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A-DEPENDENCE OF  $\pi^0$ -MESON PRODUCTION  
IN PROTON-NUCLEUS AND NUCLEUS-NUCLEUS  
COLLISIONS AT HIGH ENERGIES

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# 1 Introduction

A study of the  $A$ -dependence of particle production in  $h - A$  and  $A - A$  collisions is traditionally connected with phenomena of nuclear matter influence on particle formation. The difference between the cross sections of particle production on free and bound nucleons is normally considered as an indication of new physics phenomena (EMC-effect [1, 2],  $J/\psi$ -suppression [3, 4], and enhanced  $A$ -dependence in hadron production [5]).

The hadron with a large transverse momentum is believed to be produced by large  $p_T$  parton scatterings. The partons retain information about the collision during hadronization. Therefore, the features of single inclusive particle spectra in nuclear collisions are of interest to search for the phase transition of nuclear matter and a quantitative test of QCD. Neutral  $\pi^0$ -mesons are mainly observed via the decay  $\pi^0 \rightarrow 2\gamma$ . The decay amplitude is described by an axial anomaly [6, 7]. We assume that this mechanism can be modified by nuclear environment and can be sensitive to the phase transition. A nuclear modification of parton distributions inside nuclei is another interesting problem [8]-[11].

The  $A$ -dependence of the  $p - A$  cross section is usually presented in the form of  $\sigma = \sigma_0 \cdot A^{\alpha(x, p_\perp)}$ . Here,  $\sigma_0$  is the cross section on a free nucleon, and the factor  $\alpha(x, p_\perp)$  characterizes the influence of nuclear matter on the mechanism of particle formation. A strong dependence of  $\alpha(x, p_\perp)$  on  $p_\perp$  was found in [5]. An "anomalous nuclear enhancement",  $\alpha > 1$ , was interpreted as an effect of multiple scattering of incident partons [12].

Numerous experimental results on particle spectra measured in  $p - A$  and  $A - A$  collisions at BNL, CERN, and Fermilab [13, 14, 15] over a wide energy and transverse momentum range show that the shapes of the distributions are not a simple exponent in any representation ( $p_\perp, m_\perp, m_\perp - m_0$  etc.). The deviations from a pure exponent in  $m_\perp$ -representation are discussed in [16, 17, 18]. The slope constants used to characterize the spectra depend on the particle type, rapidity, centrality and energy of collisions, as well.

A detailed review of the  $A$ -dependence of particle production in  $h - A$  and  $A - A$  collisions can be found in [19, 20, 21],[22]-[24] (see also references therein).

One of the methods to study the properties of nuclear matter is to search for the violation of scaling laws established in elementary collisions ( $l - p, p - p, \bar{p} - p$ , etc.). In this paper, we study the  $A$ -dependence of  $z$ -scaling for  $\pi^0$ -meson production over a high  $p_\perp$  range in  $p - A$  and  $A - A$  collisions. The scaling was proposed in [25] to describe the feature of charged hadron production in  $p - p$  and  $\bar{p} - p$  collisions. The idea of the scaling was developed for the analysis of direct photon production in  $p - p$  [26],  $\bar{p} - p$  [27] and  $p - A$  [28] collisions. The scaling properties of jet production in  $\bar{p} - p$  and  $p - p$  collisions were analysed in [29].

The general concept of scaling is based on such fundamental principles as self-similarity, locality, fractality and scale-relativity. The first one reflects the dropping of certain dimensional quantities or parameters out of a physical picture of interactions. The second principle concludes that the momentum-energy conservation law is locally valid for interacting constituents. The third fractality principle says that both the structure of interacting particles and their formation mechanism are self-similar over a kinematic range. The fourth one, a scale relativity principle, states that the structures of interaction and interacting objects reveal self-similarity and fractality on any scale [31, 32].

As shown in [25, 32, 33, 34], the experimental observables, inclusive cross section

$Ed^3\sigma/dq^3$  and the multiplicity density of charged particles  $dN/d\eta|_{\eta=0} = \rho(s)$  can be used to obtain a new presentation ( $z$ -presentation) of data. The scaling function  $\psi(z)$  is found to be independent of center-of-mass energy  $\sqrt{s}$  and the angle of produced particle  $\theta$  over a wide kinematic range. The properties of the scaling are assumed to reflect the fundamental features of particle structure, interaction and production. The scaling function  $\psi(z)$  describes the probability to form the hadron with formation length  $z$ . The existence of the scaling means that the hadronization mechanism of particle production reveals such fundamental properties as self-similarity, locality, fractality and scale-relativity.

The  $A$ -dependence of  $z$ -presentation for charged hadron data [5, 19] was studied in [33]. The symmetry property of the scaling function  $\psi(z)$  under the scale transformation  $z \rightarrow \alpha z$ ,  $\psi \rightarrow \alpha^{-1}\psi$  was investigated. It was found that  $\alpha$  depends on the atomic number only. The properties of  $z$ -scaling for charged hadron production in  $p - A$  collisions are used to predict the particle yields at RHIC energies.

The  $A$ -dependence of  $z$ -scaling for  $\pi^0$ -meson production in  $p - A$  and  $A - A$  collisions is studied in this paper. Experimental data [35]-[38] and [39]-[43] are used for the analysis. The dependence  $\alpha = \alpha(A)$  is used to compare the scaling function  $\psi(z)$  for different nuclei. The properties of  $z$ -scaling for  $\pi^0$ -meson production in  $p - A$  and  $A - A$  collisions are used to predict particle yields at high energies.

The paper is organized as follows. A general concept of  $z$ -scaling and the method of constructing the scaling function for hadron production in  $p - A$  collisions is shortly described in Section 2. New results on the  $A$ -dependence of  $z$ -scaling for  $\pi^0$ -meson production on nuclei from  $\alpha$  to  $Pb$  based on the analysis of the experimental data, discussion of the obtained results, physical interpretation of the scaling function and the variable  $z$  are presented in Section 3. Conclusions are summarized in Section 4.

## 2 Method. Properties of $z$ -scaling

In this section, we would like to remind of the basic ideas of  $z$ -scaling dealing with the investigation of the inclusive process

$$P_1 + P_2 \rightarrow q + X. \quad (1)$$

The momenta and masses of colliding nuclei and inclusive particles are denoted by  $P_1, P_2, q$  and  $M_1, M_2, m_1$ , respectively. The gross features of the inclusive particle distributions for reaction (1) at high energies can be described in terms of the corresponding kinematic characteristics of the exclusive subprocess written in the symbolic form [44],

$$(x_1 M_1) + (x_2 M_2) \rightarrow m_1 + (x_1 M_1 + x_2 M_2 + m_2). \quad (2)$$

The parameter  $m_2$  is introduced in connection with internal conservation laws (for isospin, baryon number, and strangeness). The  $x_1$  and  $x_2$  are the scale-invariant fractions of the incoming four-momenta  $P_1$  and  $P_2$  of colliding objects. The cross section of process (1) is assumed to be expressed via the cross section of the corresponding parton subprocesses depending on a minimum energy which is necessary for the production of the secondary particle with mass  $m_1$  and four-momentum  $q$ .

## 2.1 Fractions $x_1$ and $x_2$

The elementary parton-parton collision is considered as a binary subprocess which is satisfied the condition

$$(x_1 P_1 + x_2 P_2 - q)^2 = (x_1 M_1 + x_2 M_2 + m_2)^2. \quad (3)$$

The equation describes the 4-momentum conservation law for an elementary subprocess. To connect kinematic and dynamic characteristics of the interaction the coefficient  $\Omega$  is introduced. It is chosen in the form

$$\Omega(x_1, x_2) = m(1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}, \quad (4)$$

where  $m$  is the mass constant and  $\delta_1$  and  $\delta_2$  are the factors relating the fractal structure of colliding objects [32]. The fractions  $x_1$  and  $x_2$  are determined to maximize the value of  $\Omega(x_1, x_2)$ , simultaneously fulfilling condition (3)

$$d\Omega(x_1, x_2)/dx_1|_{x_2=x_2(x_1)} = 0. \quad (5)$$

Prominent expressions for  $x_1$  and  $x_2$  as a function of the momenta and masses of colliding and produced particles were found in [32].

Equation (3) satisfies the 4-momentum conservation law in the whole phase space. The variables  $x_{1,2}$  are equal to unity along the phase space limit and cover the full phase space accessible at any energy. The threshold condition for process (1) can be written as follows

$$(M_1 + M_2 + m_2)^2 + E^2 - m_1^2 \leq (\sqrt{s_A} - E)^2. \quad (6)$$

Here,  $\sqrt{s_A}$  and  $E$  are the energy in the center-of-mass of the reaction and the energy of produced inclusive particle. Inequality (6) bounds kinematically the maximum energy  $E$  of the inclusive particle  $m_1$  in the c.m.s. of reaction (1).

## 2.2 Scaling function $\psi(z)$ and variable $z$

In accordance with the self-similarity principle, we search for the solution depending on a single scaling variable  $z$  in the form

$$\psi(z) \equiv \frac{1}{\langle N \rangle \sigma_{inel}} \frac{d\sigma}{dz}. \quad (7)$$

Here,  $\sigma_{inel}$  is the inelastic cross section and  $\langle N \rangle$  is the average multiplicity of charged particles. The function  $\psi(z)$  has to be dependent on the scaling variable  $z$ . We would like to note that the existence of such a solution is not evident in advance. All the quantities refer to  $pA$  interactions. The function  $\psi$  expressed via the invariant differential cross section for the production of the inclusive particle  $m_1$  is introduced as follows (see [32])

$$\psi(z) = -\frac{\pi s_A}{\rho_A \sigma_{inel}} J^{-1} E \frac{d\sigma}{dq^3}. \quad (8)$$

Here,  $s_A$  is the the center-of-mass energy squared of the corresponding  $N - A$  system and  $A$  is the atomic weight. The factor  $J$  is a known function of kinematic variables [32]. Expression (8) relates the inclusive differential cross section and the average multiplicity

density  $\rho_A(s, \eta) = d \langle N \rangle / d\eta$  to the scaling function  $\psi(z)$ . The combination  $y = 0.5 \ln(\lambda_2/\lambda_1)$  is approximated to (pseudo)rapidity  $\eta$  at high energies.

A new variable  $z$  is chosen to reflect self-similarity (scale invariance) as a general pattern of hadron production in accordance with the ansatz suggested in [32]

$$z = \frac{\sqrt{\hat{s}_\perp}}{\Omega \cdot \rho(s)}, \quad (9)$$

where  $\hat{s}_\perp^{1/2}$  is the transverse kinetic energy of subprocess (2);  $\Omega$  is the fractal measure given by Eq.(4) and  $\rho(s) = dN/d\eta|_{\eta=0}$  is the average multiplicity density of charged particles produced in the central rapidity region of the corresponding nucleon-nucleon interaction.

We would like to note that the form of  $z$ , as defined by Eqs. (9) and (4), determines its variation range. The boundaries of the range are 0 and  $\infty$ . These values are scale independent and kinematically accessible at any energy.

### 2.3 Fractality and scale-relativity

Fractality as a general principle in particle and nuclear physics means that the internal structure of particles and their interactions revealing self-similarity on any scale is described by power laws [31, 32].

Equation (3) describes the 4-momentum conservation law for an elementary subprocess. The equation, written in the form  $x_1 x_2 - x_1 \lambda_2 - x_2 \lambda_1 = \lambda_0$ , is a covariant one under the scale transformation

$$\lambda_{1,2} \rightarrow \rho_{1,2} \cdot \lambda_{1,2}, \quad x_{1,2} \rightarrow \rho_{1,2} \cdot x_{1,2}, \quad \lambda_0 \rightarrow \rho_1 \cdot \rho_2 \cdot \lambda_0. \quad (10)$$

The transformation with the scale parameters  $\rho_{1,2}$  allows us to consider the collision of complex objects in terms of a suitable subprocess of interacting elementary constituents. It is reasonable to use  $\rho_1 = 1$ ,  $\rho_2 = A_2$  and  $\rho_1 = A_1$ ,  $\rho_2 = A_2$  for the description of  $p-A$  and  $A-A$  interactions, respectively. Here,  $A_1$  and  $A_2$  are the corresponding atomic weights.

The coefficient  $\Omega$  given by Eq.(4) connects the kinematic and dynamic characteristics of the interaction. The factors  $\delta_1$  and  $\delta_2$  are the fractal dimensions of colliding objects. The fractal structure itself is defined by the structure of interacting constituents which is not an elementary one either. In this scheme, hadron-hadron, hadron-nucleus and nucleus-nucleus collisions are considered as an interaction of two fractals.

In the case of collisions of asymmetric objects, the approximation for the measure  $\Omega$  is written as

$$\Omega = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2} = (1 - \bar{x}_1)^{\bar{\delta}_1} (1 - \bar{x}_2)^{\bar{\delta}_2}. \quad (11)$$

The equation shows a correlation between the fraction  $x_i$  and the fractal dimension  $\delta_i$ . (The scale transformation can be chosen so that  $\bar{\delta}_1 = \bar{\delta}_2$ .) Thus, the measure is an invariant under simultaneity of the scale transformation of Lorenz invariants  $x_i$  and multiplicative transformation of  $\delta_i$ .

Taking into account the scale transformation Eq.(10) the measure of the interaction can be written as

$$\Omega = V^\delta, \quad (12)$$

where  $\delta$  is the coefficient (fractal dimension) describing the fractal structure of the elementary collision. The factor  $V$  is part of the full phase-space of fractions  $\{x_1, x_2\}$

corresponding to such parton-parton collisions in which the inclusive particle can be produced. The fractal property of the collision reveals itself so that only the part of all multiscatterings corresponding to the phase space  $V^\delta$  produces the inclusive particle.

### 3 $A$ -dependence of $z$ -scaling for $\pi^0$ -mesons

#### 3.1 $p - A$ collisions

Four sets of experimental data for  $\pi^0$ -meson production in  $pA$  collisions were analysed. The first one [35] includes the results of measurements of the invariant cross section  $Ed^3\sigma/dq^3$  for the production of  $\pi^0$ -mesons at a large transverse momentum  $q_\perp$  and  $\sqrt{s} = 19 \text{ GeV}$  on the  $Be, C$  and  $Al$  targets. The measurements were made over a rapidity range of  $-0.75 < y < 0.2$ . The  $q_\perp$  range for the data is  $2.15 < q_\perp < 4.75 \text{ GeV}/c$ . The second data set was obtained by E706 at the Tevatron using a proton beam with an incident momentum of  $500 \text{ GeV}/c$ . A  $Cu$  nuclear target was used [37]. The produced  $\pi^0$ -mesons were registered over a transverse momentum range of  $q_\perp = 4.0 - 8.0 \text{ GeV}/c$  and a rapidity range of  $|y| < 0.7$ . The same Collaboration measured the inclusive production of high- $p_T$  neutral mesons and direct photons by 530 and 800  $\text{GeV}/c$  protons on  $Be$  target over a the kinematic range of  $3.5 < q_\perp < 12 \text{ GeV}/c$  with central rapidities to a high accuracy [38]. The WA80 Collaboration measured  $\pi^0$ -meson spectra on heavy nucleus  $Au$  over a  $0.45 - 3.0 \text{ GeV}/c$  momentum and  $1.5 < y < 2.1$  rapidity range [36]. The same Collaboration presented [39] the final results on high energy precision neutral  $\pi^0$ -meson spectra from  $S - S$  and  $S - Au$  collisions at  $\sqrt{s} = 19.4 \text{ AGeV}$  and over a range of  $0.3 < p_\perp < 4.0 \text{ GeV}/c$  and  $2.1 < y < 2.9$ .

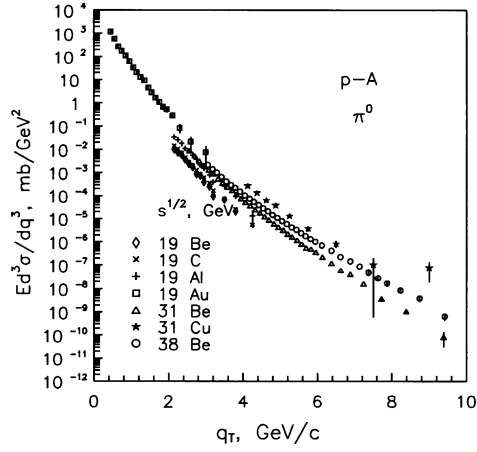
The procedure of  $z$ -analysis of the experimental data is described in detail in [32, 33]. According to the procedure, the function  $\psi$  is calculated for every nucleus using the normalization factor  $\sigma_{inel}^{pA}/\sigma_{inel}^{pp}$  [45] instead of  $\sigma_{inel}$  in the expression for the inclusive cross section. The factor  $\sigma_{inel}^{pA}$  is the total inelastic cross section for  $pA$  interactions. The relevant multiplicity densities of charged particles obtained by the Monte Carlo simulation generator HIJING [46, 47] for different nuclei ( $A = 7 - 197$ ) are parametrized by the formula  $\rho_A(s) \simeq 0.67 \cdot A^{0.18} \cdot s^{0.105}$  [32]. The symmetry transformations

$$z \rightarrow \alpha \cdot z, \quad \psi \rightarrow \alpha^{-1} \cdot \psi \quad (13)$$

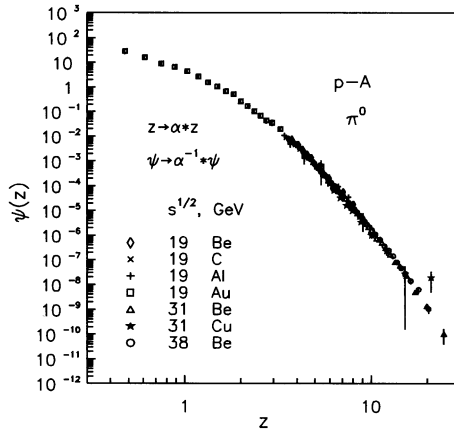
of the function  $\psi(z)$  and the argument  $z$  are used to compare the functions  $\psi$  for different nuclei.

Figure 1(a) shows the dependence of the inclusive cross section  $Ed^3\sigma/dq^3$  for  $\pi^0$ -mesons produced in  $pA$  collisions on transverse momentum  $q_\perp$  at  $\sqrt{s} = 19, 31$  and  $38 \text{ GeV}$ . The dependence of the scaling function  $\psi$  on  $z$  of the same experimental data is shown in Figure 1(b). The results presented in Figure 1(a) confirm the validity of energy independence of  $\psi(z)$  for the light ( $Be$ ) and heaviest ( $Au$ ) nuclei. In contrast to the  $z$ -presentation of experimental data, the  $q_\perp$ -presentation of the same data demonstrates a strong energy dependence. The property of the function  $\psi(z)$  allows us to say that the mechanism of hadron formation demonstrates a feature depending on general properties of nuclear matter.

It should be noted that the slope parameter  $\beta$  of  $\psi(z) \sim z^{-\beta}$  increases with  $z$ . We assume that the asymptotic behaviour  $d\beta/dz \rightarrow const$  would take place over a high- $z$

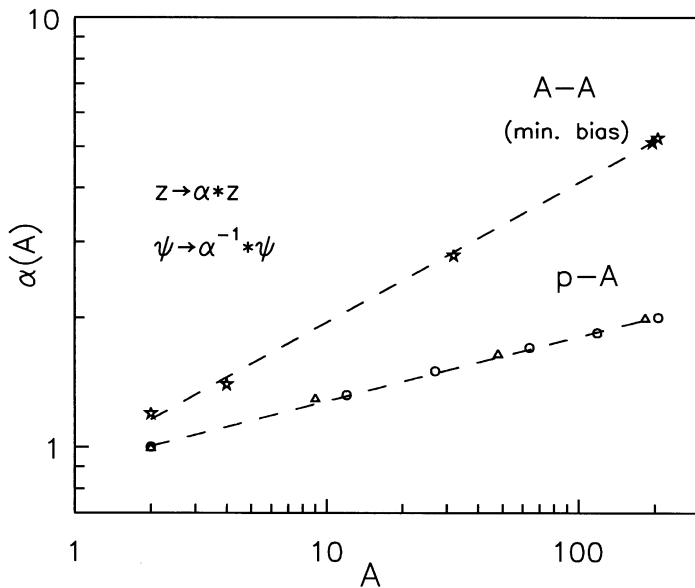


a)



b)

**Figure 1.** (a) Dependence of the inclusive cross section of  $\pi^0$ -meson production on transverse momentum  $q_{\perp}$  in  $p - A$  collisions. Experimental data are taken from [35]-[38]. (b) The corresponding scaling function  $\psi(z)$ .



**Figure 2.**  $A$ -dependence of transformation parameter  $\alpha$  of proton-nucleus and minimum bias nucleus-nucleus collisions. The line for  $p - A$  obtained by fitting of the points (o,  $\Delta$ ) is the same one as for charged ( $\pi^\pm, K^\pm, \bar{p}$ ) [33] and neutral ( $\pi^0$ ) hadrons.



range as it was found for direct- $\gamma$  [26, 28], jet [29] and  $\pi^0$  [30] production in nucleon-nucleon collisions.

The function characterizing the influence of matter on hadron formation,  $\alpha = \alpha(A)$ , found in [33] was used in our calculation. The function is independent of energy  $\sqrt{s}$  and transverse momentum  $q_{\perp}$ . The dependence of the function  $\alpha(A)$  on the atomic weight  $A$  is shown in Figure 2.

Thus, the results presented in Figures 1(b) and 2 illustrate the existence of the  $A$ -dependence of  $z$ -scaling for  $\pi^0$ -meson production in  $pA$  collisions at a high colliding energy  $\sqrt{s}$  over a high transverse momentum range and a central rapidity range. The results are in good agreement with the results obtained for charged hadrons produced in  $pA$  collisions [33].

We use the properties of  $z$ -scaling to calculate the cross section of  $\pi^0$ -meson production in  $pA$  collisions at RHIC energies. Figures 3(a,b) and 4(a,b) show the dependence of the inclusive differential cross section of  $\pi^0$ -mesons produced in  $p-Be$ ,  $p-Si$ ,  $p-Al$  and  $p-Au$  collisions on transverse momentum  $q_{\perp}$  at energy  $\sqrt{s}$  and  $\theta_{NN} \simeq 90^\circ$ . The points and solid lines are the results obtained at different  $\sqrt{s}$  ( $\circ - 63$  GeV,  $\circ - 200$  GeV,  $+ - 500$  GeV). The experimental data,  $\triangle - 19, 31$  GeV, are taken from [35, 38]. It should be noted that the obtained  $q_{\perp}$ -dependence of the cross section demonstrates a non-exponential behaviour over a high transverse momentum region ( $q_{\perp}$  up to 11 GeV/c or more) for all types of nuclear targets.

### 3.2 $A - A$ collisions

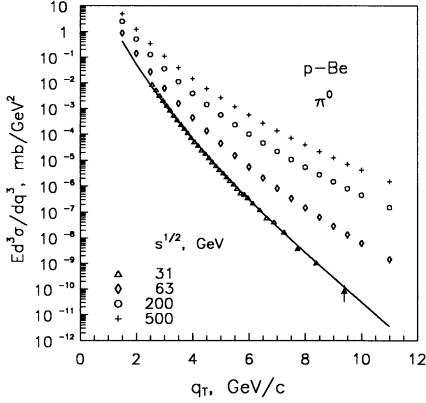
It is usually assumed that high energy-density nucleon matter produced in heavy-ion collisions could give an indication of phase transition to a quark-gluon plasma. High- $p_T$   $\pi^0$ -meson spectra should be sensitive to the transition [49]. Recently, the WA80 [39] and WA98 [40] Collaborations have measured the  $\pi^0$ -meson spectra of  $S - S$ ,  $S - Au$  and  $Pb - Pb$  collisions at  $p_{lab} = 200$  AGeV/c and 158 AGeV/c, respectively. Therefore, we would like to compare  $p_T$ -dependence of the cross section for proton-nucleus and nucleus-nucleus collisions using the available experimental data [36, 39, 40].

In our analysis, we use the data for high- $p_T$   $\pi^0$  production in light-ion ( $d - d$ ,  $\alpha - \alpha$ ) collisions obtained at the CERN ISR [41, 42, 43]. The cross sections were measured for  $p_T > 2$  GeV/c at an angle of about  $90^\circ$  and  $\sqrt{s} = 26$  and 31 GeV.

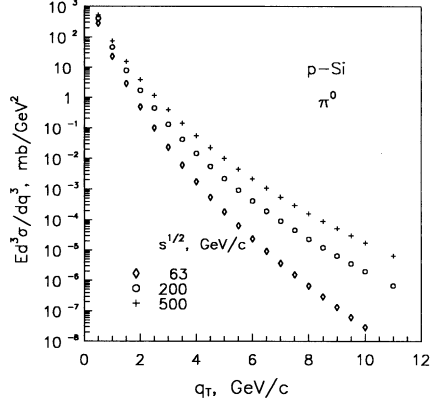
A nontrivial  $p_T$ -dependence of the spectra in  $A - A$  collisions could evidence a nuclear modification of the mechanism of particle formation.

Figure 5(a) shows the dependence of the  $\pi^0$ -meson cross section for light-ion ( $d - d$ ,  $\alpha - \alpha$ ) and minimum bias heavy-ion ( $p - Au$ ,  $S - S$ ,  $S - Au$ ,  $Pb - Pb$ ) collisions on transverse momentum. The experimental data are taken from [36, 39]-[43]. It is essential that all the data demonstrate a non-exponential behaviour as a function of  $q_{\perp}$ . The results of  $z$ -presentation of the same data are shown in Figure 5(b). To compare  $\psi(z)$  for different nucleus-nucleus collisions, the scaling transformation (13) was used. The corresponding function  $\alpha(A)$  is shown in Figure 2. For  $A - B$  collisions, the parameter  $\alpha$  should depend on the atomic weights of both nuclei  $A$  and  $B$ . We do not have enough experimental data for asymmetrical  $A - B$  collisions at high  $\sqrt{s}$  and  $p_T$ , and so the  $A$ -dependence of the transformation parameter  $\alpha$  was only found for symmetrical  $A - A$  collisions.

The results shown in Figure 5(b) demonstrate the  $A$ -dependence of the scaling function  $\psi(z)$  for  $\pi^0$ -meson production in  $A - A$  collisions at a high colliding energy  $\sqrt{s}$  over a central rapidity range. As seen from Figure 5(b), the slope parameter  $\beta$  increases with  $z$ .

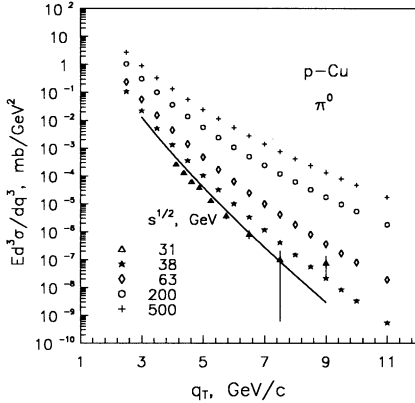


a)

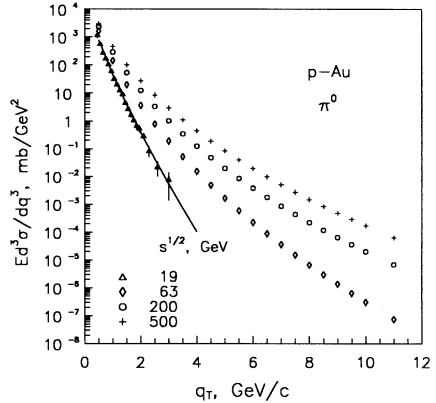


b)

**Figure 3.** Dependence of the inclusive differential cross section of  $\pi^0$ -meson production in  $p - Be$  (a) and  $p - Si$  (b) collisions on transverse momentum  $q_{\perp}$  at different colliding energy  $\sqrt{s}$  and  $\theta_{NN} \simeq 90^\circ$ . Points ( $\diamond, \circ, +$ ) and solid line are the calculated results. Experimental data,  $\Delta - 31 GeV$ , are taken from [38].

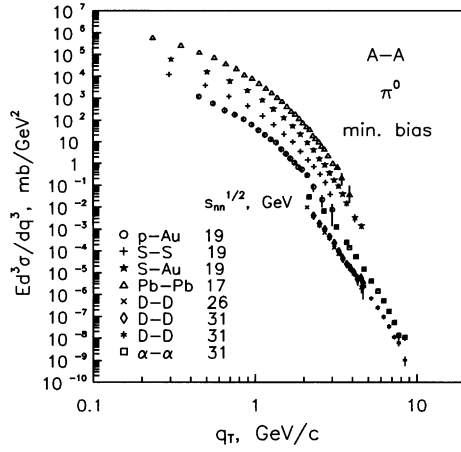


a)

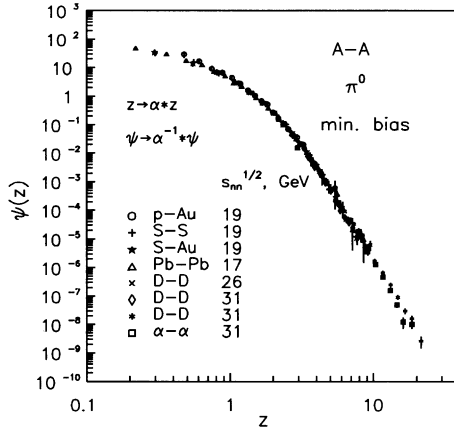


b)

**Figure 4.** Dependence of the inclusive differential cross section of  $\pi^0$ -meson production in  $p - Cu$  (a) and  $p - Au$  (b) collisions on transverse momentum  $q_{\perp}$  at different colliding energy  $\sqrt{s}$  and  $\theta_{NN} \simeq 90^\circ$ . Points ( $\diamond, \circ, +$ ) and solid lines are the calculated results. Experimental data,  $\Delta - 19$  and  $31 GeV$ , are taken from [36, 37].



a)



b)

**Figure 5.** (a) Dependence of the inclusive cross section of  $\pi^0$ -meson production on transverse momentum  $q_{\perp}$  in nucleus-nucleus minimum bias collisions. Experimental data are taken from [36],[39]-[43]. (b) The corresponding scaling function  $\psi(z)$ .

We assume that the asymptotic regime (the power law for  $\psi(z) \sim z^{-\beta}$ ) can be observed over a high- $p_T$  range, only. The results mean that the formation of  $\pi^0$ -mesons in nucleus-nucleus collisions is similar to that in proton-nucleus ones.

We use the properties of  $z$ -scaling to calculate the cross section of  $\pi^0$ -meson production in minimum bias  $A - A$  collisions at RHIC energies. Figures 6(a,b) and 7(a,b) show the dependence of the inclusive differential cross section of  $\pi^0$ -mesons produced in  $\alpha - \alpha$ ,  $S - S$ ,  $Au - Au$  and  $Pb - Pb$  collisions on transverse momentum  $q_\perp$  at energy  $\sqrt{s}$  and  $\theta_{NN} \simeq 90^\circ$ . The points and solid lines are the results obtained at different  $\sqrt{s}$  ( $\diamond - 63$  GeV,  $\circ - 200$  GeV,  $+ - 500$  GeV). The experimental data,  $\triangle - 17, 19$  and  $31$  GeV, are taken from [39, 40, 41, 43].

The verification of the predicted results is of interest for a more detailed study of the  $A$ -dependence of  $\pi^0$  spectra over a high- $p_T$  range and for a search for signatures of nuclear matter phase transition.

## 4 Discussion

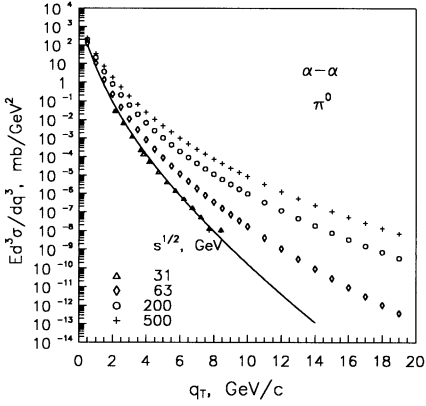
In this section, we would like to discuss the obtained results qualitatively. First of all, it should be emphasized that the scaling properties of  $\pi^0$ -meson production were observed at a high colliding energy and transverse momentum of produced hadron. Thus, the point-likeness of the constituents is defined by colliding energy  $\sqrt{s}$  and the transverse momentum of registered hadron. This means that the scaling function describes the fragmentation process of point-like produced partons into observable hadrons. Our analysis of numerous experimental data confirms the scaling. One of the important properties of the scaling function  $\psi(z)$  is a power law. The power regime is found at a high transverse momentum of  $q_\perp > 4$  GeV/ $c$ .

The power laws usually reflect the fractal structure of objects under scale transformation [31]. The power law found for hadron production over a high transverse momentum range is assumed here to describe the fractal structure of colliding objects, their constituents and interactions. The fractal measure  $\Omega(x_1, x_2)$  given by (4) determines all possible configurations of elementary interactions that lead to the production of the inclusive particle. In our case, the measure is factorized:  $\Omega(x_1, x_2) = \Omega_1(x_1) \cdot \Omega_2(x_2)$ . A single measure  $\Omega_i(x_i)$  described by a power dependence in the space of fractions  $\{x_1, x_2\}$  reflects the number of constituent configurations in the colliding object involved in the production of the inclusive particle. The measure is characterized by the fractal dimension  $\delta_i$ . The fractal dimension of the nucleus for  $\pi^0$ -meson production is expressed via the nucleon fractal dimension  $\delta_A = \delta_N \cdot A$  as for charged hadrons. Additivity of the fractal dimension of the nucleus is an interesting feature of the fractal measure  $\Omega(x_1, x_2)^*$ . As can be seen from the definition of  $z$  (4), the variable describes the energy of an elementary constituent collision per initial configuration and per produced particle.

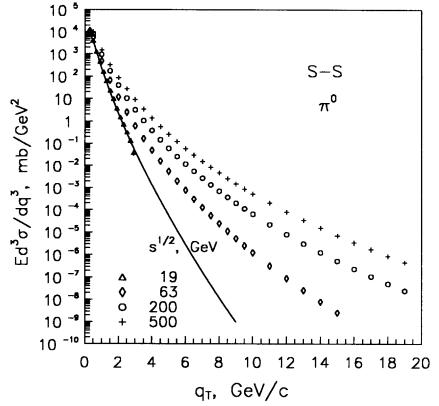
The property of scale covariance of the function  $\psi$  under scale transformation (10) shows that hadron formation (hadronization) as a process reveals self-similarity in a nuclear environment. The fractal dimension  $\delta_A$  is a quantitative characteristic of the hadron structure in surrounding matter. Therefore, the change of the fractal dimension of particle

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\* We assume that additivity of the fractal dimension of the measure  $\Omega(x_1, x_2)$  can be violated due to overlapping nucleons of colliding nucleus.

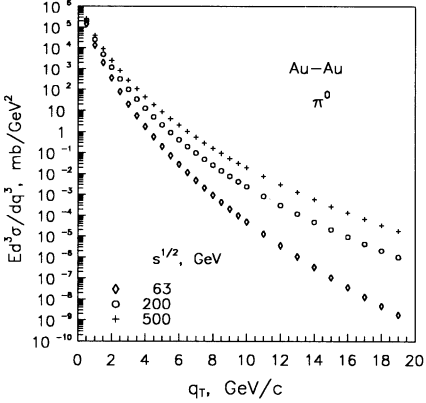


a)

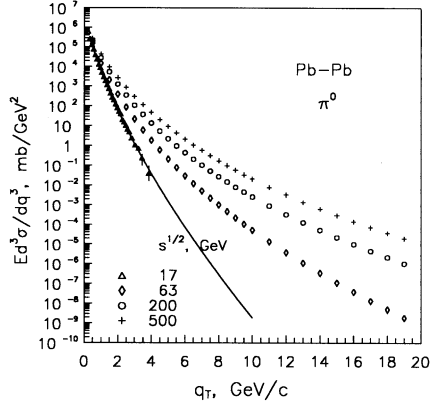


b)

**Figure 6.** Dependence of the inclusive differential cross section of  $\pi^0$ -meson production in  $\alpha - \alpha$  (a) and  $S - S$  (b) collisions on transverse momentum  $q_{\perp}$  at different colliding energy  $\sqrt{s}$  and  $\theta_{NN} \simeq 90^\circ$ . Points ( $\diamond, \circ, +$ ) and solid lines are the calculated results. Experimental data,  $\Delta - 19, 31 \text{ GeV}$ , are taken from [39, 42, 43].

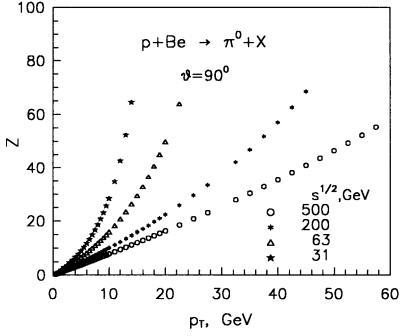


a)

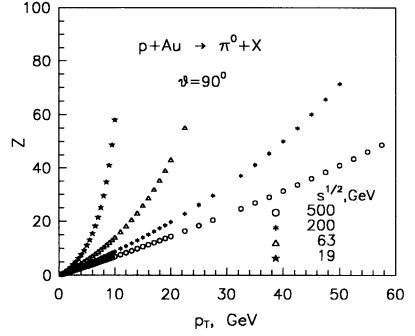


b)

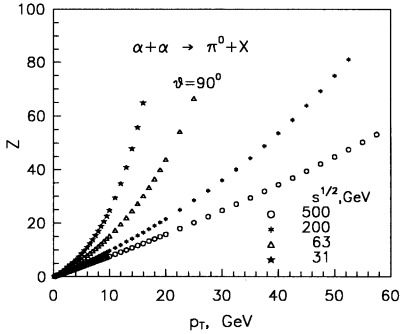
**Figure 7.** Dependence of the inclusive differential cross section of  $\pi^0$ -meson production in  $Au - Au$  (a) and  $Pb - Pb$  (b) collisions on transverse momentum  $q_{\perp}$  at different colliding energy  $\sqrt{s}$  and  $\theta_{NN} \simeq 90^\circ$ . Points ( $\diamond, \circ, +$ ) and solid line are the calculated results. Experimental data,  $\Delta - 17 \text{ GeV}$ , are taken from [40].



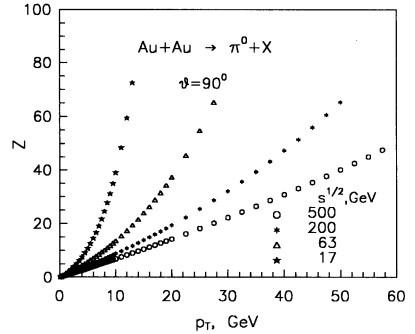
a)



b)



a)



b)

**Figure 8.** Dependence of the variable  $z$  of  $\pi^0$ -meson production in  $p$ - $Be$  (a),  $p$ - $Au$  (b),  $\alpha$ - $\alpha$  (c) and  $Au$ - $Au$  (d) collisions on transverse momentum  $p_{\perp}$  at different colliding energy  $\sqrt{s}$  and  $\theta_{NN} \simeq 90^\circ$ . Points,  $\star$  - 17, 19, 31 GeV,  $\diamond$  - 63 GeV,  $\circ$  - 200 GeV,  $+$  - 500 GeV, are the calculated results.

formation is assumed to be a signature of new physics phenomena (quark compositeness, new type of interaction, phase transition etc.).

In the framework of the proposed scenario, the interaction of colliding objects is the interaction of fractals and the mechanism of hadron formation is considered as a process of construction of complex fractal (hadron) from elementary fractal blocks. The size and structure of blocks depend on the colliding energy and transverse momentum of the produced hadron. The multiscattering of elementary constituents is the main feature of heavy-ion collisions. Fractality is the reflection of this property described by a power law.

The  $A$ -dependence of  $z$ -scaling obtained in our analysis confirms the general features of  $z$ -scaling construction found for charged hadrons [25, 32]. Thus, the influence of surrounding matter on the mechanism of particle formation is described by a smooth function  $\alpha(A)$  depending on the atomic weight  $A$ . We have found that the fractal dimension is not changed by a nuclear medium and the relation  $\delta_A = \delta_N \dot{A}$  is correct.

The  $\pi^0 \rightarrow \gamma\gamma$  process is the dominant mode of the  $\pi^0$ -meson decay. The  $\pi^0$  decay into two photons is anomalous process which takes place due to the existence of the axial anomaly [6, 7]. The decay width is expressed via the invariant amplitude  $B^{\pi^0}$  of the process as follows:  $\Gamma_{\pi^0 \rightarrow 2\gamma} = B^{\pi^0} m_\pi^3 / 64\pi$  [48]. The amplitude to the lowest order in an electric charge can be derived taking into account PCAC and  $SU(N_f)$  axial anomaly  $B^{\pi^0} = -\sqrt{2}/3\pi N_c \alpha / f_\pi$ , where  $f_\pi$  is the pion decay constant and  $N_c$  is the colour degrees of freedom. The amplitude  $B^{\pi^0}$  can be modified by nuclear matter, especially at the critical point corresponding to the nuclear phase transition. The scaling function  $\psi(z)$  describes the hadronization of point-like partons to a real  $\pi^0$ -meson. Therefore, we assume that the axial anomaly should reflect the scaling properties of the process in a nuclear medium.

The search for scaling violations of  $\pi^0$ -meson production in  $p - A$  and  $A - A$  collisions at high energies, especially in the region of high transverse momenta  $q_\perp > 4 \text{ GeV}/c$ , could be very interesting for our understanding of  $\pi^0$  formation. The obtained results show that the formation of  $\pi^0$ -meson in nucleus-nucleus collisions is similar to that in proton-nucleus ones.

Figure 8 shows a  $z - p_T$  plot for  $p - Be$ ,  $p - Au$ ,  $\alpha - \alpha$  and  $Au - Au$  collisions at  $\sqrt{s} = 19 - 500 \text{ GeV}$ . The plot at fixed  $z$  allows one to determine the transverse momentum range where the scaling can be violated. The available experimental data [35]-[38], [39]-[43] give no strong indications of the scaling violation over the range  $z = 4 - 20$ .

## 5 Conclusions

The  $A$ -dependence of  $\pi^0$ -meson production in  $p - A$  and  $A - A$  collisions at high energies in terms of  $z$ -scaling is considered. The experimental data on the inclusive cross sections for  $\pi^0$ -mesons produced on different nuclei ( $A = D, \alpha, Be, C, Al, S, Cu, Au, Pb$ ) over a high transverse momentum range ( $q_\perp = 1. - 9.4 \text{ GeV}/c$ ) were used for the analysis. The momentum of incident proton  $p_{lab}$  changes from 200 to 800  $\text{GeV}/c$ . The function  $\psi(z)$ , describing a new presentation of experimental data, is expressed via the invariant inclusive cross section  $E d^3\sigma/dq^3$  and normalized to the multiplicity density of particles produced in the collisions.

The symmetry transformations of the function  $\psi$  and its argument,  $\psi \rightarrow \alpha^{-1}\psi$  and  $z \rightarrow \alpha z$ , are used to compare  $\psi(z)$  of different  $p - A$  and  $A - A$  collisions. The  $A$ -dependence

of the transformation parameter  $\alpha$  and the fractal dimensions of nuclei  $\delta_A$  for  $p - A$  are found to be the same as for charged hadrons. The slope parameter  $\beta$  of  $\psi(z) \sim z^{-\beta}$  increases with  $z$ . An indication of the asymptotic regime,  $d\beta/dz \rightarrow const$ , is found for both  $p - A$  and  $A - A$  collisions. Our analysis of  $\pi^0$ -meson data  $z$ -presentation shows that hadron formation in proton-nucleus and nucleus-nucleus collisions is very similar.

Thus, the obtained results show that data  $z$ -presentation demonstrates general properties of the particle production mechanism such as self-similarity, locality, scale-relativity and fractality. The properties reveal themselves in  $p - A$  and  $A - A$  collisions and reflect the features of elementary constituent hadronization.

Using the properties of  $z$ -scaling, the dependence of the cross sections of  $\pi^0$ -mesons produced in  $p - A$  and  $A - A$  collisions on transverse momentum over the central range at RHIC energies is predicted.

The dependence of  $z$  on transverse momentum  $q_{\perp}$  as a function of atomic weight and colliding energy  $\sqrt{s}$  is suggested to use as a joint kinematic and dynamic criterion to select the domain where new physical phenomena can be found.

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Токарев М.В., Рогачевский О.В., Дедович Т.Г.  
А-зависимость рождения  $\pi^0$ -мезонов в протон-протонных  
и протон-ядерных взаимодействиях при высоких энергиях

E2-2000-90

Изучается А-зависимость рождения  $\pi^0$ -мезонов в протон-ядерных и ядро-ядерных взаимодействиях при больших поперечных импульсах. Развивается концепция z-скейлинга для описания особенностей рождения  $\pi^0$ -мезонов. Для анализа используются экспериментальные данные по сечениям рождения  $\pi^0$ -мезонов, полученные на ISR, SpS и Tevatron. Установлена А-зависимость масштабного преобразования  $z \rightarrow \alpha \cdot z, \psi \rightarrow \alpha^{-1} \cdot \psi$ . Получено указание на степенной режим поведения  $\psi(z) \sim z^{-\beta}$  при  $p_T > 4$  ГэВ/с. Свойства z-скейлинга используются для предсказательных расчетов сечения рождения  $\pi^0$ -мезонов в центральной области в pA- и AA-взаимодействиях при энергиях RHIC.

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A-Dependence of  $\pi^0$ -Meson Production in Proton-Nucleus  
and Nucleus-Nucleus Collisions at High Energies

E2-2000-90

The A-dependence of  $\pi^0$ -meson production in proton-nucleus and nucleus-nucleus collisions at a high transverse momentum is studied. The concept of z-scaling reflecting the general features of particle interactions is developed for the description of  $\pi^0$ -meson production. Experimental data on the cross section obtained at ISR, SpS and Tevatron are used in the analysis. The A-dependence of scale transformation  $z \rightarrow \alpha \cdot z, \psi \rightarrow \alpha^{-1} \cdot \psi$  is established. An indication of the power law,  $\psi(z) \sim z^{-\beta}$ , at high  $p_T > 4$  GeV/c is found. Based on the properties of z-scaling, the dependence of the cross section of  $\pi^0$ -mesons produced in pA and AA collisions on transverse momentum over the central rapidity range at RHIC energies is predicted.

The investigation has been performed at the Laboratory of High Energies, JINR.

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