\mathcal{A})$

Theorem 30 (Completeness) *For any $\mathcal{BALC}_l$-concept C in NNF, $\mathcal{TBALC}_l$ produces a complete tree with a clash-free branch from the Abox $\{C\}$ iff C is satisfiable in $\mathcal{BALC}_l$.*

6 Related Works

Some recent works concerned with TDLs are surveyed below. In [4], Baader et al. considered the case where linear-time temporal operators are allowed to occur only in front of DL axioms over $\mathcal{ALC}$ (i.e., ABox assertions and general concept inclusion axioms), but not inside of concepts descriptions. They showed that reasoning in the presence of rigid roles becomes considerably simpler in this setting. The decision procedures described in [4] were developed for the purpose of showing worst-case complexity upper bounds: with rigid roles, satisfiability is 2EXPTIME-complete, without rigid roles, the complexity decreases further to EXPTIME-complete (i.e., the same complexity as reasoning in $\mathcal{ALC}$ alone). They also considered two other ways of decreasing the complexity of satisfiability to EXPTIME. Compared with [4], our approach is mainly intended to obtain: (1) reusable TDLs, i.e., the existing $\mathcal{ALC}$-based satisfiability testing algorithms are reusable and (2) "light-weight" TDLs, i.e., the complexity of satisfiability testing is the same as that of $\mathcal{ALC}$.

In [2], Baader et al. extended the known approaches to LTL runtime verification. In this approach, they used an $\mathcal{ALC}$-based temporal description logic, $\mathcal{ALC}$-LTL, instead of the propositional LTL. They also considered the case where states may be described in an incomplete way by $\mathcal{ALC}$-ABoxs, instead of assuming that the observed system behavior provides us with complete information about the states of the system. Compared with [2], applications of our proposed logics have not yet been proposed. In particular, it is not clear if the boundedness of the time domain in $\mathcal{BALC}_l$ is really useful for ontological reasoning.

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