

**SYSTEMS OF VARIATIONAL INEQUALITIES
WITH HIERARCHICAL VARIATIONAL INEQUALITY
CONSTRAINTS FOR LIPSCHITZIAN
PSEUDOCONTRACTIONS**

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Abstract. In this paper, we consider the problem of solving a general system of variational inequalities (GSVI) with a hierarchical variational inequality (HVI) constraint for countably many uniformly Lipschitzian pseudocontractive mappings and an accretive operator in a real Banach space. We propose an implicit composite extragradient-like method based on the Mann iteration method, the viscosity approximation method and the Korpelevich extragradient method. Convergence results for the proposed iteration method are also established under some suitable assumptions.

Key Words and Phrases: Implicit composite extragradient-like method, general system of variational inequalities, fixed point, accretive operator, uniform convexity, uniform smoothness.

2010 Mathematics Subject Classification: 49J30, 47H09, 47H10, 47J20.

*This research was partially supported by the Innovation Program of Shanghai Municipal Education Commission (15ZZ068), Ph.D. Program Foundation of Ministry of Education of China (20123127110002) and Program for Outstanding Academic Leaders in Shanghai City (15XD1503100)

**This research was partially supported by the grant MOST 106-2923-E-039-001-MY3.

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Received: December 17, 2017; Accepted: February 9, 2018.

