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I.Zborovský[1], M.V.Tokarev[2], Yu.A.Panebratsev[3],
G.P.Škoro[4]

Z-SCALING, FRACTALITY AND PRINCIPLE OF RELATIVITY IN RELATIVISTIC COLLISIONS OF HADRONS AND NUCLEI

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[1]Nuclear Physics Institute, Academy of Sciences of the Czech Republic, Řež, Czech Republic; E-mail: zborovsky@ujf.cas.cz
[2]E-mail: tokarev@sunhe.jinr.ru
[3]E-mail: panebrat@sunhe.jinr.ru
[4]Faculty of Physics, University of Belgrade, Institute of Nuclear Sciences «Vinča», Belgrade, Yugoslavia; E-mail: goran@ff.bg.ac.yu

Z-scaling in the inclusive particle production

Motivation : Search for universal phenomenological description of particle production cross sections at high energies.
The approach is based on the principles of : *locality, self-similarity,* and *fractality* of hadronic interactions.

1. Locality principle : Gross features of the single particle distributions for the reaction

$$M_1 + M_2 \to m_1 + X$$

can be expressed in terms of the constituent interaction

$$(x_1 M_1) + (x_2 M_2) \to m_1 + (x_1 M_1 + x_2 M_2 + m_2)$$

subjected to the condition

$$(x_1 P_1 + x_2 P_2 - q)^2 = (x_1 M_1 + x_2 M_2 + m_2)^2.$$

2. Self-similarity principle : Dropping of certain quantities out of the physical picture of the interactions. We search for the solution

$$\psi(z) \equiv \frac{1}{N\sigma_{inel}} \frac{d\sigma}{dz}$$

which depends on a single scaling variable z. The scaling function ψ is expressed via the invariant differential cross section $Ed^3\sigma/dq^3$ and the average multiplicity density $dN/d\eta$ of particles produced in the reaction as follows

$$\psi(z) = \frac{\pi s A_1 A_2}{[dN/d\eta]\sigma_{inel}} J^{-1} E \frac{d^3\sigma}{dq^3}.$$

The symbol J stands for the corresponding Jacobian.

3. Fractality principle :
The scaling variable z is a fractal measure

$$z = z_0 \varepsilon^{-\delta}$$

representing formation length of particles produced in the constituent interactions. It depends on the anomalous (fractal) dimension of the particle trajectory δ and tends to infinity with increasing resolution ε^{-1}. Here

$$\varepsilon(x_1, x_2) = (1 - x_1)^{A_1}(1 - x_2)^{A_2}$$

is the relative uncertainty with which one can single out the binary subprocess from the system of two colliding nuclei. In the single nucleon interaction regime, the quantity

$$\varepsilon(x_1, x_2) = (1 - \bar{x}_1/A_1)^{A_1}(1 - \bar{x}_2/A_2)^{A_2} \sim (1 - \bar{x}_1)(1 - \bar{x}_2)$$

is approximated in terms of the momentum fractions $\bar{x}_i = A_i x_i$ of the interacting nuclei expressed in units of nucleon mass. This is no longer valid for cumulative processes with $\bar{x}_i > 1$, which correspond to the joining of partons from different nucleons of nuclei. The region is interesting for study of fractality at small scales. Here we focus on small distances under the condition that simultaneously still larger amount of energy is deposited in it.

Structure of the momentum fractions x_1 and x_2 :
The fractions are defined in a way to minimize the resolution ε^{-1} with respect to the constituent interactions in which the inclusive particle m_1 can be produced. The form of the momentum fractions

$$x_1 = \lambda_1 + \chi_1, \qquad x_2 = \lambda_2 + \chi_2$$

corresponds to the symbolic notation

$$(\lambda_1 + \chi_1) + (\lambda_2 + \chi_2) \to (\lambda_1 + \lambda_2) + (\chi_1 + \chi_2)$$

of the underlying subprocess.

The symbols indicate

$$\lambda_1 = \frac{(P_2q)+M_2m_2}{(P_1P_2)-M_1M_2}, \qquad \lambda_2 = \frac{(P_1q)+M_1m_2}{(P_1P_2)-M_1M_2},$$

$$\chi_1 = \sqrt{\mu_1^2+\omega_1^2}-\omega_1, \qquad \chi_2 = \sqrt{\mu_2^2+\omega_2^2}+\omega_2.$$

The factors μ_i and ω_i are given by

$$\mu_1^2 = \alpha\lambda^2\frac{(1-\lambda_1)}{(1-\lambda_2)}, \qquad \mu_2^2 = \alpha^{-1}\lambda^2\frac{(1-\lambda_2)}{(1-\lambda_1)},$$

$$\omega_1 = \frac{(\alpha-1)}{2}\frac{\lambda^2}{(1-\lambda_2)}, \qquad \omega_2 = \frac{(\alpha-1)}{2\alpha}\frac{\lambda^2}{(1-\lambda_1)},$$

where

$$\lambda^2 = \lambda_1\lambda_2+\lambda_0, \qquad \lambda_0 = \frac{0.5(m_2^2-m_1^2)}{(P_1P_2)-M_1M_2}.$$

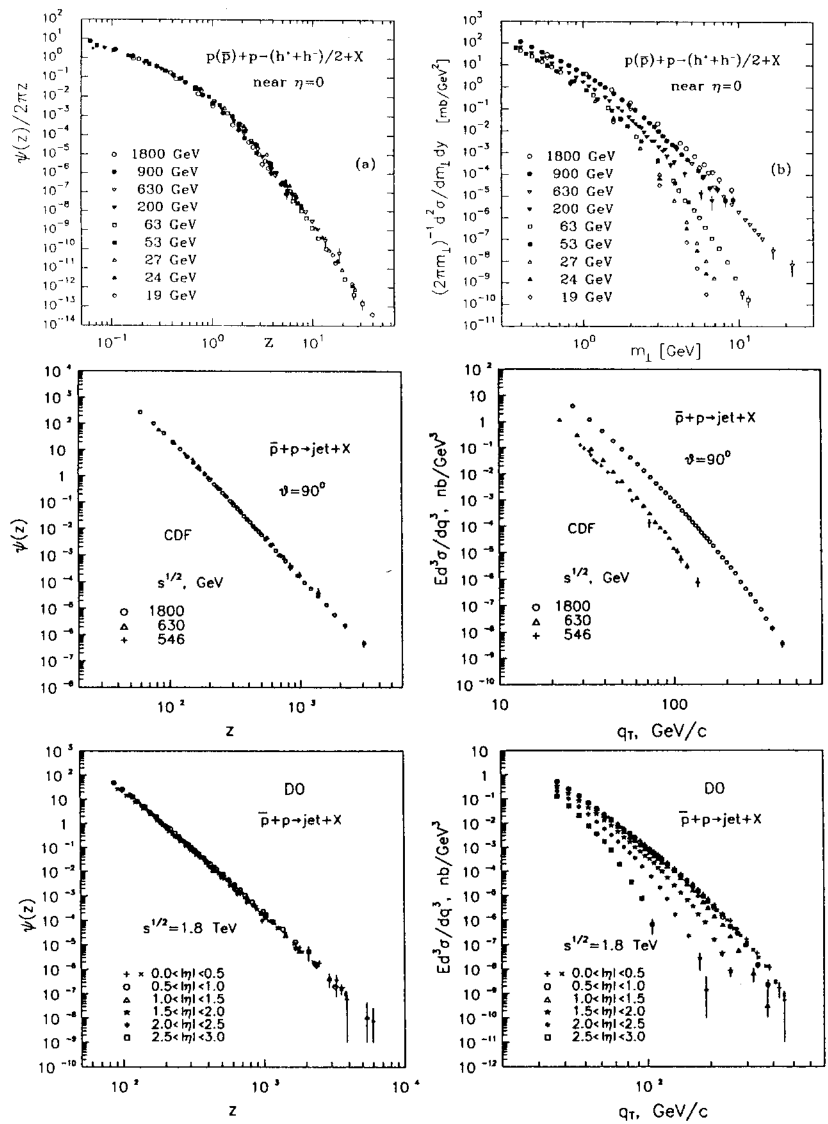
The parameter $\alpha \equiv \delta_2/\delta_1 = A_2/A_1$ is ratio of the anomalous (fractal) dimensions of the interacting (hadrons or) nuclei. The finite part of the scaling variable

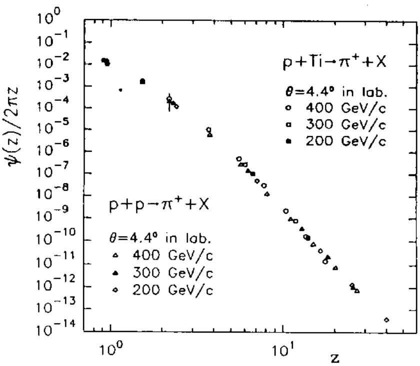
$$z_0 = \frac{\hat{s}^{1/2}_{\perp kin}}{m\rho(s)},$$

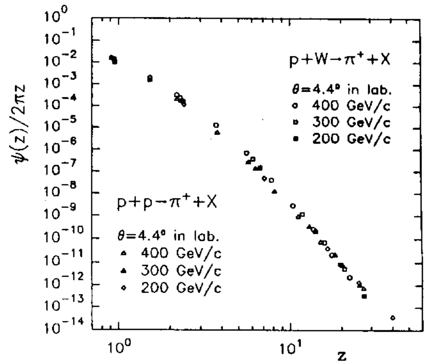
$$\hat{s}^{1/2}_{\perp kin} = \hat{s}^{1/2}_{\lambda} + \hat{s}^{1/2}_{\chi} - m_1 - (x_1M_1 + x_2M_2 + m_2),$$

$$\hat{s}_\lambda = (\lambda_1P_1+\lambda_2P_2)^2, \qquad \hat{s}_\chi = (\chi_1P_1+\chi_2P_2)^2$$

is characterized by transverse kinetic energy of the underlying subprocess. The m is nucleon mass and the factor $\rho(s) = dn(0)/d\eta$ is the multiplicity density of particles produced in the central region of the corresponding NN interaction. The m_2 is used in connection with internal conservation laws (isospin, strangeness, etc.).


p(p̄)+p→(h⁺+h⁻)/2+X
near η=0
(a)
ψ(z)/2πz
z
1800 GeV
900 GeV
630 GeV
200 GeV
63 GeV
53 GeV
27 GeV
24 GeV
19 GeV
p(p̄)+p→(h⁺+h⁻)/2+X
near η=0
(b)
(2πm⊥)⁻¹ d²σ/dm⊥dy [mb/GeV²]
m⊥ [GeV]
p̄+p→jet+X
ϑ=90⁰
CDF
s^1/2, GeV
1800
630
546
ψ(z)
z
Ed³σ/dq³, nb/GeV³
qT, GeV/c
D0
p̄+p→jet+X
s^1/2=1.8 TeV
0.0<|η|<0.5
0.5<|η|<1.0
1.0<|η|<1.5
1.5<|η|<2.0
2.0<|η|<2.5
2.5<|η|<3.0





The z-scaling regularity is illustrated in the figures. The energy and angular independence of the scaling function is contrasted with the differential cross sections as demonstrated above.

Fractality and particle formation

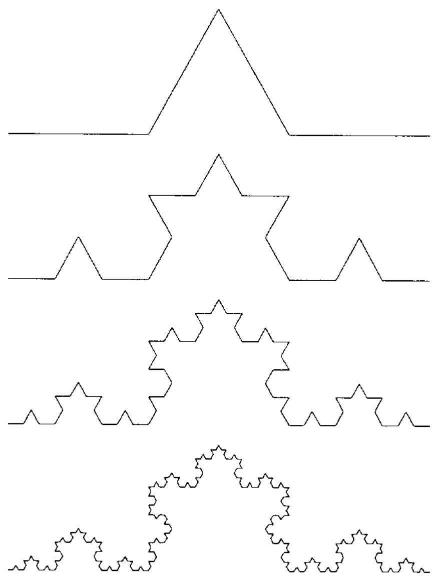
Using the concept of z scaling we aim at grasping the universal principles that influence the hadronic interactions with large momentum transfer at high energies. The scaling construction is based on hadron interaction self-similarity at constituent level. Its substantial element is idea of the fractality of the hadron and nucleus constituents and their interactions. This means that structure of the objects and the underlying processes are assumed to posses the properties of fractals. Fractal character in the initial state regards parton (quark and gluon) composition of the colliding hadrons and nuclei. Fragmentation of the point-like partons into the observable hadrons in the final state is considered to be a fractal process characterized by the scaling function $\psi(z)$. It refers to construction of a complex fractal (dressed constituent quark, hadron) from the more elementary fractal blocks. In this sense, the fractality of hadronic interactions is assumed to possess the universal character.

One of the main characteristic of fractals is the divergence of their measures in terms of the increasing resolution. The divergence is characterized by the fractal anomalous dimension δ.

Illustration : The von Koch curve with the fractal measure

$$z_\varepsilon = z_0 \varepsilon^{D_T - D}$$

representing its length.



n-th approximation is composed of p^n segments, each of the length $z_0 q^{-n}$. The curve has the length

$$z_n = z_0 (p/q)^n, \qquad p = 4, \quad q = 3.$$

This can be rewritten to the form

$$z_n = z_0 \big(q^{-n}\big)^{1 - lnp/lnq},$$

which gives $D_T = 1$, $D = \ln p / \ln q$, $\varepsilon = q^{-n}$, and

$$\delta = D - D_T > 0.$$

Fractal objects and fractal space-time

There exists suggestions that universal properties of the matter and its interactions are attributed to the structure of space-time itself. In the theory of relativity both special and general, it concerns the Lorentz transformation and the curvature of the trajectories. Free particles are moving along smooth geodesical lines, characteristic for the classical (curved) space-time. The situation changes at scales typical for the quantum world of the elementary particles. Common property is the unpredictability of motion at small distances. In this region particles follow irregular trajectories becoming non-differentiable with increasing resolution. The geometry of motion lines can be attributed to the properties of fractals which are extremely irregular objects fragmented at all scales. As an example one can mention quantum-mechanical path of a particle in the sense of Feynman trajectories. It was Feynman who first discovered the fractal character of the trajectories. Their fractal dimension

$$D = 1 + \delta = 2$$

is a direct consequence of the Heisenberg uncertainty relation.

Essential hypothesis concerning the universality of the assumptions is expressed in the statement: *Presence of the interacting fractal objects deforms the structure of surrounding space at small distances. As a consequence, space-time becomes locally fractal with geodesical lines acquiring an extremely irregular scale-dependent shape.* The secondary partons, produced in fractal space-time at small scales, follow extremely irregular geodesics starting from the regions they have been created. Formation of a particle from the bare parton realizes along a fractal-like trajectory characterized by its length.

Addressed questions:

- **possibility of erratic nature of the particle motion**
- **increasing length of the trajectory with increasing resolution $\rightarrow$ superluminal propagation of energy along fractal-like geodesics**
- **where are the masses from? 'dressing' of a particle - changing its mass with the resolution**

It is possible if one assumes space-time to be fractal at small scales. In such space-time particle is reduced to and identified with its own fractal-like trajectory. The rest mass m_0 being itself a geometrical fractal structure of the particle's trajectory.

The reflection invariance and scale-motion relativity

For any resolution ε^{-1}, fractal space-time F can be approximated by a Riemann space R_ε defined within a differentiable geometry. The family of the Riemann spaces is characterized by metric tensors with fluctuating curvatures. The fluctuations increase with decreasing scale breaking the reflection invariance at small distances. We show that even in this case, the Lorentz invariance need not to be violated locally. Let us consider the relativistic boost in 1+1 dimensions.

(i) Linearity of the relativistic transformation is expressed by

$$x' = \gamma(u)[x - ut], \qquad t' = \gamma(u)[A(u)t - B(u)x].$$

(ii) Group structure of the transformations requires

$$A(u)=1+2au, \quad B(u)=u, \quad \gamma(u) = \frac{1}{\sqrt{1+2au-u^2}},$$

and results in the composition law for the velocities

$$v = \frac{v' + u + 2auv'}{1 + uv'}.$$

(iii) breaking of the reflection invariance - non-zero values of a.

Space-time asymmetry in 3+1 dimensions

We search for linear and homogeneous transformations with group properties required by the principle of relativity. The parameter of the group is the velocity $\vec{u}$ of a system S' in the S reference frame. We introduce the notations

$$\gamma = \frac{1}{\sqrt{(1+\vec{a}\cdot\vec{u})^2 - (1+a^2)u^2}}, \qquad g = \frac{(1+\vec{a}\cdot\vec{u})\gamma - 1}{u^2},$$

$a^2 = \vec{a}\cdot\vec{a}$ and $u^2 = \vec{u}\cdot\vec{u}$. We also define

$$\gamma_\pm = gu^2 \pm \gamma\vec{a}\cdot\vec{u}, \qquad g_\pm = \gamma(1+a^2) \pm g\vec{a}\cdot\vec{u}.$$

It can be shown that the transformations

$$\vec{x}' = \vec{x} - \vec{u}\left[\gamma(t + \vec{a}\cdot\vec{x}) - g\vec{u}\cdot\vec{x}\right],$$

$$t' = t + \left[\gamma_+(t + \vec{a}\cdot\vec{x}) - g_+\vec{u}\cdot\vec{x}\right]$$

comply the requirements. The inverse relations

$$\vec{x} = \vec{x}' + \vec{u}\left[\gamma(t' + \vec{a}\cdot\vec{x}') + g\vec{u}\cdot\vec{x}'\right],$$

$$t = t' + \left[\gamma_-(t' + \vec{a}\cdot\vec{x}') + g_-\vec{u}\cdot\vec{x}'\right]$$

are obtained by the interchange $\vec{x} \leftrightarrow \vec{x}'$, $t \leftrightarrow t'$, $\vec{u} \leftrightarrow \vec{u}'$, and by the substitution

$$\vec{u}' = -\frac{\vec{u}}{1 + 2\vec{a}\cdot\vec{u}}.$$

The formula connects the velocity $\vec{u}'$ of the system S in the S' frame with the velocity $\vec{u}$ of the system S' in the S reference system. Because of the asymmetry $\vec{a}$, the magnitudes of the two velocities are not equal. The transformation formulae in 1+1 dimensions are recovered for $\vec{u} = (u, 0, 0)$ and $\vec{a} = (a, 0, 0)$.

In the 4-vector notation $x^\mu = \{\vec{x}, t\}$, the transformations can be rewritten as follows

$$x' = D(\vec{u})x,$$

$$D(\vec{u}) = \begin{pmatrix} \delta_{ij} + g u_i u_j - \gamma u_i a_j & -\gamma u_i \\ -g_+ u_j + \gamma_+ a_j & 1+\gamma_+ \end{pmatrix}.$$

The transformation matrix has the form

$$D(\vec{u}) = A_x^{-1}(\vec{a})\Lambda(\vec{\beta})A_x(\vec{a}),$$

where

$$A_x(\vec{a}) = \begin{pmatrix} \sqrt{1+a^2}\delta_{ij} & 0 \\ a_j & 1 \end{pmatrix}, \qquad \Lambda(\vec{\beta}) = \begin{pmatrix} \delta_{ij} + g_0 \beta_i \beta_j & -\gamma_0 \beta_i \\ -\gamma_0 \beta_j & \gamma_0 \end{pmatrix},$$

$$\gamma_0 = \frac{1}{\sqrt{1-\beta^2}}, \qquad g_0 = \frac{\gamma_0 - 1}{\beta^2}, \qquad \vec{\beta} = \sqrt{1+a^2}\frac{\vec{u}}{1 + \vec{a}\cdot\vec{u}}.$$

The group composition is given by

$$\Omega_x(\vec{\phi})D(\vec{v}) = D(\vec{v}')D(\vec{u}), \qquad \Omega_x(\vec{\phi}) = A_x^{-1}R(\vec{\phi})A_x,$$

provided

$$\vec{v} = \frac{\vec{v}' + \vec{u}\left[\gamma(1 + \vec{a}\cdot\vec{v}') + g\vec{u}\cdot\vec{v}'\right]}{1 + \gamma_-(1 + \vec{a}\cdot\vec{v}') + g_-\vec{u}\cdot\vec{v}'}.$$

The matrix

$$R(\vec{\phi}) = \begin{pmatrix} R^{\varphi}_{ij} & 0 \\ 0 & 1 \end{pmatrix}, \qquad \vec{\phi} = \vec{v}' \times \vec{u}$$

describes the Thomas precession. Region of the accessible values of velocities is given by the ellipsoid

$$(v_\parallel - a)^2 + (1+a^2)v_\perp^2 = (1+a^2).$$

The focus of the ellipsoid is in the point $\vec{v} = 0$. The ellipsoid is invariant with respect to the above transformations of the velocities.

The relativistic transformations preserve the invariant

$$t^2 - \vec{x}^2 + 2t\vec{a}\cdot\vec{x} - (\vec{a} \times \vec{x})^2 \equiv \tau^2$$

which corresponds to the metrics

$$\hat{a} = \begin{pmatrix} -d_{ij} & a_i \\ a_j & 1 \end{pmatrix}, \qquad d_{i,j} = (1+a^2)\delta_{i,j} - a_i a_j .$$

Energy and momentum

The position and momentum of a particle in the 4-space are given by $x^\mu = \{\vec{x}; t\}$ and $p^\mu = \{\vec{P}, E\}$, respectively. Let us define an 'elementary' particle as an object which reveals no internal structure at any resolution considered. We comprehend the notion of elementarity as a relative concept which relies on the scales we are dealing with. For the infinite resolution it should be a perfect point whose trajectory is a fractal curve. For an arbitrary small but still finite resolution ε^{-1} the perfect point is approximated by a particle which we call 'elementary' with respect to this resolution. It is therefore natural to assume that the concepts of the momentum, energy, mass and the velocity of the 'elementary' particle have good physical meaning also at the scales where space-time is expected to break down its isotropy.

The 4-momentum of such a particle should comply the relation given by the metrics of space-time in which the particle is situated. The relation is characterized by the space-time asymmetry $\vec{a}$:

$$p^2 \equiv \hat{a}_{\mu\nu} p^\mu p^\nu = E^2 - \vec{P}^2 + 2E\vec{a}\cdot\vec{P} - (\vec{a}\times\vec{P})^2 \equiv m_0^2 .$$

How do the variables $\vec{P}$ and E depend on the particle velocity? How do the variables transform?

First, we introduce the mechanical variables $\pi^\mu = \{\vec{\pi}, \pi_0\}$ which transform as follows

$$\pi' = D^\dagger(\vec{u})\pi.$$

The π^μ have properties of the 4-momentum of a particle with the space component oriented in the direction of the asymmetry $\vec{a}$. The general form of the 4-momentum is defined in the way to preserve the same metric invariant as the coordinates and time. There exists two sets of the variables $p_s^\mu = \{\vec{P}_s, E\}$, $s = L, R$ defined by

$$\pi = A_s(\vec{a})p_s, \qquad A_s(\vec{a}) = \begin{pmatrix} \delta_{ij} \pm \varepsilon_{ijk}a_k & 0 \\ 0 & 1 \end{pmatrix},$$

which comply the requirement. The plus (minus) sign corresponds to $s{=}L$ ($s{=}R$), respectively.
4-momentum p_s^μ with $s = L$ - left-handed motion.
4-momentum p_s^μ with $s = R$ - right-handed motion.
Explicitly:

$$\vec{\pi} = \vec{P}_s \pm \vec{P}_s \times \vec{a}, \qquad \vec{P}_s = \frac{\vec{\pi} \pm \vec{a}\times\vec{\pi} + (\vec{a}\cdot\vec{\pi})\vec{a}}{1 + a^2}.$$

It is convenient to introduce the variables $\vec{U}_s$, $s = L, R$,

$$\vec{U}_s = \vec{u} \mp \vec{u}\times\vec{a}, \qquad \vec{u} = \frac{\vec{U}_s \mp \vec{a}\times\vec{U}_s + (\vec{a}\cdot\vec{U}_s)\vec{a}}{1 + a^2}.$$

Using the notations $G = g/(1{+}a^2)$ and $G_\pm = g_\pm/(1{+}a^2)$, we get

$$p_s' = \Delta(\vec{U}_s)p_s,$$

where

$$\Delta(\vec{U}) = \begin{pmatrix} \delta_{ij}{+}GU_iU_j{-}G_-a_iU_j & GU^2a_i{-}G_+U_i \\ -\gamma U_j & 1{+}\gamma_+ \end{pmatrix}.$$

The transformation matrix can be written in the way

$$\Delta(\vec{U}_s) = A_{ps}^{-1}(\vec{a})\Lambda(\vec{\beta})A_{ps}(\vec{a}),$$

where

$$A_{ps}(\vec{a}) = \frac{1}{\sqrt{1+a^2}} \begin{pmatrix} \delta_{ij} \pm \varepsilon_{ijk} a_k & -a_i \\ 0 & \sqrt{1+a^2} \end{pmatrix}.$$

The group composition is given by

$$\Omega_p(\vec{\phi})\Delta(\vec{V}) = \Delta(\vec{V}')\Delta(\vec{U}), \quad \Omega_p(\vec{\phi}) = A_{ps}^{-1} R(\vec{\phi}) A_{ps},$$

provided

$$\vec{V} = \frac{\vec{V}' + \vec{U}\left[\gamma + G_+ \vec{a}\cdot\vec{V}' + G\vec{U}\cdot\vec{V}'\right]}{1 + \gamma_- + GU^2\vec{a}\cdot\vec{V}' + G\_\vec{U}\cdot\vec{V}'}.$$

The 4-vectors x^μ and p_s^μ have mutually different transformation properties for $\vec{a} \neq 0$. The transformations preserve the same metric form $D^\dagger(\vec{u})\hat{a}D(\vec{u}) = \Delta^\dagger(\vec{U})\hat{a}\Delta(\vec{U}) = \hat{a}$. This implies the dependence of the energy (positive energy solution) on the momentum as follows

$$E = \sqrt{1+a^2}\mathcal{E} - \vec{a}\cdot\vec{P}, \qquad \mathcal{E} = \sqrt{\vec{P}^2 + M_0^2}.$$

$E \geq 0$ for arbitrary $\vec{a}$ and $\vec{P}$. It has a minimum

$$\vec{P}_0 = M_0\vec{a}, \qquad E(\vec{P}_0) = M_0, \qquad M_0 = \frac{m_0}{\sqrt{1+a^2}}.$$

The energy includes, a priori, the potential energy contained in the scale structure involved. The structure is already present even in the rest frame of the particle; the rest mass m_0 being itself a geometrical fractal structure of the particle trajectory. The particle is identified with its own trajectory, which is the fractal-like trajectory of a point-like 'elementary' object moving chaotically with the momentum $\vec{P}$ and having the mass (minimum energy) $M_0 = E_{min}$.

The 4-momentum conservation

$$m_0^2 = \hat{a}_{\mu\nu}(p_1 + p_2)^\mu (p_1 + p_2)^\nu = m_1^2 + m_2^2 + 2\hat{a}_{\mu\nu} p_1^\mu p_2^\nu$$

gives $E = E_1 + E_2$, $\vec{P} = \vec{P}_1 + \vec{P}_2$, and $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$.
The form invariant relation between p_s^μ and $\vec{v}$ reads

$$\vec{P}_s = M[\vec{v} + (1+2\vec{a}\cdot\vec{v})\vec{a}] \mp M(\vec{v}\times\vec{a}), \qquad s = L, R$$
$$E = (1 + \vec{a}\cdot\vec{v})M.$$

The inertial mass depends on the velocity as follows :

$$M = M_0\gamma(\vec{v}), \qquad \gamma(\vec{v}) = \frac{1}{\sqrt{(1 + \vec{a}\cdot\vec{v})^2 - (1+a^2)v^2}}.$$

Particle trajectory is defined by the form invariant equation

$$tE - \vec{x}\cdot\vec{P}_s \mp \vec{a}\cdot(\vec{x}\times\vec{P}_s) + 2\vec{a}\cdot\vec{x}E = \tau M_0.$$

Its solution is $\vec{x} = \vec{v}t$, $t = \tau\gamma$, where

$$M\vec{v} = \frac{\vec{P}_s \pm \vec{P}_s\times\vec{a} - \vec{a}E}{1 + a^2}.$$

We conjecture that the vector asymmetry $\vec{a}$ is special case of a 'field of the space-time asymmetry' assumed at small scales. In our approximation, the 'field' implies the commutation relation

$$A_{ps}^\dagger \eta A_x - A_x^\dagger \eta A_{ps} = \begin{pmatrix} \pm\epsilon_{ijk}2a_k & -2a_i \\ 2a_j & 0 \end{pmatrix}.$$

The space-time asymmetries are caused by the vacuum fluctuations. The fluctuations possess erratic character which influences both metrics of space-time and motion of the elementary particles. The aim of the scale-motion relativity is to give a new approach to the quantum mechanics in which quantum behaviour would result from space-time structure at small scales.

The assumptions about stochastic and scale dependent nature of the space-time asymmetry $\vec{a}$ give

- **possibility of fractal-like motion of particles with respect to their momenta**
- **possibility of superluminous propagation of energy along fractal-like trajectories**
- **generation of particle masses in dependence on the character of the space-time fluctuations**

Interactions of asymmetric fractal objects

The natural question arises whether one can organize a region in which stochastic character of the space-time asymmetries could be somehow oriented. We argue that such region can be created in the interactions of hadrons and nuclei. This concerns high energies where the objects reveal fractal composition in terms of the parton content involved. One can imagine that the chaoticity of the space-time anisotropy can be oriented and space-time 'polarized' by the interactions of fractals possessing mutually different anomalous dimensions. We denote the asymmetry corresponding to the 'polarized' region by $\vec{\mathbf{a}} = (0, 0, \mathbf{a})$. The energy of the recoil particle in the constituent interaction is expressed as follows

$$\frac{2E}{\sqrt{s}} = \chi_1 + \chi_2 = \sqrt{\omega_1^2 + \mu_1^2} + \sqrt{\omega_2^2 + \mu_2^2} - (\omega_1 - \omega_2)\,.$$

This gives

$$\sqrt{(1+\mathbf{a}^2)(\chi_z^2 + \chi_\perp^2)} - \mathbf{a}\chi_z = \sqrt{\omega_1^2 + \mu_1^2} + \sqrt{\omega_2^2 + \mu_2^2} - (\omega_1 - \omega_2)\,,$$

where

$$\chi_z = \frac{2P_z}{\sqrt{s}} = \frac{P_z}{E_{max}}, \quad \chi_\perp = \frac{2M_\perp}{\sqrt{s}} = \frac{M_\perp}{E_{max}}, \quad M_\perp^2 = P_\perp^2 + M_0^2.$$

It follows from the conservation of the energy and momentum that

$$\sqrt{(1+\mathbf{a}^2)(\chi_z^2+\chi_\perp^2)} = \sqrt{\omega_1^2+\mu_1^2}+\sqrt{\omega_2^2+\mu_2^2},$$

$$\mathbf{a}\chi_z = \omega_1 - \omega_2.$$

The $\vec{P}$ and E are connected by the relation expressed in the way

$$\left(\frac{\chi_z}{\chi_1+\chi_2}-\mathbf{a}\right)^2 + (1+\mathbf{a}^2)\left(\frac{\chi_\perp}{\chi_1+\chi_2}\right)^2 = 1+\mathbf{a}^2,$$

where

$$\frac{P_z}{E} = \frac{\sqrt{s}}{2E}\chi_z = \frac{\chi_z}{\chi_1+\chi_2}, \qquad \frac{M_\perp}{E} = \frac{\sqrt{s}}{2E}\chi_\perp = \frac{\chi_\perp}{\chi_1+\chi_2}.$$

This altogether gives the solution

$$\chi_z = \mu_1 - \mu_2, \qquad \chi_\perp = 2\sqrt{\mu_1\mu_2},$$

with the asymmetry

$$\mathbf{a} = \mathbf{a}_0\lambda_c,$$

where

$$\mathbf{a}_0 = \frac{\alpha-1}{2\sqrt{\alpha}}, \qquad \lambda_c = \sqrt{\frac{\lambda_1\lambda_2}{(1-\lambda_1)(1-\lambda_2)}}, \qquad \lambda_c \le 1,$$

$\mathbf{a}_0$ - induced asymmetry of space-time.
A heuristic estimate for 400 GeV/c pA data: $\mathbf{a} \le 0.09 \div 0.13$.
Collisions of asymmetric fractal objects ($\alpha \neq 1$) result in creation of a domain in which the isotropy of space-time is violated. The induced asymmetry of space-time is due to the richer parton content of one fractal with respect to the other one. It is expressed by mutually different anomalous dimensions δ_i of the interacting fractals (e.g. hadrons and nuclei).

We identify the induced asymmetry with the space component of four velocity

$$\frac{\mathcal{V}}{\sqrt{1-\mathcal{V}^2}} = \mathbf{a}_0.$$

The velocity can be expressed in the form

$$\mathcal{V} = \frac{\alpha - 1}{\alpha + 1}, \qquad \alpha = \delta_2/\delta_1,$$

representing the 'velocity of space-time drift' induced by the interaction of the parton fractals. The quantity represents no real motion but characterizes local polarization of the vacuum. It satisfies the scale-relativity composition rule

$$\mathcal{V} = \frac{\mathcal{V}_1 + \mathcal{V}_2}{1 + \mathcal{V}_1\mathcal{V}_2},$$

provided $\alpha = \alpha_1\alpha_2$.

Exploiting the experimentally established relation $\delta_A = A\delta$, we get

$$\frac{A_3}{A_1} = \frac{A_3}{A_2}\frac{A_2}{A_1}.$$

This is the natural scaling property consisting in the following: If one examines the nucleus A_3 with a probe A_2, and then the probe nucleus A_2 with another probe A_1 one arrives at similar fractal structure as if examining the nucleus A_3 with the probe A_1.

Summary

- The z-scaling represents an observed regularity for the inclusive particle production in hadronic interactions with large momentum transfer at high energies. The scaling variable z is the formation length of particles and has character of a fractal measure. The scaling function $\psi(z)$ is probability density to form a particle with the formation length z. The existence of the z-scaling is confirmation of the hadron interaction ***self-similarity***, ***locality***, and ***fractality*** at the constituent level.

- Fractality of hadrons, nuclei and their interactions is universal property connected with fractal structure of space-time at small scales. A formalism concerning local relativistic transformations of the coordinates and momenta of a particle in space-time with vector asymmetry $\vec{a}$ was elaborated. The relations between the 4-momentum and velocity of the particle in space-time with the asymmetry $\vec{a}$ were obtained.

- Increase of stochasticity of the asymmetry $\vec{a}$ with decreasing scales would result in erratic fractal-like motion of particles with respect to their momenta. This implies change of the rest mass M_0 in dependence on the value of $\vec{a}$ and possibility of motion with superluminal velocities along fractal-like trajectories. Anisotropy of space-time induced in the interaction depends on the ratio of the anomalous fractal dimensions of the objects (hadrons and nuclei) colliding at high energies.

- The anomalous fractal dimensions δ for single hadrons and jets are found to be different. The experiment gives $\delta \sim 0.8$ for charged particles (pions) and $\delta \sim 1$ for the production of jets. Change of these values can be considered as a possible signal of new physics.

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Z-скейлинг, фрактальность и принцип относительности
при релятивистских столкновениях адронов и ядер

Обсуждается связь понятия длины формирования частицы, рожденной при релятивистских столкновениях адронов и ядер, с такими фундаментальными принципами физики, как локальность, фрактальность и самоподобие. Она проявляется в существовании z-скейлинга, новой закономерности, отражающей свойства поведения инклюзивных дифференциальных сечений рождения частиц при высоких энергиях. Скейлинговая переменная z отражает свойство длины формирования элементарной частицы как фрактальной меры. Обсуждается применимость принципа относительности для анизотропного пространства-времени. Получены релятивистские преобразования кинематических переменных (импульса, координаты, скорости) в анизотропном пространстве. Приводятся аргументы в пользу того, что такая анизотропия появляется при взаимодействии адронов и ядер.

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Z-Scaling, Fractality and Principle of Relativity
in Relativistic Collisions of Hadrons and Nuclei

The formation length of particles produced in the relativistic collisions of hadrons and nuclei has relevance to the fundamental principles of physics at small interaction distances. The relation is phenomenologically expressed by a z-scaling observed in the differential cross sections for the inclusive reactions at high energies. The scaling variable reflects the length of the elementary particle trajectories in terms of a fractal measure. Characterizing the fractal approach, we demonstrate the relativity principle in space-time with broken isotropy. We derive relativistic transformations accounting for the asymmetry of space-time induced in the interactions by various parton fractal structures of hadrons and nuclei.

The investigation has been performed at the Laboratory of High Energies, JINR.

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