

D15-2002-125

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**METHOD OF DETERMINATION
OF MUON CATALYZED FUSION PARAMETERS
IN H-T MIXTURE**

Submitted to «European Physical Journal D»

1 Introduction

The pt -reaction is one of the least known of all processes of muon catalyzed fusion (μCF) in the mixture of hydrogen isotopes. It is very important to gain information on reaction characteristics of all muonic processes in the H-T mixture (e.g., the rate of muon transfer from $p\mu$ atom to triton, the rate of transition between hyperfine levels of $t\mu$ atoms, the rate of formation of the $pt\mu$ molecule, and the rate of nuclear synthesis in it) to interpret correctly the results of experiments in the triple mixture of hydrogen isotopes H-D-T and to describe the kinetics of all processes occurring in the mixture. From the theoretical point of view, the experiments investigating μCF processes in hydrogen-tritium mixture will allow one to test an algorithm describing a three-body system of particles interacting according to Coulomb rule.

It is necessary to emphasize the importance of the μCF study in H-T mixture in order to obtain the information about characteristics of pt -reaction at ultra low energy range ($\sim keV$)¹.

The investigation of the reaction between light nuclei at infra-low energies ($\sim keV$) is very important for verification of fundamental symmetries in strong interactions [1-3], the contribution of meson exchange currents [4-7] and to solve some astrophysical problems [8-10]

With classical accelerators, it is practically impossible to study the pt -reaction in direct collision at very low energies ($\sim keV$) because the cross sections of it and intensities of proton (triton) beams are very small [11-14].

At present, there are only two experiments [15-16] that investigate characteristics of μCF in an H-T mixture². Only one [15] was performed with an H-T mixture and the second [16] with triple mixture H-D-T (no doubt, exact measurements of the parameters of muon catalyzed fusion of the pt reaction can be achieved only with the double mixture H-T).

In this paper we give the detail description of the kinetics of μCF for what is essential for data analysis of experimental results with the H-T mixture. Besides, the aim of this paper is to choose optimal conditions of the experiment for precision investigation of muonic processes in the H-T mixture.

2 Kinetics

The scheme of μ -atomic and μ -molecular processes in the H-T mixture after the negative muons stopped, is shown in Fig.1. As a result of the muon transition from $p\mu$ -atom to tritium nuclei

¹ In nuclear fusion reactions in the muonic molecules of hydrogen isotopes the astrophysical range of energies ($\sim keV$) is realized [11-14].

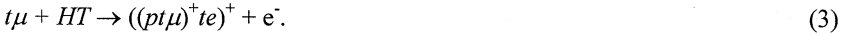
² Recently, on the TRIUMF meson facility the investigation of the processes of muonic atom ($p\mu$, $d\mu$, $t\mu$) interaction with hydrogen lattice at temperature of 3K have been performed. The preliminary results of $pt\mu$ molecule formation rate have obtained [17-18].

$$p\mu + t \rightarrow t\mu + p + 183 \text{ eV} \quad (1)$$

$t\mu$ atoms are formed with a kinetic energy of about 45 eV (the scheme on Fig.1 corresponds to a very low tritium concentration in the H-T mixture ($\leq 1\%$), which allows one to neglect direct capture of the muon by tritium). The ground state of the $t\mu$ atom is split into two hyperfine structure levels with $\vec{F} = \vec{S}_t + \vec{S}_\mu$ being the total spin of the $t\mu$ atom ($S_t = S_\mu = 1/2$ are the spins of triton and muon, respectively) equal to $F=1$ ($(\vec{S}_t, \vec{S}_\mu) \equiv (\uparrow\uparrow)$) and $F=0$ ($(\vec{S}_t, \vec{S}_\mu) \equiv (\uparrow\downarrow)$). The energy of hyperfine splitting of the $t\mu$ atom equals 0.24 eV. The initial population of hyperfine levels is assumed statistically to be:

$$\eta = 3/4 (F = 1), \quad \eta = 1/4 (F = 0).$$

In a collision of $t\mu$ atoms with H_2 or HT molecules



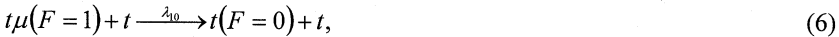
the $pt\mu$ molecule is formed by the electric dipole transition EI in excited state (J, ν) , (where J, ν are rotational and vibrational quantum numbers of the pt – system in $pt\mu$ molecule, respectively).

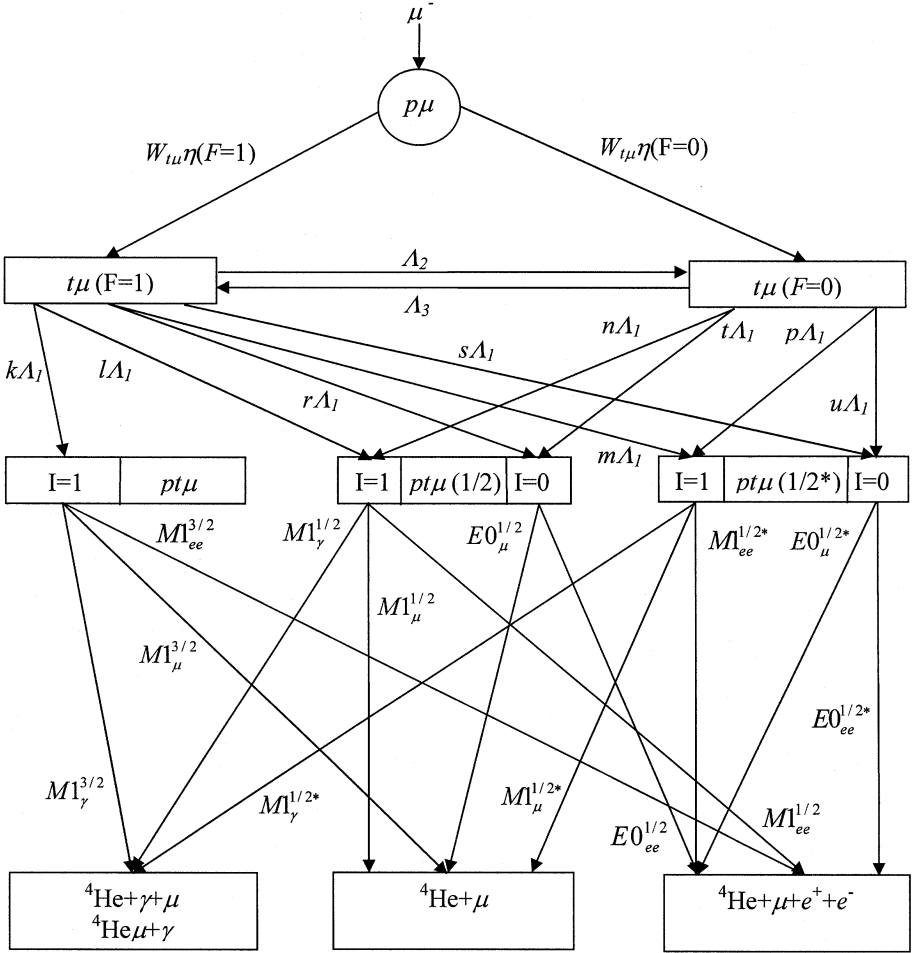
At the collision of the $t\mu$ atom with triton from the T_2 or HT molecule, the formation of a $tt\mu$ molecule is possible



due to EI dipol transition.

The competitive processes to the formation of a $pt\mu$ molecule are: free muonic decay ($\mu \rightarrow e^- + \nu_\mu + \bar{\nu}_e$), $tt\mu$ molecule formation (processes (4) and (5)) and the $t\mu$ atom transition between hyperfine levels.





$$\begin{aligned}
 M_{\gamma}^{3/2} &= W_{3/2}^1 \lambda_{f,\gamma}^{\mu} (I=1), M_{\mu}^{3/2} = W_{3/2}^1 \lambda_{f,\mu}^{\mu} (I=1), M_{ee}^{3/2} = W_{3/2}^1 \lambda_{f,ee}^{\mu} (I=1); \\
 M_{\gamma}^{1/2} &= W_{1/2}^1 \lambda_{f,\gamma}^{\mu} (I=1), M_{\mu}^{1/2} = W_{1/2}^1 \lambda_{f,\mu}^{\mu} (I=1), M_{ee}^{1/2} = W_{1/2}^1 \lambda_{f,ee}^{\mu} (I=1), \\
 E_{\mu}^{1/2} &= W_{1/2}^0 \lambda_{f,\mu}^{\mu} (I=0), E_{ee}^{1/2} = W_{1/2}^0 \lambda_{f,ee}^{\mu} (I=0); \\
 M_{\gamma}^{1/2*} &= W_{1/2*}^1 \lambda_{f,\gamma}^{\mu} (I=1), M_{\mu}^{1/2*} = W_{1/2*}^1 \lambda_{f,\mu}^{\mu} (I=1), M_{ee}^{1/2*} = W_{1/2*}^1 \lambda_{f,ee}^{\mu} (I=1), \\
 E_{\mu}^{1/2*} &= W_{1/2*}^0 \lambda_{f,\mu}^{\mu} (I=0), E_{ee}^{1/2*} = W_{1/2*}^0 \lambda_{f,ee}^{\mu} (I=0); \\
 \Lambda_I &= \phi C_p \lambda_{p\mu}, \Lambda_2 = C_t \phi \lambda_{10}, \Lambda_3 = C_t \phi \lambda_{01}.
 \end{aligned}$$

Fig.1. Kinetics of the muonic processes in H-T mixture.

The transition (7) is possible only when the energy of the $t\mu$ atom fulfills the condition: $E_{t\mu} > \Delta E = 0.24\text{eV}$ (ΔE is energy of hyperfine splitting of the ground state of the $t\mu$ atom). The probability of the transition of a $t\mu$ atom between hyperfine levels due to the collision of a $t\mu$ atom with a proton



according to [21, 22] is very small (because of the small rate of spin-flip reactions due to spin-spin interactions compared to the rate of charge exchange reactions).

The transition of $pt\mu$ molecule from the state with $(J\nu) = (10)$ to the ground state $(J\nu) = (00)$ proceeds very quickly ($\sim 10^{-11}\text{s}$) and the energy difference between two states is carried out by conversion electron.

The ground state of the $pt\mu$ molecule is split into three sublevels with total angular momentum $J=I+S = 3/2, 1/2, 1/2^*$ [19, 20] (see Fig.2).

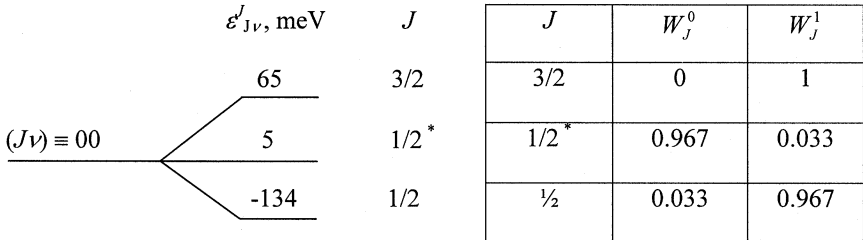


Fig. 2. Scheme of the energy sublevels of $pt\mu$ molecule ground state: W_J^0, W_J^1 are the probabilities that the sum of spins of proton and triton in a $pt\mu$ molecule, $(\vec{I} = \vec{I}_p + \vec{I}_t)$, in the state with total angular momentum J equals 0 and 1.

The binding energy of the ground state of the $pt\mu$ molecule (in the non relativistic case) equals $\varepsilon_{00} = 214\text{keV}$.

Table 1. The population of $pt\mu$ molecule levels, formed in the collision of a $t\mu$ atom in the para ($F=0$) or ortho-state ($F=1$) [20] with a proton.

J	ν	$\epsilon_{J\nu}, \text{eV}$	J	$\epsilon'_{J\nu}, \text{eV}$	$a'_{J\nu}(\uparrow\downarrow)$	$a'_{J\nu}(\uparrow\uparrow)$
0	0	-214.0	1/2*	0.0046	0.1120	0.2960
			1/2*	-0.1344	0.8880	0.0373
			3/2	0.0649	0	0.6667
1	0	-99.0	1/2	0.0053	0.0256	0.1026
				-0.1249	0.3076	0.0086
				0.0555	0.0001	0.1111
			3/2	0.0083	0.0548	0.2039
				-0.1262	0.6119	0.0183
				0.0608	0	0.2222
			5/2	0.0594	0	0.3333

where $\epsilon_{J,\nu}$ is the energy of the stationary state of the $pt\mu$ molecule ($J \nu$) in the non relativistic case; $\epsilon'_{J\nu}$ the energy of the stationary state of the molecule $pt\mu$ ($J\nu$) with total angular momentum J deduced from $\epsilon_{J\nu}$; and $a'_{J\nu}(\uparrow\downarrow), a'_{J\nu}(\uparrow\uparrow)$ are the populations of the state (J, ν, J) of the $pt\mu$ molecule, created in the collision of a $t\mu$ atom in the para - ($F=0$) or ortho - ($F=1$) state, respectively, with a proton.

As it is seen from Fig. 2 and Tab.1, the probability of the formation of a $pt\mu$ molecule in the state with $I=1$ in the collision of a $t\mu$ atom in a ortho-state ($F=1$) with a proton is smaller than during the collision of a $t\mu$ atom in the para-state ($F=0$):

$$W_{pt\mu}^{F=0}(J=1/2^*; I=1) = a_{00}^{1/2^*}(\uparrow\downarrow) \cdot W_{1/2^*}^1 = 3.7 \cdot 10^{-3};$$

$$W_{pt\mu}^{F=0}(J=1/2; I=1) = a_{00}^{1/2}(\uparrow\downarrow) \cdot W_{1/2}^1 = 8.59 \cdot 10^{-1};$$

$$W_{pt\mu}^{F=0}(J=1/2; 1/2^*; I=1) = W_{pt\mu}^{F=0}(J=1/2^*; I=1) + W_{pt\mu}^{F=0}(J=1/2; I=1) = 8.62 \cdot 10^{-1};$$

$$W_{pt\mu}^{F=1}(J=1/2; I=1) = a_{00}^{1/2}(\uparrow\uparrow) \cdot W_{1/2}^1 = 3.6 \cdot 10^{-2};$$

$$W_{pt\mu}^{F=1}(J=1/2^*; I=1) = a_{00}^{1/2^*}(\uparrow\uparrow) \cdot W_{1/2^*}^1 = 9.8 \cdot 10^{-3};$$

$$W_{pt\mu}^{F=1}(J=1/2; 1/2^*; I=1) = W_{pt\mu}^{F=1}(J=1/2; I=1) + W_{pt\mu}^{F=1}(J=1/2^*; I=1) = 4.58 \cdot 10^{-2};$$

where $i \equiv \gamma, \mu, e^+e^-, 2\gamma$; ρ is the density of the probability that the distance between the proton and triton in the $pt\mu$ molecule equals 0 and K_0^i, K_1^i are pt reaction constants for S wave in the nuclear states with $I = 0$ (singlet) and $I = 1$ (triplet).

For the theoretical description of the pt reaction we use the resonant model of the existence of ${}^4\text{He}$ nuclei in excited state 0^+ near the threshold of this reaction. It is seen from Fig.1 that transitions (6)-(7) change the populations of the state of the $pt\mu$ molecule (the population of the state with $J=3/2$ decreases, therefore together with the $pt\mu$ molecule formation, the process of thermalization of $t\mu$ atoms proceeds) which can change not only the yield of the reaction products (9) but also the ratio between the partial probabilities for different channels of the reaction.

Below, the kinetics of the $pt\mu$ cycle is presented under the assumption that the rates of all muonic processes in the H-T mixture do not depend on energy and that thermalization of $t\mu$ atoms occurs sufficiently fast (the average time of thermalization of $t\mu$ atoms $t_{\text{term}} \approx 20 - 30$ ns is considerably smaller than characteristic times of all other muonic processes in the H-T mixture).

The yields and time distributions of γ quanta with energy 19.8 MeV and the conversion muons with energy 19.2 MeV, formed in the pt reaction, can be described by the following expressions:

$$\frac{dN_\gamma}{dt} = A_1^\gamma e^{-\lambda_1 t} + A_2^\gamma e^{-\lambda_2 t} + A_3^\gamma e^{-\lambda_3 t} + A_4^\gamma e^{-\lambda_4 t}, \quad (11)$$

$$A_1^\gamma = A \left(\frac{1}{\lambda_1 - \lambda_2} \left(\frac{k+l+m}{\lambda_1 - \lambda_4} \right) + \frac{1}{\lambda_1 - \lambda_3} \left(\frac{n+p}{\lambda_1 - \lambda_4} \right) \left(\frac{1}{3} - \frac{\lambda_{10} \varphi C_t}{\lambda_1 - \lambda_2} \right) \right), \quad (12)$$

$$A_2^\gamma = \frac{A}{(\lambda_1 - \lambda_2)(\lambda_2 - \lambda_4)} \left((k+l+m) - \frac{\lambda_{10} \varphi C_t}{\lambda_2 - \lambda_3} (n+p) \right), \quad (13)$$

$$A_3^\gamma = -\frac{A}{\lambda_1 - \lambda_3} \left(\frac{n+p}{\lambda_3 - \lambda_4} \right) \left(\frac{1}{3} + \frac{\lambda_{10} \varphi C_t}{\lambda_2 - \lambda_3} \right), \quad (14)$$

$$A_4^\gamma = \frac{A}{\lambda_1 - \lambda_4} \left(\frac{k+l+m}{\lambda_2 - \lambda_4} + \frac{n+p}{\lambda_3 - \lambda_4} \left(\frac{1}{3} + \frac{\lambda_{10} \varphi C_t}{\lambda_2 - \lambda_4} \right) \right), \quad (15)$$

$$A = \frac{3}{4} N_{\text{stop}} \cdot \lambda_{pt} \varphi C_t \cdot \lambda_{p\mu} \varphi C_p \cdot \lambda_{f,\gamma}^{\text{pt}} (I=1) \cdot \varepsilon_\gamma, \quad (16)$$

$$k = W_{p\mu}^{F=1} (J=3/2; I=1) = 6.67 \cdot 10^{-1},$$

$$l = W_{\rho\mu}^{F=1}(J=1/2; I=1) = 3.6 \cdot 10^{-2},$$

$$m = W_{\rho\mu}^{F=1}(J=1/2^*; I=1) = 9.8 \cdot 10^{-3},$$

$$n = W_{\rho\mu}^{F=0}(J=1/2; I=1) = 8.59 \cdot 10^{-1},$$

$$p = W_{\rho\mu}^{F=0}(J=1/2^*; I=1) = 3.7 \cdot 10^{-3},$$

$$C_p + C_t = 1, \quad (17)$$

$$\frac{dN_\mu}{dt} = A_1^\mu e^{-\lambda_1 t} + A_2^\mu e^{-\lambda_2 t} + A_3^\mu e^{-\lambda_3 t} + A_4^\mu e^{-\lambda_4 t} + A_5^\mu e^{-\lambda_5 t}, \quad (18)$$

$$A_1^\mu = A1 \left[\frac{1}{\lambda_1 - \lambda_2} \left(\frac{\lambda_{f,\mu}^{\rho I}(I=1)}{\lambda_1 - \lambda_4} (k+l+m) + \frac{\lambda_{f,\mu}^{\rho I}(I=0)}{\lambda_1 - \lambda_0} (r+s) \right) + \left(\frac{1}{3} - \frac{\lambda_{10} \varphi C_t}{\lambda_1 - \lambda_2} \right) \times \right. \\ \left. \times \frac{1}{\lambda_1 - \lambda_3} \times \left(\frac{\lambda_{f,\mu}^{\rho I}(I=1)}{\lambda_1 - \lambda_4} (n+p) + \frac{\lambda_{f,\mu}^{\rho I}(I=0)}{\lambda_1 - \lambda_4} (t+u) \right) \right], \quad (19)$$

$$A_2^\mu = \frac{A1}{\lambda_1 - \lambda_2} \left(\frac{\lambda_{f,\mu}^{\rho I}(I=1)}{\lambda_2 - \lambda_4} (k+l+m) + \frac{\lambda_{f,\mu}^{\rho I}(I=0)}{\lambda_2 - \lambda_0} (r+s) \right) - \frac{\lambda_{10} \varphi C_t}{\lambda_2 - \lambda_3} \times \\ \times \left(\frac{\lambda_{f,\mu}^{\rho I}(I=1)}{\lambda_2 - \lambda_4} (n+p) + \frac{\lambda_{f,\mu}^{\rho I}(I=0)}{\lambda_2 - \lambda_0} (t+u) \right), \quad (20)$$

$$A_3^\mu = -\frac{A_1}{\lambda_1 - \lambda_3} \left(\frac{\lambda_{f,\mu}^{\rho I}(I=1)}{\lambda_3 - \lambda_4} (n+p) + \frac{\lambda_{f,\mu}^{\rho I}(I=0)}{\lambda_3 - \lambda_0} (t+u) \right) \times \left(\frac{1}{3} + \frac{\lambda_{10} \varphi C_t}{\lambda_2 - \lambda_3} \right), \quad (21)$$

$$A_4^\mu = \frac{A_1 \cdot \lambda_{f,\mu}^{\rho I}(I=1)}{\lambda_1 - \lambda_4} \left[\frac{1}{\lambda_2 - \lambda_4} (k+l+m) + \frac{1}{\lambda_3 - \lambda_4} (n+p) \times \left(\frac{1}{3} + \frac{\lambda_{10} \varphi C_t}{\lambda_2 - \lambda_4} \right) \right], \quad (22)$$

$$A_5^\mu = \frac{A1 \cdot \lambda_{f,\mu}^{\rho I}(I=0)}{\lambda_1 - \lambda_0} \left[\frac{r+s}{\lambda_2 - \lambda_0} + \frac{t+u}{\lambda_3 - \lambda_0} \times \left(\frac{1}{3} + \frac{\lambda_{10} \varphi C_t}{\lambda_2 - \lambda_0} \right) \right], \quad (23)$$

$$A1 = \frac{3}{4} N_{\mu stop} \cdot \lambda_{pt} \varphi C_t \cdot \varepsilon_{\mu}, \quad (24)$$

$$\lambda_1 = \lambda_0 + \lambda_{pp\mu} \varphi C_p + \lambda_{pt} \varphi C_t, \quad (25)$$

$$\lambda_2 = \lambda_0 + \lambda_{pt\mu} \varphi C_p + \lambda_{t\mu} \varphi C_t + \lambda_{10} \varphi C_t, \quad (26)$$

$$\lambda_3 = \lambda_0 + \lambda_{pt\mu} \varphi C_p + \lambda_{t\mu} \varphi C_t, \quad (27)$$

$$\lambda_4 = \lambda_0 + \lambda_f^{pt} (I = 1), \quad (28)$$

$$\lambda_5 = \lambda_0 + \lambda_f^{pt} (I = 0), \quad (29)$$

$$\lambda_0 = 0.455 \cdot 10^6 \text{ s}^{-1},$$

$$r = W_{pt\mu}^{F=1} \left(J = \frac{1}{2}; I = 0 \right) = 1.22 \cdot 10^{-3}, \quad s = W_{pt\mu}^{F=1} (J = 1/2^*; I = 0) = 0.286,$$

$$t = W_{pt\mu}^{F=0} \left(J = \frac{1}{2}; I = 0 \right) = 3.7 \cdot 10^{-3}, \quad u = W_{pt\mu}^{F=0} (J = 1/2^*; I = 0) = 0.108,$$

$$N_{\gamma} = \frac{A_1^{\gamma}}{\lambda_1} + \frac{A_2^{\gamma}}{\lambda_2} + \frac{A_3^{\gamma}}{\lambda_3} + \frac{A_4^{\gamma}}{\lambda_4}, \quad (30)$$

$$N_{\mu} = \frac{A_1^{\mu}}{\lambda_1} + \frac{A_2^{\mu}}{\lambda_2} + \frac{A_3^{\mu}}{\lambda_3} + \frac{A_4^{\mu}}{\lambda_4} + \frac{A_5^{\mu}}{\lambda_5}, \quad (31)$$

where $N_{\mu stop}$ is the number of muons stopped in the H-T mixture, N_{γ} , N_{μ} are the yields of γ quanta and conversion muons, respectively; $\lambda_0 = 0.455 \cdot 10^6 \text{ s}^{-1}$ is the free muon decay rate; λ_{pt} , λ_{10} , $\lambda_{pt\mu}$ are the rates of the muon transition from $p\mu$ atom to triton, of the transition of $t\mu$ atom from the state with $F = 1$ to the state with $F = 0$, and of the $pt\mu$ molecule formation, respectively (the above values are reduced to liquid hydrogen density, $n_0 = 4.25 \cdot 10^{22} \text{ cm}^{-3}$); $\lambda_{f,\mu}^{pt} (I = 0)$, $\lambda_{f,\mu}^{pt} (I = 1)$ are partial rates of nuclear synthesis in the $pt\mu$ molecule with muon production for the total spin of proton and triton equal to 0 and 1, respectively, and $\lambda_{f,\mu}^{pt} (I = 1)$ is the rate of nuclear synthesis in the $pt\mu$ molecule in the state $I = 1$ with γ quanta production; $\lambda_f^{pt} (I = 0)$, $\lambda_f^{pt} (I = 1)$ are the rates of nuclear synthesis in the $pt\mu$ molecule for the total spin of proton and triton

equal to 0 and 1, respectively; $\lambda_{f,ee}^{pt}$ ($I = 0$), $\lambda_{f,ee}^{pt}$ ($I = 1$) are the rates of nuclear synthesis in the $pt\mu$ molecule with the formation of an electron-positron pair for the total spin of p and t equal to 0 and 1, respectively; C_p and C_t are atomic concentrations of protium and tritium in H-T mixture; φ is the density of the H-T mixture reduced to liquid hydrogen density; and ε_γ , ε_μ are the efficiencies of the detection of γ quanta from reactions (9a)-(9d) and conversion muons from (9e), respectively.

The measurement of the synthesis rate in the $pt\mu$ molecule with the production of conversion muons, $\lambda_{f,\mu}^{pt}$ ($I = 0$) is very important and will allow one to verify the validity of the hypothesis of the existence of a threshold resonance in the fusion channel (and to check the charge distribution in the system with $A=4$).

Having time distributions of γ quanta with energy 19.8 MeV (reactions (9a)-(9d)) and conversion muons with energy 19.2 MeV (reaction (9e)) or electron-positron pair (reaction (9f)) for different tritium concentration C_t , using equations (11), (18), (30) and (31), one can derive unknown parameters: λ_{pt} , λ_{t0} , $\lambda_{pt\mu}$, $\lambda_{f,r}^{pt}$ ($I = 1$) and $\lambda_{f,\mu}^{pt}$ ($I = 0$). We assume the values of parameters $\lambda_{pp\mu}$, $\lambda_{tt\mu}$, k , l , m , n , p , r , s , t and u are known (the value $\lambda_{pp\mu}$ was taken as an average from papers [18,23-27], $\lambda_{tt\mu}$ from [28] and the remaining parameters were taken from [19-20]).

This approach is valid, because the yields and time distributions of the products from different channels of the pt reaction require the same μCF parameters, which, on the one hand can guarantee correct interpretation of the results and correct estimation of systematic errors, and on the other hand can increase the accuracy of measured parameters.

3 Optimization and results description

The existing theoretical and experimental parameters describing the $pt\mu$ -cycle are presented in Table 2.

As shown, there is big difference between experimental and theoretical values of some parameters like λ_{pt} , λ_f^{pt} ($I = 1$), and λ_f^{pt} ($I = 0$). Regarding the rate of the $pt\mu$ molecule formation ($\lambda_{pt\mu}$) there is strong agreement between theory and experiment.

It is shown from Tab.2 that it is necessary to measure fundamental characteristics of μCF in the H-T mixture to explain the nature of the difference between theoretical and experimental values. Fig.3 shows the dependence of the yield of γ quanta and conversion muons as a function of the tritium concentration C_t (calculated according to formulae (30) and (31)) for a density of the H-T mixture equal to the density of liquid hydrogen, $\varphi=1$. Comparing obtained dependences with corresponding values from paper [16] one can notice differences not only in shape but also in absolute values of conversion muon yield for the same values of C_t . The reason for such discrepancies is not clear.

Table 2 . The experimental and theoretical values of the parameters of the μCF process in the H-T mixture.

Value	Experiment				Theory	
	H/T [15]	H/D/T [16]	H/T [17]	H/T [18]	[14]	[19]
$\lambda_{pt}, 10^9 \text{ s}^{-1}$		9.3±1.5	5.86 ± (0.10) _{stat}	5.8±0.4		
$\lambda_{I0}, 10^9 \text{ s}^{-1}$	6.0±0.5	1.0±0.2				
$\lambda_{p\mu}, 10^9 \text{ s}^{-1}$		7.5±1.3			0.4	
$\lambda_f^{pt} (I = 1), 10^4 \text{ s}^{-1}$	6.5±0.7	7.0±1.2			≈1800	7
$\lambda_f^{pt} (I = 0), 10^2 \text{ s}^{-1}$		(15±4)·10 ²				8.6
$\lambda_{f,\gamma}^{pt} (I = 1), 10^6 \text{ s}^{-1}$						0.07
$\lambda_{f,ee}^{pt} (I = 1), 10^2 \text{ s}^{-1}$						2.4
$\lambda_{f,ee}^{pt} (I = 0), 10^2 \text{ s}^{-1}$						3.6
$\lambda_{f,\mu}^{pt} (I = 0), \text{ s}^{-1}$						0.35
$\lambda_{f,\mu}^{pt} (I = 0), 10^2 \text{ s}^{-1}$					10 ³ -10 ⁴	5±1
$\omega(K_\alpha)\%$		11±3				
$\frac{\lambda_{f,\mu}^{pt} (I = 1)}{\lambda_{f,\mu}^{pt} (I = 1)}$					10 ⁻⁵	5·10 ⁻⁶
$\frac{\lambda_{f,\mu}^{pt} (I = 0)}{\lambda_{f,ee}^{pt} (I = 0)}$					≈1	0.73

$\omega (K_\alpha)$ is the yield of K_α - line of X-radiation from the $^4\text{He}\mu$ atom (reactions (9c) and (9d)). There also exists some other, single theoretical estimates of the above parameters not shown in the table:

$$\lambda_{pt}=7.5 \cdot 10^9 \text{ c}^{-1} \text{ [22]}, (7.0-8.0) \cdot 10^9 \text{ c}^{-1} \text{ at } T=300-30 \text{ K [29]}, 5.8 \cdot 10^9 \text{ c}^{-1} \text{ [30]},$$

$$5.5 \cdot 10^9 \text{ c}^{-1} \text{ [31]}, 5.7 \cdot 10^9 \text{ c}^{-1} \text{ [32]};$$

$$\lambda_{I0}=0.89 \cdot 10^9 \text{ c}^{-1} \text{ [22]}, 0.91 \cdot 10^9 \text{ c}^{-1} \text{ [33]}, 1.3 \cdot 10^9 \text{ c}^{-1} \text{ [34]};$$

$$\lambda_{p\mu}=6.5 \cdot 10^6 \text{ c}^{-1} \text{ [35]}, 6.38 \cdot 10^6 \text{ c}^{-1} \text{ [36]};$$

$$\lambda_f^{pt} (I = 1)=0.5 \cdot 10^6 \text{ c}^{-1} \text{ [37]}^3, 0.13 \cdot 10^6 \text{ c}^{-1} \text{ [38]}^4, 0.008 \cdot 10^6 \text{ c}^{-1} \text{ [40]}^4;$$

$$\omega(K_\alpha)=9.5\% \text{ [43]}.$$

³ These values were obtained according to the formula: $\lambda_f^{pt} (I = 1) = 4/3 \cdot K_0 \rho_0$ using pt reaction constants K_0 from papers [37] and [38], respectively and ρ_0 from [39].

⁴ The estimate of this value was obtained using the cross section $\sigma(n, \gamma) = 55 \pm 3 \mu\text{b}$ of the mirror reaction $^3\text{He}(n, \gamma)^4\text{He}$ [41, 42].

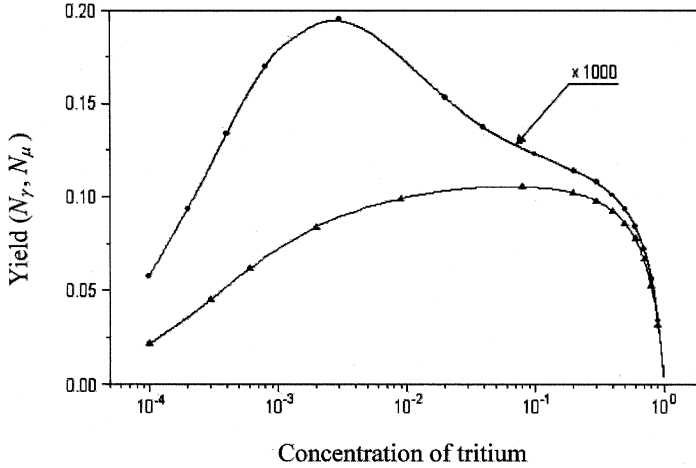


Fig. 3. The dependences of γ quanta (circles) and conversion muon (squares) yields from pt fusion as a function of tritium concentration.

According to [16] the maximum values of the γ quantum and conversion muon yields of calculated for one muon stopped in the H-T mixture equals $N_\gamma^{\max} \approx 0.11$ ($C_t \approx 6 \cdot 10^{-2}$) and $N_\mu^{\max} = 0.015$ (for $C_t = 3 \cdot 10^{-3}$), and in present paper $N_\gamma^{\max} \approx 0.10$ ($C_t \approx 8 \cdot 10^{-2}$), $N_\mu^{\max} \approx 1.5 \cdot 10^{-4}$ ($C_t \approx 1.5 \cdot 10^{-2}$).

The dependence of the ratio of the conversion muon and γ quanta yields as a function of tritium concentration is shown in Fig.4. The distinguishing feature of this dependence is that the ratio N_μ / N_γ is practically constant for a tritium concentration larger than 0.2. There are six unknown parameters ε_γ , ε_μ , $\lambda_{f,\mu}^{pt}$ ($I=0$), λ_{10} , $\lambda_{pt\mu}$, λ_{pt} in the expressions (11), (18), (30), (31) and to determine them with sufficient accuracy, three exposures of muon beam in the H-T mixture for three tritium concentrations are required. Really there are seven unknown parameters but the quantity λ_f^{pt} ($I=1$) is determined from the slope of exponent with index λ_d : λ_f^{pt} ($I=1$) = $\lambda_d - \lambda_0$ (see the expression (18)).

According to [19] the partial rates of nuclear M1 transition in $pt\mu$ molecule with emission of conversion muon ($\lambda_{f,\mu}^{pt}$ ($I=1$)) and electron – positron pair ($\lambda_{f,ee}^{pt}$ ($I=1$)) are negligible in comparison with $\lambda_{f,\gamma}^{pt}$ ($I=1$). Therefore the following ratio is valid:

$$\lambda_{f,\gamma}^{pt} (I=1) \approx \lambda_f^{pt} (I=1) = \lambda_{f,\gamma}^{pt} (I=1) + \lambda_{f,\mu}^{pt} (I=1) + \lambda_{f,ee}^{pt} (I=1).$$

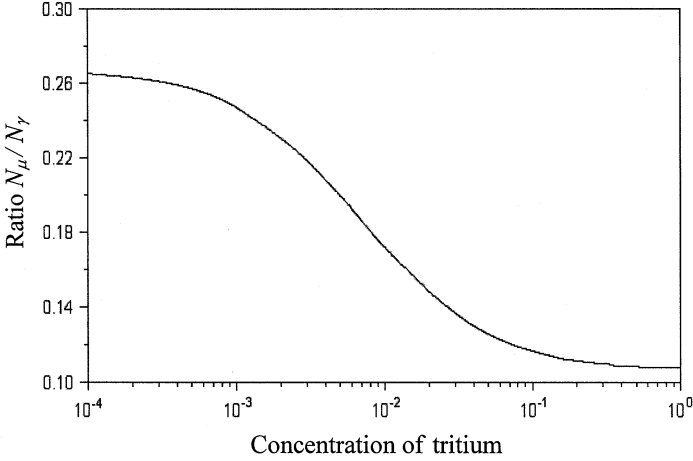


Fig. 4. Ratio between yields of conversion muon (N_μ) and γ quanta (N_γ) as a function of tritium concentration.

The accuracy of estimating these parameters depends on the statistic of detected events in the experiment. In principle, the rates of the processes $\lambda_{f,\gamma}^{\mu}$ ($I=1$), λ_{10} , $\lambda_{pI\mu}$, λ_{pI} can be estimated from the slopes of exponents with indexes λ_1 , λ_2 , λ_3 , λ_4 (expressions (25)-(28)). The value $\lambda_{f,\mu}^{\mu}$ ($I=0$) can not be experimentally found from the slope of exponent with index λ_5 ($\lambda_5 = \lambda_{f,\mu}^{\mu}$ ($I=0$) + λ_0) because the value $\lambda_{f,\mu}^{\mu}$ ($I=0$) is very small (λ_f^{μ} ($I=0$) = $5 \cdot 10^2$ s $^{-1}$ [19]) compared to λ_0 ($\lambda_5 \approx \lambda_0$). Therefore, the value $\lambda_{f,\mu}^{\mu}$ ($I=0$) can only be found analyzing the factor A_5^{μ} (23) before the exponent with index $\lambda_5 \approx \lambda_0$ in expression (18).

Below as an example of H-T experiment optimization it will be considered the performance of experiment using the muon channel $\mu E4$ of PSI meson facility (Switzerland). As a target it is supposed to use the liquid hydrogen with tritium concentration C_t less than 10%. The maximum tritium concentration is dictated by safety conditions.

The optimization of the planned experiment requires finding three tritium concentrations and corresponding times of the exposures, on the muon channel so that the errors of the determination of unknown parameters will be minimal (this means that the sum of the squares of the relative errors of the desired parameters is minimal in the interval $C_t = 0 - 0.1$).

As input data, the following values were used:

$$N_{\mu stop} = 10^4 \text{ s}^{-1} [44]; \quad \varepsilon_\gamma = 5 \cdot 10^{-3}; \quad \varepsilon_\mu = 7 \cdot 10^{-2}; \quad \lambda_{\rho t\mu} = 7.5 \cdot 10^6 \text{ s}^{-1} [16];$$

$$\lambda_{f,\gamma}^{pt} (I = 1) = \lambda_f^{pt} (I = 1) = 7 \cdot 10^4 \text{ s}^{-1} [16,19];$$

$$\lambda_{f,\mu}^{pt} (I = 0) = 5 \cdot 10^2 \text{ s}^{-1} [19]; \quad \lambda_{t\mu} = 1.8 \cdot 10^6 \text{ s}^{-1} [28];$$

$$\lambda_{\rho t} = 9.3 \cdot 10^9 \text{ s}^{-1} [16]; \quad \lambda_{t0} = 1.0 \cdot 10^9 \text{ s}^{-1} [16]; \quad \varphi = 1.0; \quad C_t = 0 - 0.10.$$

For the purpose of choosing optimal experimental conditions, it was assumed that the total time of exposure for three different tritium concentrations was 700 h.

The time for each of three exposures is determined as:

$$t_1 : t_2 : t_3 = \sqrt{n_\gamma^{(3)}} : \sqrt{n_\gamma^{(2)}} : \sqrt{n_\gamma^{(1)}},$$

where $n_\gamma^{(1)}, n_\gamma^{(2)}, n_\gamma^{(3)}$ are the yields of γ quanta per one second in the exposures 1 – 3, respectively.

As a result of the combined χ^2 analysis of the calculated six time distributions of γ quanta and conversion muons (the Monte Carlo method was used for each of the three exposures for obtains the simulated experimental time distributions of γ quanta and conversion muon), we have found three optimal values of tritium concentrations⁵: $C_t = 5 \cdot 10^{-4}, 6 \cdot 10^{-2}, 10^{-1}$.

The Figs.5, 6 show the calculated time distributions of the detected γ quanta with energy 19.8 MeV and conversion muons for three chosen tritium concentrations. Fig.7 shows the dependences of calculated parameter errors as a function of the statistic of detected events.

These parameter error dependences correspond to the approximation of the simulated γ quanta and conversion muon experimental time distributions by the expressions (11), (18), (30), (31) with unknown parameters $\varepsilon_\gamma, \varepsilon_\mu, \lambda_{\rho t\mu}, \lambda_{t0}, \lambda_{\rho t}, \lambda_{f,\gamma}^{pt} (I = 1)$. It should be pointed out that in such case the $\lambda_{f,\mu}^{pt} (I = 0)$ was fixed and equal $5 \cdot 10^2 \text{ s}^{-1}$ [19].

As seen, the sufficient total time of statistics gathering for determination $\lambda_{\rho t\mu}, \lambda_{f,\gamma}^{pt} (I = 1), \lambda_{\rho t}$ parameters with accuracy of $\sim 10\%$ is ~ 300 hours.

As for transition rate between hyperfine level of $t\mu$ atom λ_{t0} (curve 2 on Fig.7) the uncertainty of this magnitude is $\sim 100\%$ for the same time gathering statistics. At the statistics gathering time of 600 h the accuracy of λ_{t0} falls to 75%.

From the proceeding it may be seen that the result of joint analysis of γ quanta and conversion muon time distributions received at three chosen tritium concentrations is weakly sensitive to the value of λ_{t0} .

⁵ The minimum of χ^2 for different combinations of three tritium concentrations corresponds to the chosen set of three C_t .

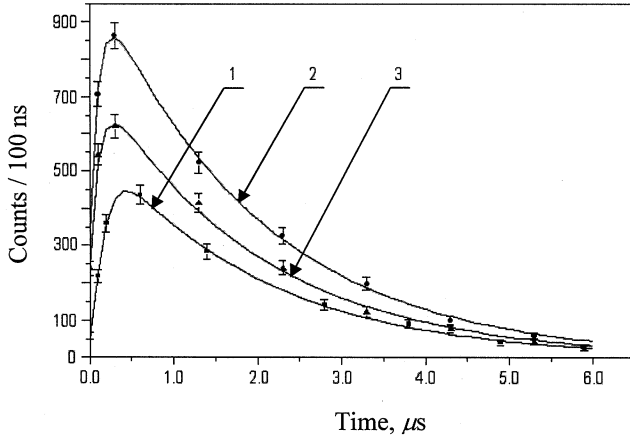


Fig.5. The time distributions of pt fusion γ quanta for three different values of tritium concentrations:

1 - $C_T=5 \cdot 10^{-4}$; 2 - $C_T=6 \cdot 10^{-2}$; 3 - $C_T=10^{-1}$.

The solid lines are the result of fitting of the simulated time spectra. The indicated bars are the statistics errors.

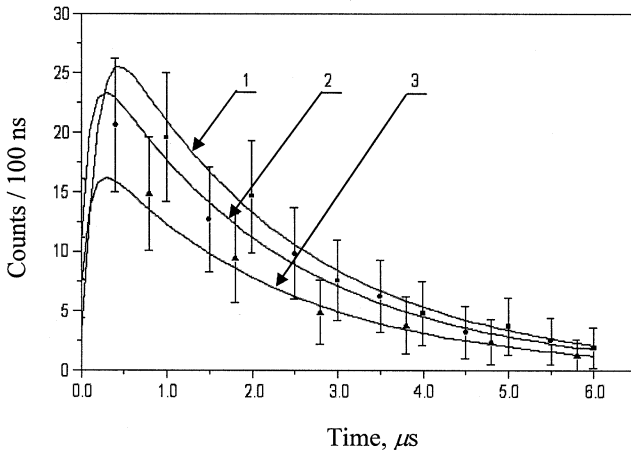


Fig. 6. Conversion muon time spectra for three chosen tritium concentrations:
1 - $C_T=5 \cdot 10^{-4}$; 2 - $C_T=6 \cdot 10^{-2}$; 3 - $C_T=10^{-1}$.

The solid lines are the result of fitting of the simulated time spectra. The indicated bars are the statistics errors.

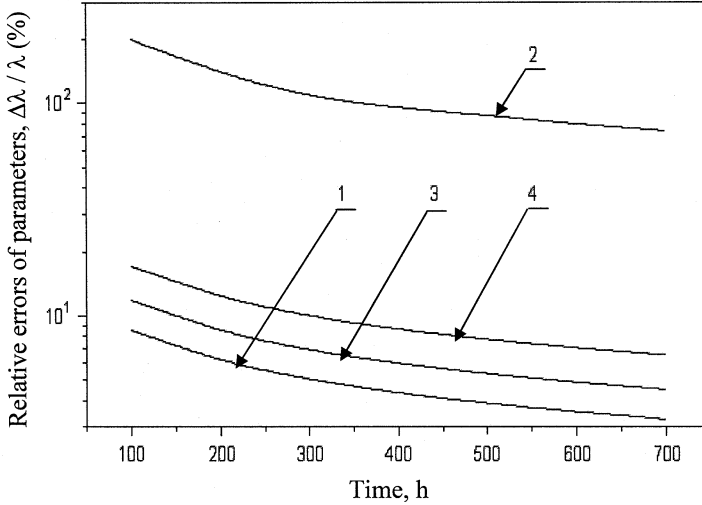


Fig. 7. Relative errors of $\lambda_{p\mu}$, λ_{I0} , $\lambda_{f,y}^{pt}$ ($I=1$) and λ_{pt} as a function of the statistics gathering time (ε_γ , ε_μ , $\lambda_{p\mu}$, λ_{I0} , λ_{pt} , $\lambda_{f,y}^{pt}$ ($I=1$) are the variable parameters; $\lambda_{f,\mu}^{pt}$ ($I=0$) = $5 \cdot 10^2 \text{ s}^{-1}$):
 1 - $\Delta \lambda_{pt} / \lambda_{pt}$; 2 - $\Delta \lambda_{I0} / \lambda_{I0}$; 3 - $\Delta \lambda_{p\mu} / \lambda_{p\mu}$; 4 - $\Delta \lambda_{f,y}^{pt} (I=1) / \lambda_{f,y}^{pt} (I=1)$; 5 - $\lambda_{f,\mu}^{pt} (I=0) / \lambda_{f,\mu}^{pt} (I=0)$.

More precise measurement of λ_{I0} is possible at essential increasing of the collection statistics and the range of variation of H-T mixture density and tritium concentration.

The next step of the μCF parameter errors calculation has been done setting ε_γ and ε_μ are known with accuracy of 5% from additional experiments. The results of these calculations are presented in Fig. 8. As seen, it is appeared the possibility to determine for the first time the information about fusion rate $\lambda_{f,\mu}^{pt}$ ($I=0$). This circumstance is very important for correct description of the pt reaction mechanism. The relative errors of other μCF parameters in H-T mixture at this optimization less than the corresponding values from previous optimization at the same gathering times of statistics.

From the presented analysis of the μCF kinetic in the H-T mixture, one can conclude that from the experiment performed for three different tritium concentrations, the unknown parameters of muon catalyzed ($\lambda_{p\mu}$, $\lambda_{f,y}^{pt}$ ($I=1$), λ_{pt}) fusion can be obtained with sufficient accuracy. Simultaneous measurement of yields and time distributions of γ quanta and conversion muons will allow one not only to find the ratio of probabilities for the radiation and non radiation channel of pt reaction, but also their exact values. So the possibility exists

to measure the fusion rate occurred in the ground state of the $p\mu$ molecule due to EO and MI transitions with the conversion of muons and γ quanta, respectively.

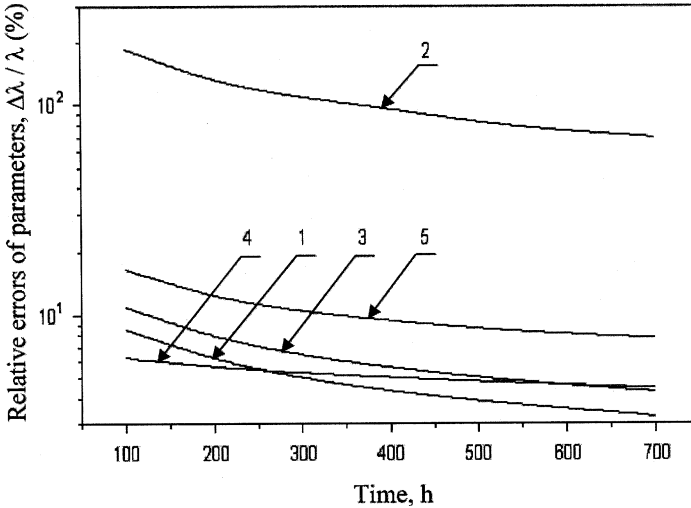


Fig. 8. The dependences of the μCF parameter relative errors from statistic gathering time (ε_γ and ε_μ are know from additional experiment). The numbers at curves correspond to the Fig. 7.

The measurement of γ quanta and conversion muon efficiencies in the additional experiments will allow to obtain the value of $\lambda_{f,\mu}^{\mu}$ ($I=0$) and to decrease the relative errors of μCF parameters such as $\lambda_{p\mu}$, λ_{10} , λ_{μ} , $\lambda_{f,\gamma}^{\mu}$ ($I=1$).

In addition, the accuracy of λ_{10} can be improved due to the measurement and joint analysis of γ quanta, conversion muon and Auger electrons emitted at deexcitation of $p\mu$ molecules formed in $(J\nu)=(10)$ state.

Acknowledgment

Authors would like to express their sincerely gratitude to Dr. M. Filipowicz, E. Gula and A. Krylov and J. Wozniak for arithmetic of H-T experiment optimization and to Dr. Mulhauser, P. Knowles for fruitful discussions.

This work was supported by the Russian Fund for Fundamental Researches (under grant #N 01-02-16483).

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Received on May 27, 2002.

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D15-2002-125

Метод определения параметров процесса мюонного катализа
в H-T-смеси

Предложен метод измерения параметров процесса мюонного катализа ядерных реакций синтеза μCF в H-T-смеси. Приведено описание кинетики мю-атомных и мю-молекулярных процессов, предшествующих протеканию pt -реакции в $p\mu$ -молекуле. Получены аналитические выражения для временных распределений γ -квантов и конверсионных мюонов, образующихся в результате протекания реакции ядерного синтеза в $p\mu$ -молекуле. Показано, что информация об искомым параметрах μCF может быть найдена путем совместного анализа временных распределений γ -квантов и конверсионных мюонов, полученных в эксперименте при трех и более различных значениях атомарной концентрации трития C_t в H-T-смеси. Проведена оптимизация планируемых экспериментов на мезонной фабрике PSI (Швейцария) с целью выбора трех значений C_t для прецизионного определения параметров μCF в H-T-смеси.

Работа выполнена в Лаборатории ядерных проблем им. В. П. Дзелепова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2002

Bystritsky V. M., Gerasimov V. V.

D15-2002-125

Method of Determination of Muon Catalyzed Fusion Parameters
in H-T Mixture

A method for measurement of the muon catalyzed fusion parameters μCF in the H-T mixture is proposed. The kinetics of the mu-atomic and mu-molecular processes preceding the pt reaction in the $p\mu$ molecule is described. Analytical expressions are obtained for the yields and time distributions of γ quanta and conversion muons formed in nuclear fusion reactions in $p\mu$ molecules. It is shown that information on the desired parameters can be found from the joint analysis of the time distributions of γ quanta and conversion muons obtained in experiments with the H-T mixture at three (and more) appreciable different atomic concentrations of tritium. The planned experiments with the H-T mixture at the meson facility PSI (Switzerland) are optimized to gain the precise information about the desired μCF parameters.

The investigation has been performed at the Dzhelepov Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2002

Макет *Т. Е. Попеко*

ЛР № 020579 от 23.06.97.

Подписано в печать 27.06.2002.

Формат 60 × 90/16. Бумага офсетная. Печать офсетная.

Усл. печ. л. 1,62. Уч.-изд. л. 2,54. Тираж 180 экз. Заказ № 53376.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.