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V.M.Dubovik[1], A.G.Galperin[2], A.A.Lushnikov[3],
V.S.Richvitsky[4]

CLUSTER GROWTH KINETICS

[1]E-mail: dubovik@thsun1.jinr.ru
[2]E-mail: galperin@vxjinr.jinr.ru
[3]Karpov NIFCHI, E-mail: alush@nifchi
[4]E-mail: rqvtsk@cv.jinr.ru

One understands a cluster as either "a number of things of the same kind growing together or a number of ... particles, objects, etc., in a small, close group" [1]. If a number of objects in a cluster or the cluster mass increases in time, then this process is called coagulation (aggregation). Usually coagulation process is considered as the merging of two colliding particles. We will consider also a dissociation (fragmentation) as a process inverse to coagulation. Those processes are united in terms nucleation or clusterisation very often. That originates from the generality of mathematical description for kinetics of such phenomena rather from actual microscopic details. On the one hand, we can see those approaches in course of investigations on molecular and submolecular level, in theories of condensed matter, nuclei and nuclear chains [2]. On the other hand, disperse systems are considered in astronomy (forming of cosmic objects), atmospheric science, chemistry, etc. [3].

For example, expanding universe is formed not at once. The clusters grow by coalescence of smaller clusters. Their growth kinetics is similar to the kinetics of coagulation. In what follows we formulate basic equations and outline the methods for their solution. Moreover, one could expect that theoretical tools which have been developed to describe physical systems, may be exploited in other fields, such as the ecology of computation [4] or biology, economics, transport problem, etc. [5, 6].

Consider the kinetics of formation of G cluster using a picture of one-track motorway. We assume that the starting configuration is G independent cars on motorway, the leading one being the slowest, and no one can pass the other. Initially each cluster contains one car.

The process begins at $t = 0$. On passing some time t, the G cars group in clusters containing $g_1, g_2, \ldots, g_s$ cars. These clusters continue to coalesce. The problem is formulated as follows: to determine the time evolution of the probability $w_s(g_1, g_2, \ldots, g_s; t)$ to find in the system s clusters $g_1, g_2, \ldots, g_s$ whose masses are subjected to the constraint:

$$\sum_{k=1}^{s} g_k = G. \tag{1}$$

Thus, we study the systems of constant mass or of finite number of particles.

The paper is organized as follows. In Section 1 we explain the stochastic motion of our objects, obtain the probability of finding out the system in the state of exactly s clusters and dependent on time average number of clusters by

means of introducing a generating function. Later on, we formulate a master equation governing the time evolution of the probability of finding the clusters of various masses (Sect. 2). We solve that problem applying Laplace's transform with respect to t to the master equation (Sect. 3). Then we find the probability to detect a cluster of assigned mass g by summation of w_s over all g_k irrespective of the distribution of other participants, excepting the selected one (Sect. 4). Some properties of those convolutions are due to morphism between a set of generating functions with a product and a set of $w(g)$ with convolution (see Appendix, too), that depends on the structure of semigroup with unit. In the following we consider processes of similar fragmentation (Sect. 5), aggregation and similar fragmentation going together (Sect. 6), and the process with an arbitrary fragmentation (Sect. 7). In conclusion, we discuss the results.

1. NUMBER OF CLUSTERS: PURE AGGREGATION PROCESS

Let γ be the rate of elementary coalescence act; two adjacent clusters produce a single one (for instance, a dimer is formed when a car catches up another one). We assume that γ is g–independent. Then we may characterize the situation by the number of intervals between adjacent clusters. If there are s clusters in the system, the number of intervals is $s-1$. Each coalescence act annihilates one interval. The number of ways to do this is exactly equal to the number of intervals.

Let $W(s,t)$ be the probability of meeting exactly s clusters at the time t, then:

$$\frac{dW(s,t)}{dt} = \gamma[sW(s+1,t) - (s-1)W(s,t)]. \tag{2}$$

This equation should be supplemented with the initial conditions:

$$W(s,0) = W_0(s). \tag{3}$$

In particular, if initially there were exactly G independent cars, the function

$$W_0(s) = \Delta(s-G), \tag{4}$$

with Δ being Kroneker's delta:$\Delta(0) = 1$ and $\Delta = 0$ otherwise.

Equation (2) may be solved by introducing the generating function:

$$F(z,t) = \sum_s W(s,t) z^{s-1}. \tag{5}$$

Combining of Eqs. (2) and (5) gives:

$$\partial_t F = (1-z)\partial_z F. \tag{6}$$

The rate γ is included into the definition of time. The initial condition for initially monodisperse system is rewritten in terms of z as:

$$F(z,0) = z^{G-1}. \tag{7}$$

The solution of Eq. (6) with the initial condition (Eq. (7)) has the form:

$$F(z,t) = [1 - e^{-t}(1-z)]^{G-1}. \tag{8}$$

The probability $W(s,t)$ is thus expressed in terms of binomial distribution:

$$W(s,t) = C_{G-1}^{s-1} e^{-(s-1)t}(1-e^{-t})^{G-s}. \tag{9}$$

There are no problems to find the time dependence of average number of clusters:

$$\bar{s}(t) = \partial_z z F(z,t)|_{z=1} = 1 + (G-1)e^{-t}. \tag{10}$$

2. MASS DISTRIBUTION AT PURE AGGREGATION PROCESS

In analogy with the kinetics of disperse systems we shall refer to g_k as the cluster mass. Our objective now is to formulate the master equation governing the time evolution of the probability $w_s(g_1, g_2 \dots g_s; t)$ to find the clusters of masses $g_1, g_2 \dots$ at the time t. This equation is formulated as follows:

$$\frac{dw_s}{dt} = \sum_{[g'],k} w_{s+1}(g_1 \dots g'_k, g'_{k+1}, g'_{k+2} \dots g'_{s+1}) \Delta(g_k - g'_k - g'_{k+1}) \times$$

$$\times \Delta(g'_{k+2} - g_{k+1}) \dots \Delta(g'_{s+1} - g_s) - (s-1)w_s. \tag{11}$$

The meaning of the terms on the r.h.s. of Eq. (11) is rather apparent. The rate of losses is simply proportional to the number of empty intervals (the rate constant γ is included in the definition of time). The gain occurs each time when two clusters of masses g'_k and g'_{k+1} coalesce producing a new cluster of mass g_k. Other Δ-s simply restore the numeration of g_i clusters with $i < k$ for the system of s clusters.

Of course, initial conditions to Eq. (11) should also be specified. We again assume that initially there were G single cars:

$$w_G(1, 1 \dots 1, t = 0) = 1 \tag{12}$$

and all other probabilities are 0.

3. SOLUTION TO BASIC EQUATION

On applying Laplace's transform with respect to t gives instead of Eq. (11):

$$(p+s-1)\bar{w}_s(g_1,g_2\ldots)=\sum_{[g'],k}\bar{w}_{s+1}(g_1\ldots g'_k,g'_{k+1},g'_{k+2}\ldots g'_{s+1})\times$$

$$\times\Delta(g_k-g'_k-g'_{k+1})\Delta(g'_{k+2}-g_{k+1})\ldots\Delta(g'_{s+1}-g_s), \tag{13}$$

where barred $\bar{w}$ stands for the Laplace transform of $w(g_1,g_2\ldots;t)$. The last equation of this set is readily solved (Eq. (9)) to yield:

$$\bar{w}_G=\tfrac{1}{p+G-1}. \tag{14}$$

Now let us try to seek for a solution to Eq. (11) in the form:

$$\bar{w}_s(g_1,g_2\ldots;,p)=\tfrac{1}{(p+G-1)(p+G-2)\ldots(p+s-1)}A_s(g_1,g_2\ldots), \tag{15}$$

where the coefficients A are independent of p and satisfy the following set of recurrence relations:

$$A_s(g_1,g_2\ldots)=\sum_{[g'],k}A_{s+1}(g_1\ldots g'_k,g'_{k+1},g'_{k+2}\ldots g'_{s+1})\times$$

$$\times\Delta(g_k-g'_k-g'_{k+1})\Delta(g'_{k+2}-g_{k+1})\ldots\Delta(g'_{s+1}-g_s). \tag{16}$$

A useful sum rule immediately follows from Eq. (16):

$$Q_s=sQ_{s+1}, \tag{17}$$

where $Q_s=\sum A_s(g_1,g_2\ldots g_s)$ (summation goes over all g), or

$$Q_s=\tfrac{(G-1)!}{(s-1)!}. \tag{18}$$

In fact, the expression $A_s(g_1,\ldots,g_s)$ depends on s only. It does not depend on a distribution of numbers $g_1,\ldots,g_s$ upon conditions $\sum_{k=1}^{s}g_k=G,\quad g_k\geq 1$ are conserved.

It can be seen from Eqs. (11), (13), (9) and (2) that the problem under consideration splits into two subproblems. The first one is a time evolution problem. It deals with transitions between different states of the system and connects each three nearest adjacent states. Second subproblem is of scrutinizing mass spectra. It is a pure combinatorial task. In fact, we have to do with some population dynamics. Mass spectra at instant t originate from the interchange of generations at G given and the proper weights depend on the whole set of possible transitions from $s+1$-states into the s-state under consideration.

Under induction method one has:

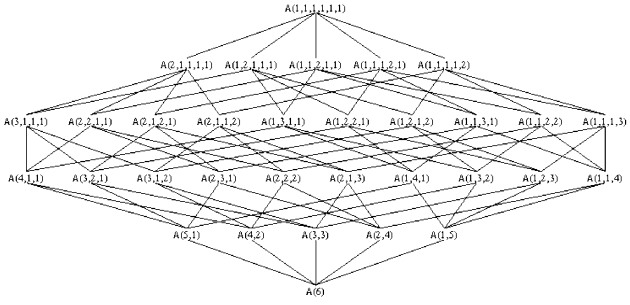

Fig. 1. Generation scheme for G=6

- $A_G(1,\dots,1)=1$ is a unique value;
- the number of terms in formula (Eq. (16)) is equal to

$$\sum_{k=1}^{s} g_k - 1 = \sum_{k=1}^{s} g_k - \sum_{k=1}^{s} 1 = G - s$$

for each s fixed and k fixed. Under the inductive assumption one can write down $A_{s+1}(g_1',\dots,g_{s+1}') = A_{s+1}$. From this it follows that $A_s(g_1,\dots,g_s) = (G-s)A_{s+1}$ irrespective of a specific distribution of $g_1,\dots,g_s$.

The recurrence equations obtained just now $A_s = (G-s)A_{s+1}$, $A_G = 1$ have solutions

$$A_s = (G-s)! \tag{19}$$

The time dependence may be readily restored on using the inversion

$$\frac{1}{(p+s-1)(p+s)\dots(p+G-1)} \longrightarrow \frac{1}{(G-s)!} e^{-(s-1)t}(1-e^{-t})^{G-s}. \tag{20}$$

Equations (16), (17) and (20) readily reproduce Eq. (9) .

The final result is formulated as follows:

$$w_s(g_1,g_2,\dots g_s;t) = e^{-(s-1)t}(1-e^{-t})^{G-s}\Delta(G-g_1-g_2-\dots g_s). \tag{21}$$

4. SINGLE-CLUSTER DISTRIBUTION AT PURE AGGREGATION PROCESS

Here we find the probability to find a cluster of mass g irrespective of the distribution of other participants. To this end we sum w_s over all g_k except one

(g_1, for example):

$$w(g,t) = \sum_{g_k} w_s(g, g_2, \dots g_s; t)$$
$$= e^{-(s-1)t}(1-e^{-t})^{G-s} \sum_{g_k} \Delta(G - g - g_2 - \dots g_s). \tag{22}$$

Using the identities

$$\Delta(q) = \left\{ \begin{array}{l} 1,\ q=0 \\ 0,\ q=1,2,... \end{array} \right\} = \frac{1}{2\pi i} \oint \frac{dz}{z^q z},$$
$$\left\{ \begin{array}{l} 0,\ q=0 \\ \\ 1,\ q=1,2,... \end{array} \right\} \tag{23}$$
$$= \frac{1}{2\pi i} \oint \frac{zdz}{z^q(1-z)},$$
$$\frac{1}{2\pi i} \oint \frac{dz}{z^{r+1}(1-z)^{R+1}} = C^r_{R+r} = C^R_{R+r},$$

one finds the convolution in Eq. (22):

$$w(g,t) = e^{-(s-1)t}(1-e^{-t})^{G-s} \frac{1}{2\pi i} \oint \frac{z^{s-1}dz}{z^{G-g}(1-z)^{s-1}z} =$$
$$= C^{s-2}_{G-g-1} e^{-(s-1)t}(1-e^{-t})^{G-s}. \tag{24}$$

Some important properties of the convolutions are discussed in Appendix.

5. SIMILAR FRAGMENTATION PROCESS

Let us consider process of pure fragmentation (dissociation, decay) of clusters. We assume inner cluster structure at $t = 0$ to be similar to the original picture at $t = 0$ with G cars of unit-mass and, thus, with $G-1$ intervals between those (Sect. 1). An analogous assumption relates to all the s clusters at $t \neq 0$. It means the similarity of inner and outer cluster structures. Let us define a cluster size as the number of particles confined into the cluster. If γ is the rate of elementary fragmentation act and a fragmentation rate is proportional to the cluster size without unit, i.e., the number of possible rupture places is equal to the number of inner intervals, the equation

$$\frac{dW(s,t)}{dt} = \gamma[(G-s+1)W(s-1,t) - (G-s)W(s,t)], \tag{25}$$

with initial conditions

$$W(1,0)=1; \quad W(s,0)=0, \quad if\ s\neq 1, \tag{26}$$

describes the process announced. The r.h.s. of Eq. (25) consists of a gain term due to decay of clusters belonging to $s-1$-cluster state and a loss term due to decays of those clusters belonging to s-cluster state, that produces the clusters pertaining to $s+1$-cluster state. Equation (25) can be solved using generating function introduced by equation

$$\frac{\partial}{\partial t}\,\mathrm{F}(z,\,t)-(-z^2\,\gamma+z\,\gamma)\,\frac{\partial}{\partial z}\,\mathrm{F}(z,\,t)=\gamma\,(z\,G-z-G+1)\,\mathrm{F}(z,\,t), \tag{27}$$

with the solution

$$F(z,t)=-\frac{(z+e^{(-\gamma\,t)}-e^{(-\gamma\,t)}\,z)^G}{-z-e^{(-\gamma\,t)}+e^{(-\gamma\,t)}\,z}. \tag{28}$$

Of cource, one recognizes a usual Poissonian process here. It seems contextual and, hence, quite reasonable to name such a process as a similar fragmentation (process).

For example, for $G=5$

$$\mathrm{W}_1=e^{(-4\,\gamma\,t)}, \tag{29}$$

$$\mathrm{W}_2=4\,e^{(-3\,\gamma\,t)}-4\,e^{(-4\,\gamma\,t)}, \tag{30}$$

$$\mathrm{W}_3=6\,e^{(-2\,\gamma\,t)}-12\,e^{(-3\,\gamma\,t)}+6\,e^{(-4\,\gamma\,t)}, \tag{31}$$

$$\mathrm{W}_4=4\,e^{(-\gamma\,t)}-12\,e^{(-2\,\gamma\,t)}+12\,e^{(-3\,\gamma\,t)}-4\,e^{(-4\,\gamma\,t)}, \tag{32}$$

$$\mathrm{W}_5=1-4\,e^{(-\gamma\,t)}+6\,e^{(-2\,\gamma\,t)}-4\,e^{(-3\,\gamma\,t)}+e^{(-4\,\gamma\,t)}. \tag{33}$$

The average number of clusters read as

$$\bar{s}(t)=\partial_z z F(z,t)|_{z=1}=e^{(-\gamma\,t)}+G\,(1-e^{(-\gamma\,t)}). \tag{34}$$

6. PROCESS OF AGGREGATION AND SIMILAR FRAGMENTATION

Let us consider a process when a cluster growth goes conjointly with a similar fragmentation. Let γ_1 and γ_2 be the constant rates of elementary coalescence act and elementary fragmentation act, respectively,

$$\begin{array}{c}\frac{dW(s,t)}{dt}=\\ \gamma_1[sW(s+1,t)-(s-1)W(s,t)]-\\ \gamma_2[(G-s)W(s,t)-(G-s+1)W(s-1,t)],\end{array} \tag{35}$$

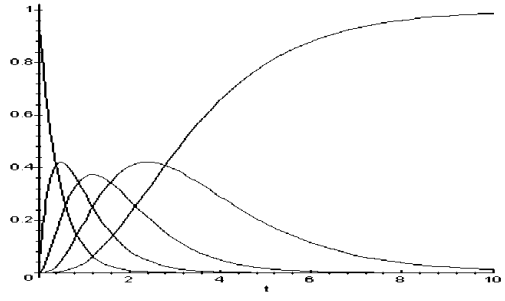

Fig. 2. W functions versus time for pure aggregation process. $G = 5$

with initial conditions

$$W(1,0) = 1; \qquad W(s,0) = 0, \quad if\ s \neq 1. \tag{36}$$

The r.h.s. of Eq. (35) consists of gain terms due to coagulation of clusters from the $(s+1)$-cluster state and dissociation of those clusters belonging to $(s-1)$-cluster state, and loss terms due to simultaneous coalescence and dissociation of clusters belonging to s-cluster state. To make things more clear we could rewrite Eq. (35) in the form

$$\frac{dW(s,t)}{dt} = (\gamma_1 s W(s+1,t) + \gamma_2 (G-s+1) W(s-1,t)) -$$

$$(\gamma_1 (s-1) W(s,t) + \gamma_2 (G-s) W(s,t)). \tag{37}$$

These equations can be solved using generating function introduced by the equation

$$\begin{array}{c} \frac{\partial}{\partial t}\,\mathrm{F}(z,\,t) - (-z^2\,\gamma_2 - z\,\gamma_1 + z\,\gamma_2 + \gamma_1)\,\frac{\partial}{\partial z}\,\mathrm{F}(z,\,t) = \\ \gamma_2\,(z\,G - z - G + 1)\,\mathrm{F}(z,\,t), \end{array} \tag{38}$$

which solution is

$$F(z,t) =$$

$$\begin{array}{c} \gamma\,(\frac{z\,\gamma_2 + \gamma_1\,e^{(-t\,\gamma)}\,z - \gamma_1\,e^{(-t\,\gamma)} + \gamma_1}{\gamma})^G / \\ (z\,\gamma_2 + \gamma_1\,e^{(-t\,\gamma)}\,z - \gamma_1\,e^{(-t\,\gamma)} + \gamma_1), \end{array} \tag{39}$$

where

$$\gamma = \gamma_1 + \gamma_2. \tag{40}$$

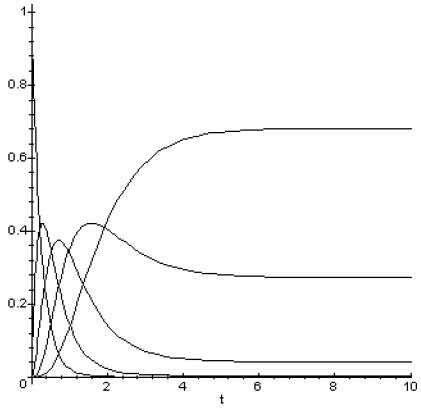

Fig. 3. W functions. Process of aggregation and similar fragmentation. $G = 5, \gamma = 1.1$, $\gamma 1 = 1, \gamma 2 = 0.1$

For example, for $G = 5$

$$\mathrm{W}_1 = .6830134554\,(e^{(-1.1t)} - 1)^4, \tag{41}$$

$$\mathrm{W}_2 = -2.732053822\,(e^{(-1.1t)} - 1)^3\,(.1 + e^{(-1.1\,t)}), \tag{42}$$

$$\mathrm{W}_3 = 4.098080732\,(e^{(-1.1t)} - 1)^2\,(.1 + e^{(-1.1\,t)})^2, \tag{43}$$

$$\mathrm{W}_4 = -2.732053822\,(e^{(-1.1t)} - 1)\,(.1 + e^{(-1.1\,t)})^3, \tag{44}$$

$$\mathrm{W}_5 = .6830134554\,(.1 + e^{(-1.1\,t)})^4. \tag{45}$$

The average number of clusters is read as

$$\begin{aligned} \bar{s}(t) &= \partial_z z F(z,t)|_{z=1} \\ &= 1 - \tfrac{\gamma_2 + \gamma_1\, e^{(-t\,\gamma)}}{\gamma} + \tfrac{G\,(\gamma_2 + \gamma_1\, e^{(-t\,\gamma)})}{\gamma}. \end{aligned} \tag{46}$$

7. PROCESS WITH AN ARBITRARY FRAGMENTATION

Let us assume an inner cluster structure to be beside itself under aggregation, i.e., it does not depend on the cluster size and hence, number of inner intervals (see Sect. 6), except for the unique case of the unit size that stops the process. Roughly speaking, cluster becomes structureless under aggregation. In that case a rational supposition is that a fragmenation rate depends solely on the cluster state index or, the same, on the integer number of clusters in that state. Let γ_1 and γ_2 be constant rates of elementary coalescence act and fragmentation act, respectively. The corresponding equation is read as

$$\begin{gathered}\frac{dW(s,t)}{dt} = \gamma_1[sW(s+1,t) - (s-1)W(s,t)] - \\ \gamma_2[(1-\Delta(G-s))sW(s,t) - (s-1)W(s-1,t)],\end{gathered} \tag{47}$$

with initial conditions

$$W(1,0) = 1; \quad W(s,0) = 0, \quad if\ s \neq 1. \tag{48}$$

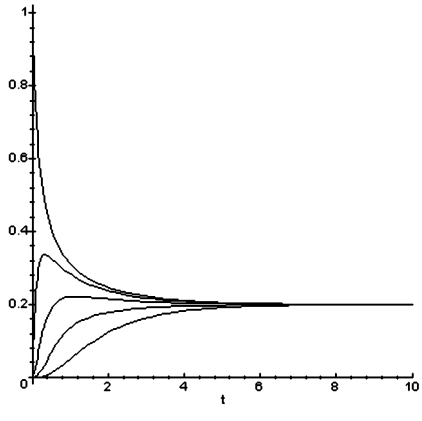

Fig. 4. W functions. Aggregation with an arbitrary fragmentation. $G = 5$, $\gamma = 1.1$, $\gamma 1 = 1$, $\gamma 2 = 0.1$

This system may be solved for each G. If, for example, G=5,

$$W(s,t) = \sum_{i=1}^{G} C_i e^{\lambda_i t} A_{si}, \tag{49}$$

$$C = [.4472135953,\ -.4472135955,\ -.4472135956,\ -.4472135954,\ -.4472135958], \tag{50}$$

$$\lambda = [-10.95389430,\ -5.731178750,\ -2.571635009,\ -.7432919279,\ 0], \tag{51}$$

$$A = \begin{bmatrix} .01096020981\ ,\ .1137666455\ ,\ -.4309667436\ ,\ .7753739038\ ,\ -.4472135953 \\ -.1090967701\ ,\ -.5382503358\ ,\ .6773224210\ ,\ .1990447401\ ,\ -.4472135950 \\ .4283919843\ ,\ .6781456172\ ,\ .3605539786\ ,\ -.1630940161\ ,\ -.4472135954 \\ -.7774690197\ ,\ .1935516685\ ,\ -.1596960607\ ,\ -.3641110314\ ,\ -.4472135955 \\ .4472135955\ ,\ -.4472135956\ ,\ -.4472135953\ ,\ -.4472135953\ ,\ -.4472135957 \end{bmatrix}. \tag{52}$$

8. RESUME

In the present paper we have been concentrated on kinetics of cluster formation. It does not matter whether a motorway, a computational network, space dust or glass is a real environment to perform such a script. The known technical and natural phenomena of block (jam) or aggregation (coagulation) and the progress of those have been considered as processes of clusterisation (nucleation).

Our kinetics models of the latters take into account both aggregation (coagulation) and fragmentation (decay) processes. Previous studies of nonlinear Lotka–Volterra systems [7] brought us to a search for the possible way of a linear description of those very complicated and nonlinear situations. One can substitute a dynamical description of some system with a stochastic one. When we deal with a system of a finite number of particles, a natural way the above substitution to be done is in the use of a language of that system enumerated states. A state is characterized via population number and probability function to reveal the system itself in this state exactly. Of course, that probability ought to depend upon a probability of something else to be befall. In the case considered it could be an act of coalescence or decay (dissociation, fragmentation, etc., where term used depends on an applied province). Thus, we have just met a product of probabilities. What can one do?

A probability (rate) of the above acts could be dependent on or independent of the system states or its particular attributes. In a case of such a dependence one can say nothing without a special investigation. On the contrary, the rates independence from the above circumstances makes the situation a linear one in

respect of a state probability function. Let us take into account that the rates may be independent not only of space coordinates but cluster size, too (e.g., if coalescence/fragmentation depends on only valency of some chemical clusters). It leads to our key mathematical assumption. It consists in constancy of the above rates. This is why we get linear analytically solved master equations of cluster growth kinetics, which are also an evolutionary type equations.

Use of generating function is the very method to solve most of those problems. But it becomes out of use when a structure uniformity of terms in the r.h.s. of equations discussed is violated as the result of combining aggregation and fragmentation terms, i.e., owing to violation of Markovian semigroup structure of those r.h.s. The latter, as well as symmetry, momenta and other questions not included in this sketch will be considered in more detail in papers to follow, as well as some details of the utilization of a MAPLE-program, which have been used to obtain some analytical results. It should be memorized, that we considered two types of fragmentation: similar fragmentation and arbitrary one. They lead to different results. Such a difference can be realized in terms of a manifestation of distinction between collective and more discrete additive properties of cluster on a very abstract level far off a specific nature of the latters.

Present work is one of the stream originated from famous Smoluchovski's articles [8]. A number of details and some literature index could be found out in lectures of Lushnikov [9].

9. ACKNOWLEDGEMENTS

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APPENDIX

Let

$$\begin{gathered} N_0 = 0, 1, 2, ..., \\ a(n) = a_n, \ n \in N_0, \\ A = \{a : N_0 \to R\}, \\ F = \{f : C \to C, f(z) = \sum_{n \in N_0} a_n z^n\}. \end{gathered} \tag{53}$$

The reversible mapping $Z : A \to F$

$$f = Z(a) \ \ so \ as \ \ f(z) = \sum_{n \in N_0} a(n) z^n \tag{54}$$

has the inverse mapping Z^{-1}

$$a = Z^{-1}(f) \quad so\ as \quad a(n) = \oint f(z)\frac{dz}{z^{n+1}}, \quad n \in N_0. \tag{55}$$

The convolution $\circ$ is operation on the set A such as

$$c = a \circ b \quad iff \quad c(k) = \sum_{j=0}^{k} a(k-j)b(j). \tag{56}$$

The mapping Z is a morphism of semigroup $(A, \circ)$ to semigroup $(F, \cdot)$. Z maps the convolution $\circ$ to product $\cdot$:

$$c = a \circ b \to Z(c) = Z(a)Z(b). \tag{57}$$

The associative and commutative semigroup $(A, \circ)$ has the unit Δ:

$$\begin{array}{c} \Delta(0) = 1,\ \Delta(n) = 0,\ n \neq 0, \\ Z(\Delta) = 1,\ Z^{-1}(1) = \Delta. \end{array} \tag{58}$$

Here is a useful formula

$$(a_1 \circ a_2 \circ \ldots \circ a_n)(m) = \sum_{\{j_k \in N_0\}} a_1(j_1) \ldots a_n(j_n)\Delta(m - j_1 - \ldots - j_n) \tag{59}$$

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Кинетика роста кластеров

Исследуются аналитически решаемые модели кинетики роста кластеров с учетом процессов агрегации и фрагментации. Предположение постоянства констант коагуляции и фрагментации позволяет сформулировать модели, линейные по функциям вероятности обнаружения системы в состоянии с заданным числом кластеров.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова, Лаборатории высоких энергий, Лаборатории информационных технологий ОИЯИ и НИФХИ им. Л.Я.Карпова.

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Cluster Growth Kinetics

Processes of some traffic blocking coming into existence are considered as probabilistic ones. We study analytic solutions for models for the dynamics of both cluster qrowth and cluster growth with fragmentation in the systems of finite number of objects. Assuming rates constancy of both coalescence and fragmentation, the models under consideration are linear on the probability functions.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, the Laboratory of High Energies, the Laboratory of Information Technologies of JINR and at the Karpov NIFCHI.

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