Record dynamics in spin glassses, superconductors and biological evolution.

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The question:

Is intermittent, logarithmically slow, dynamics, driven by record events, typical of complex systems?

List of content:

- Dynamics of complex systems
- Three models

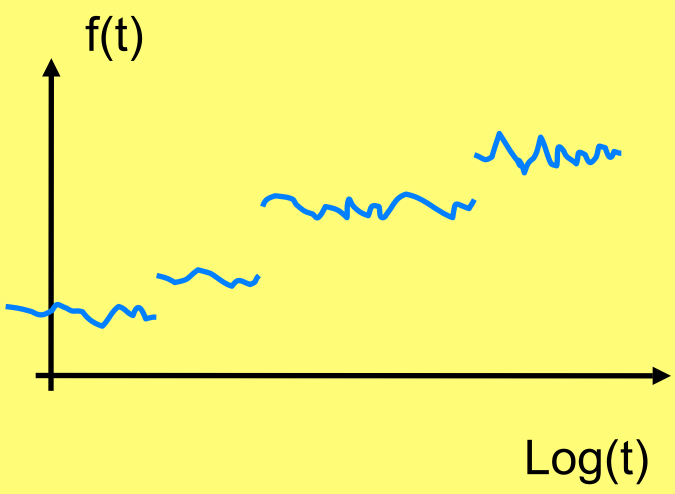
  definition and dynamics
- Manifestation of record dynamics
- Consequences
- Conclusion/summary

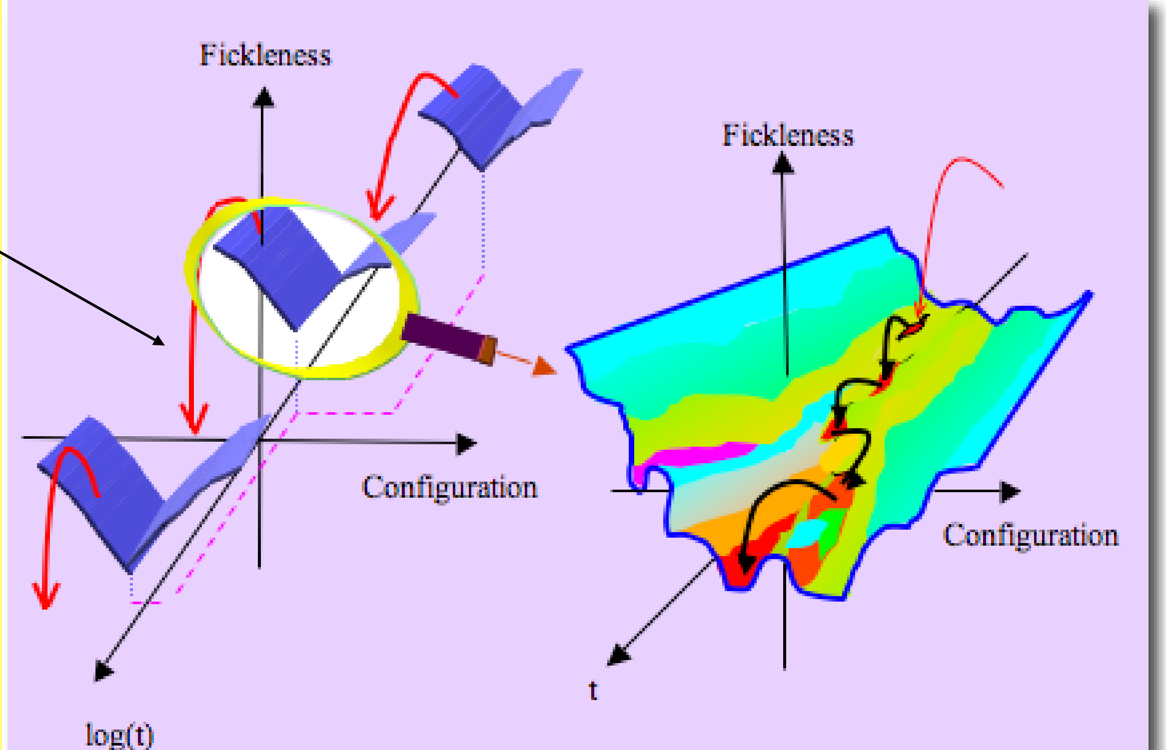
Complex dynamics:

Intermittent, non-stationary

Jumping through collective adaptation space: quake driven

Transitions





Motion within one quasi-stable epoch

The models:

Tangled Nature Model of co-evolving biological species

Restricted Occupancy Model of vortex dynamics in type II superconductors.

Edward-Anderson Spin Glass nearest neighbour Gaussian couplings

The relaxation

Tangled Nature model

collective adaptation: configurations increasingly coupled together.

ROM model

magnetic pressure

Spin Glass

thermal quench

First Model:

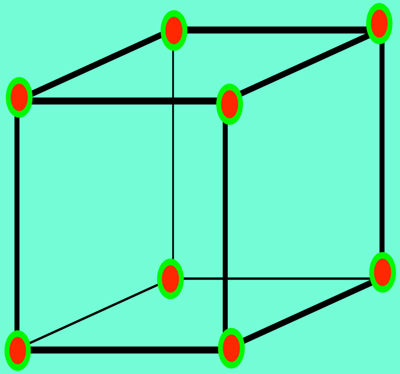
Tangled Nature

Tangled Nature model of evolution

Definition:

* Individuals $\mathbf{S}^{\alpha} = (S_1^{\alpha}, S_2^{\alpha}, \ldots, S_L^{\alpha})$, where $S_i^{\alpha} = \pm 1$

and $\alpha = 1, 2, \ldots, N(t)$



$L = 3$

* Dynamics – a time step:

☻ **Annihilation:**

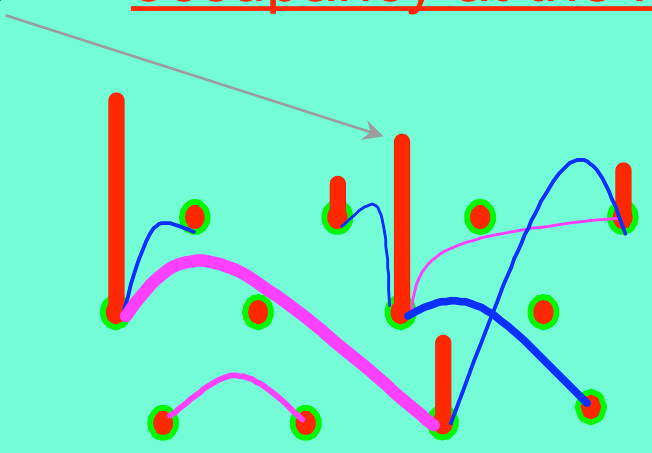
Choose indiv. at random, remove with probability $p_{kill} = const$

☻ Reproduction:

- Choose indiv. at random
- Determine

$$H(\mathbf{S}^{\alpha},t) = \frac{1}{cN(t)} \sum_{\mathbf{S}} J(\mathbf{S}^{\alpha},\mathbf{S})\, n(\mathbf{S},t) - \mu\, N(t)$$

$n(\mathbf{S},t) =$ occupancy at the location $\mathbf{S}$

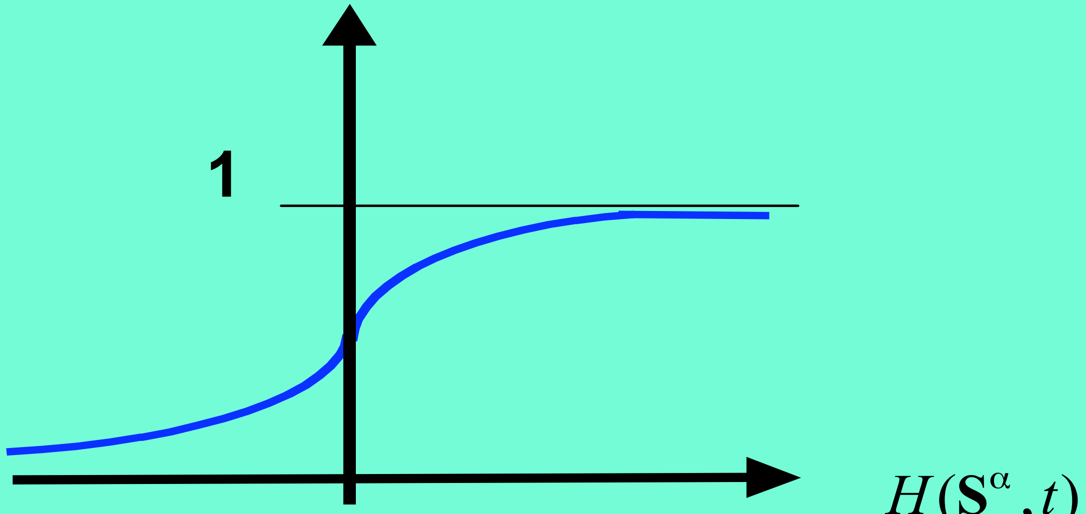


The coupling matrix $J(S, S')$

- Either consider $J(S, S')$ to be uncorrelated
- or to vary smoothly through type space.

from $H(\mathbf{S}^{\alpha}, t)$ reproduction probability

$$p_{off}(\mathbf{S}^{\alpha}, t) = \frac{\exp[H(\mathbf{S}^{\alpha}, t)]}{1 + \exp[H(\mathbf{S}^{\alpha}, t)]} \in [0,1]$$



☻ Asexual reproduction:

Replace

$\mathbf{S}^{\alpha} \rightarrow \mathbf{S}_1^{\alpha}, \; \mathbf{S}_2^{\alpha}$

by two copies

with probability $p_{off}(\mathbf{S}^{\alpha}, t)$

Mutations

- Mutations occur with probability

p_{mut} , i.e.

$$S_i^{\gamma} \mapsto -S_i^{\gamma}$$

Phenomenology

- Long time dynamics
- The evolved networks

Segregation in genotype space

Non Correlated

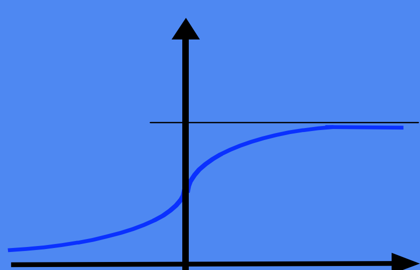
Initiation

Only one genotype

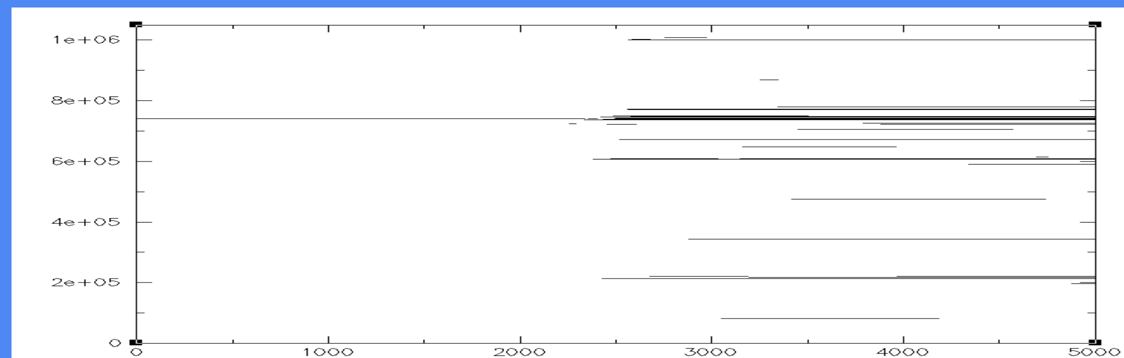
Jn term = 0

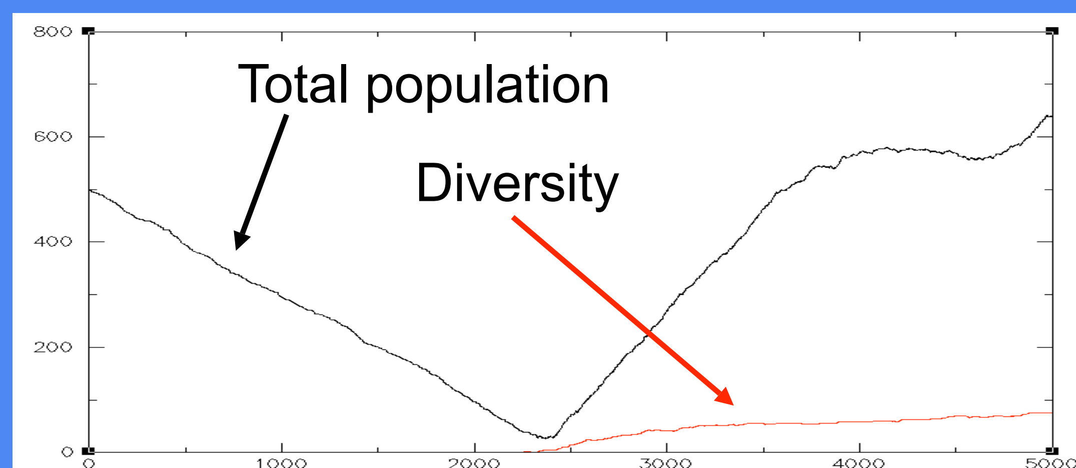
$$H = \frac{k}{N(t)} \sum_{s} Jn - \mu N(t)$$

N(t) adjusts



$$H = -\mu N(t)$$



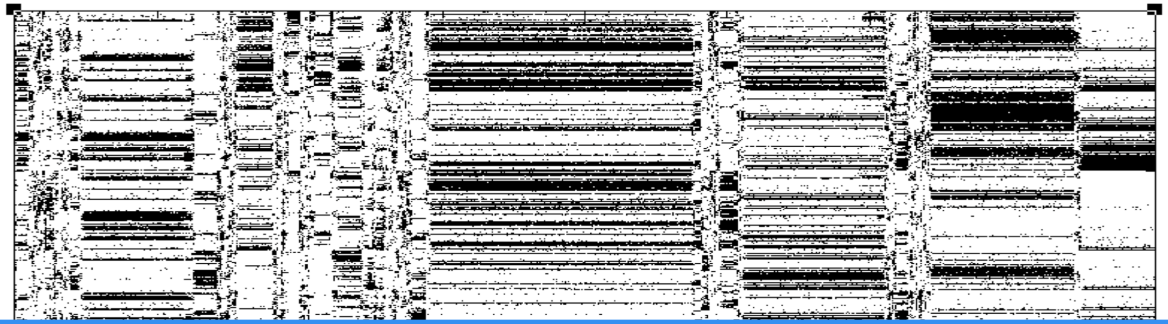


Matt Hall

Intermittency at systems level:

Non Correlated

Type label



 # generations

1 generation
= $N(t)/p_{kill}$

Matt Hall

Intermittency at systems level:

Correlated

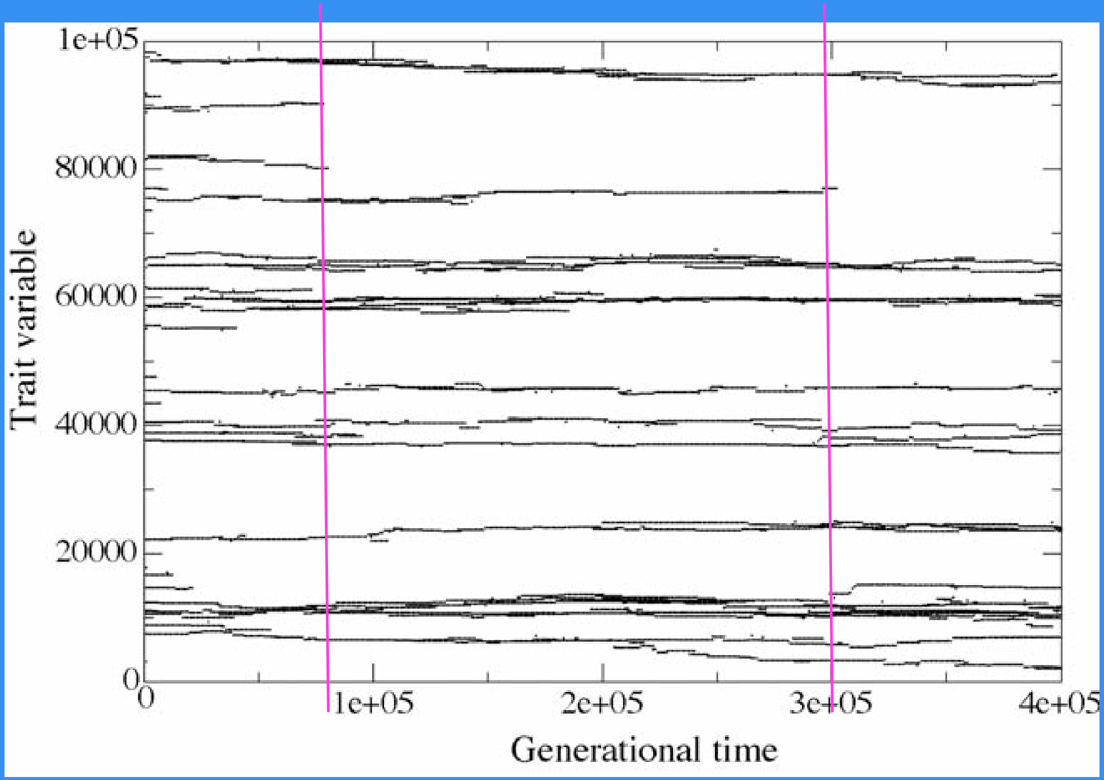


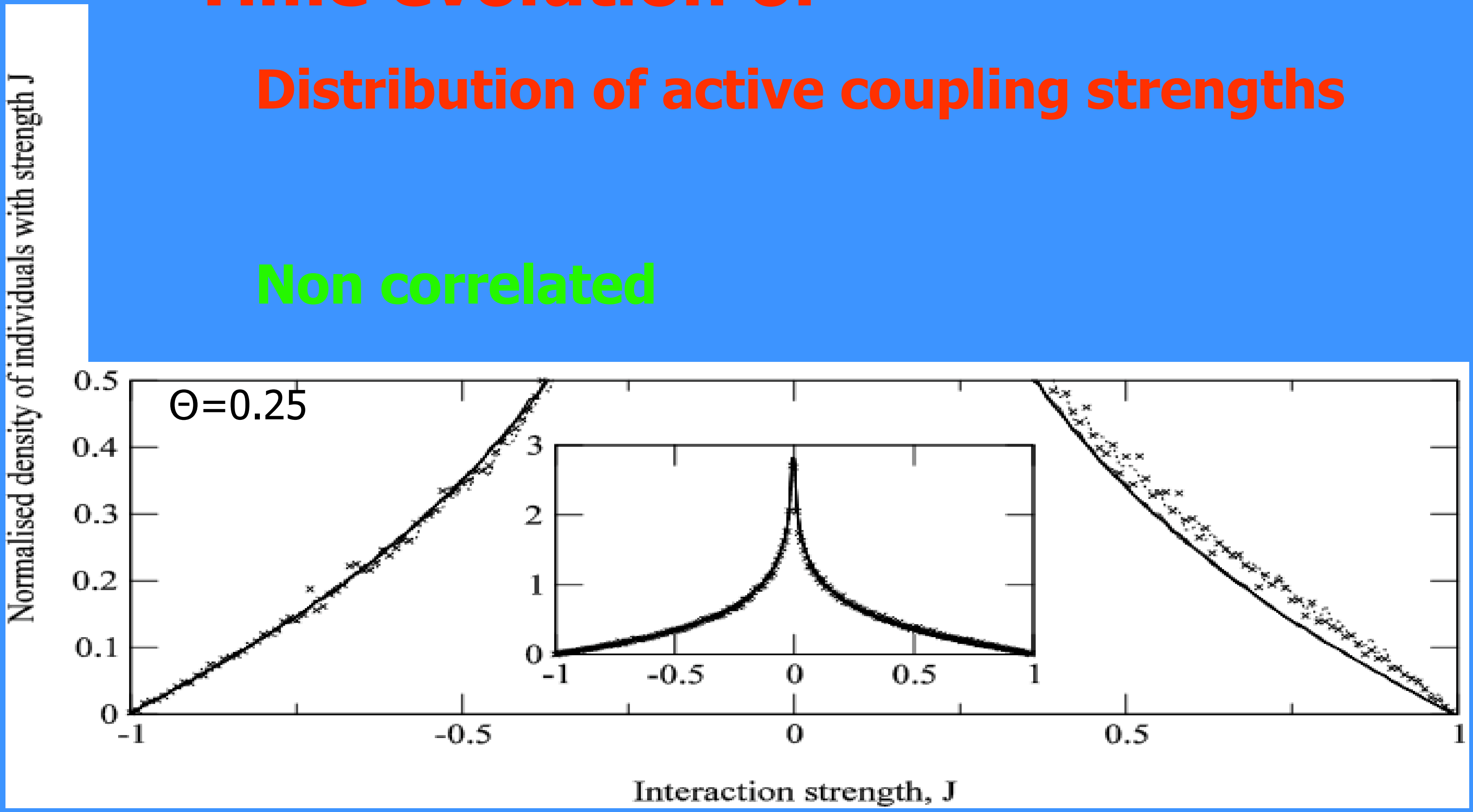
Fig. 1 – An occupation plot of a single run for a system with *R* = 10,000. For each timeslice a point appears where a phenotype is in existence but as the full space is in 16 dimensions a projection onto a single trait is used.

Simon Laird

Time evolution of

Distribution of active coupling strengths

Non correlated

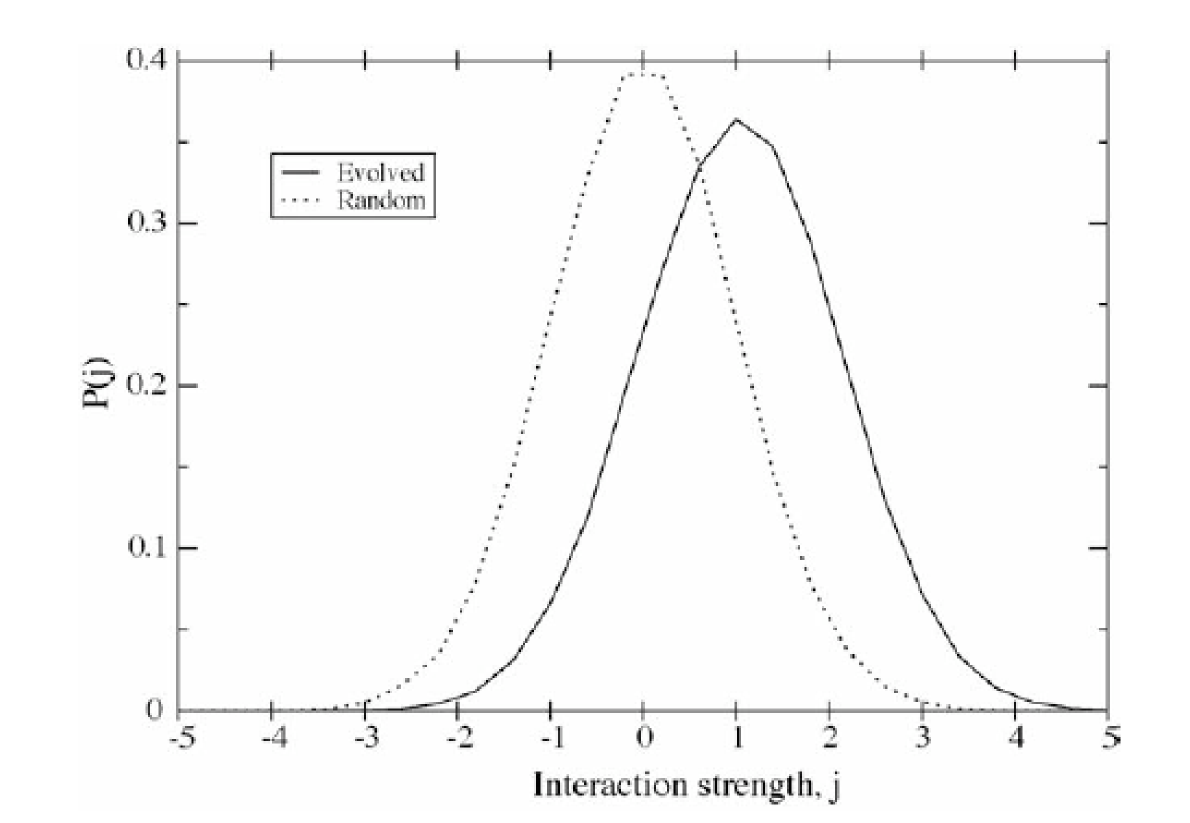


Paul Anderson

Time evolution of

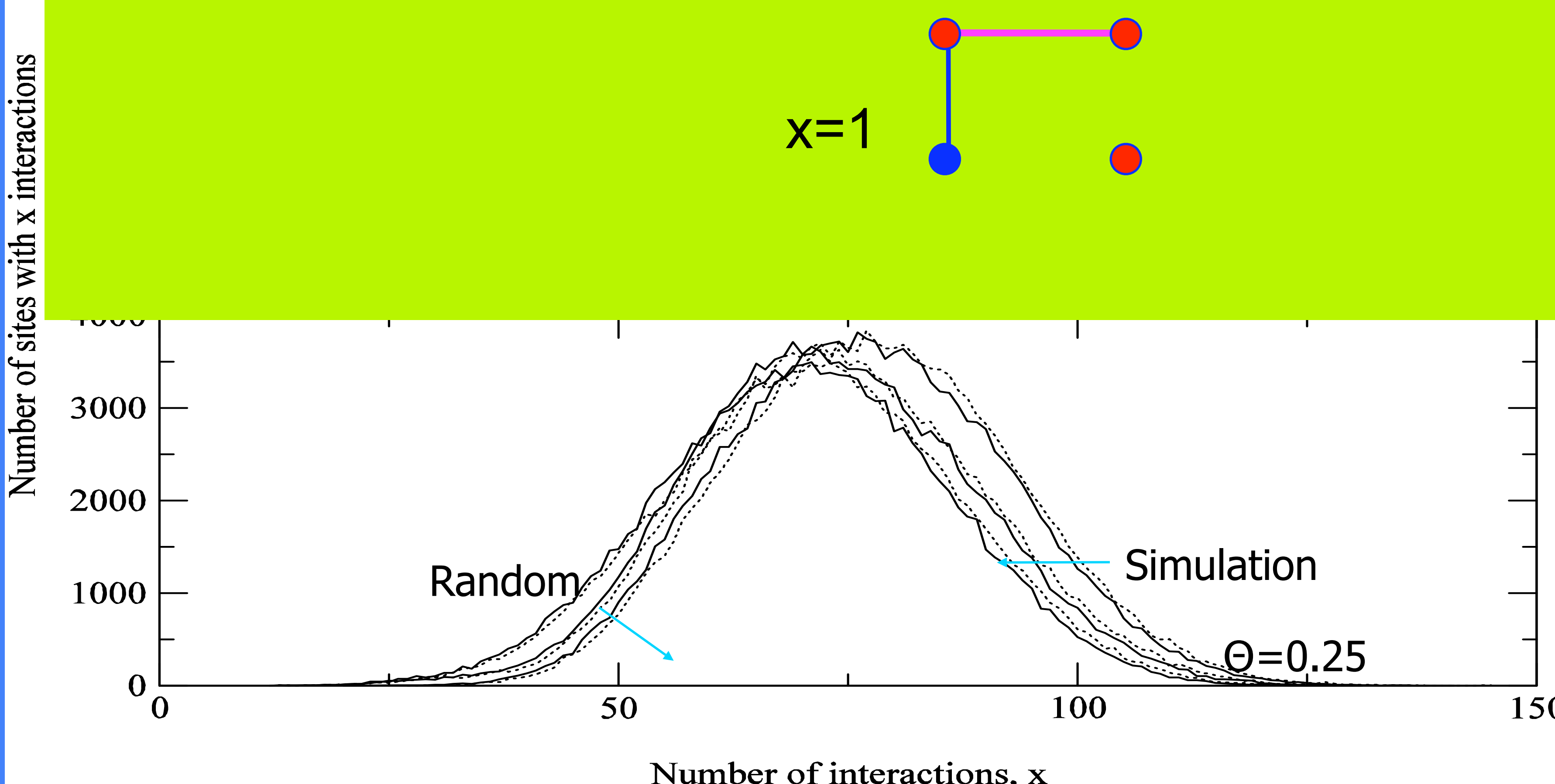
Distribution of active coupling strengths

Correlated



Simon Laird

Increasing complexity ?

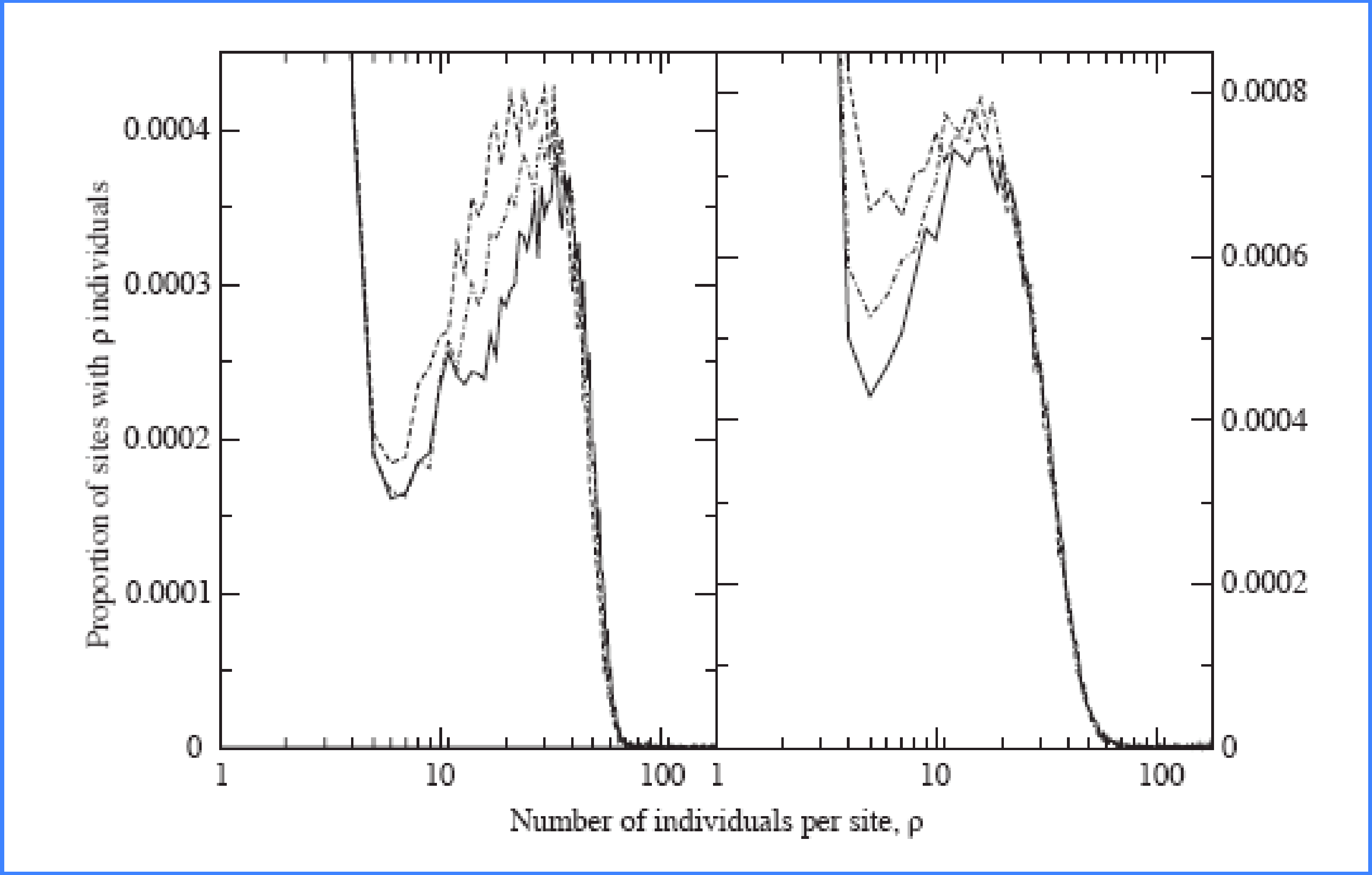


Note: Effect is significant for correlated type space

Time evolution of

Species abundance distribution

Non Correlated



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Low connectivity

High connectivity

The evolved degree distribution

Correlated

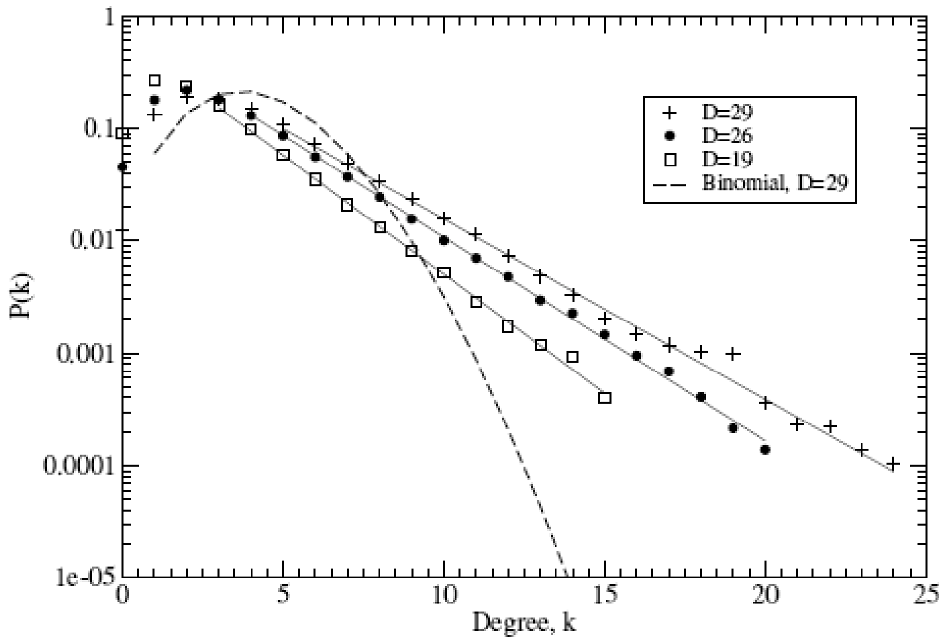


Figure 1: Degree distributions for the Tangled Nature model simulations. Shown are ensemble averaged data taken from all networks with diversity, $D = \{19, 26, 29\}$ over 50 simulation runs of 10^6 generations each. The exponential forms are highlighted by comparison with a binomial distribution of $D = 29$ and equivalent connectance, $C \simeq 0.145$ to the simulation data of the same diversity.

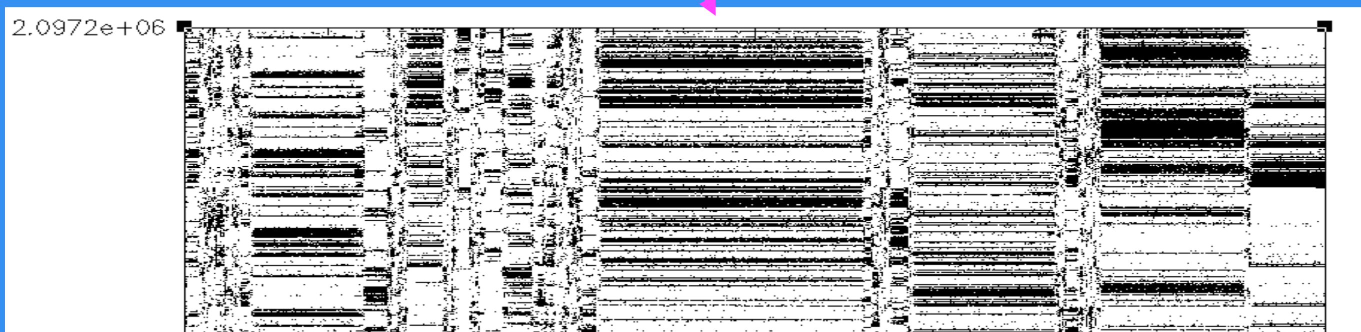
Simon Laird

Exponential becomes 1/k in limit of vanishing mutation rate

Intermittent dynamics

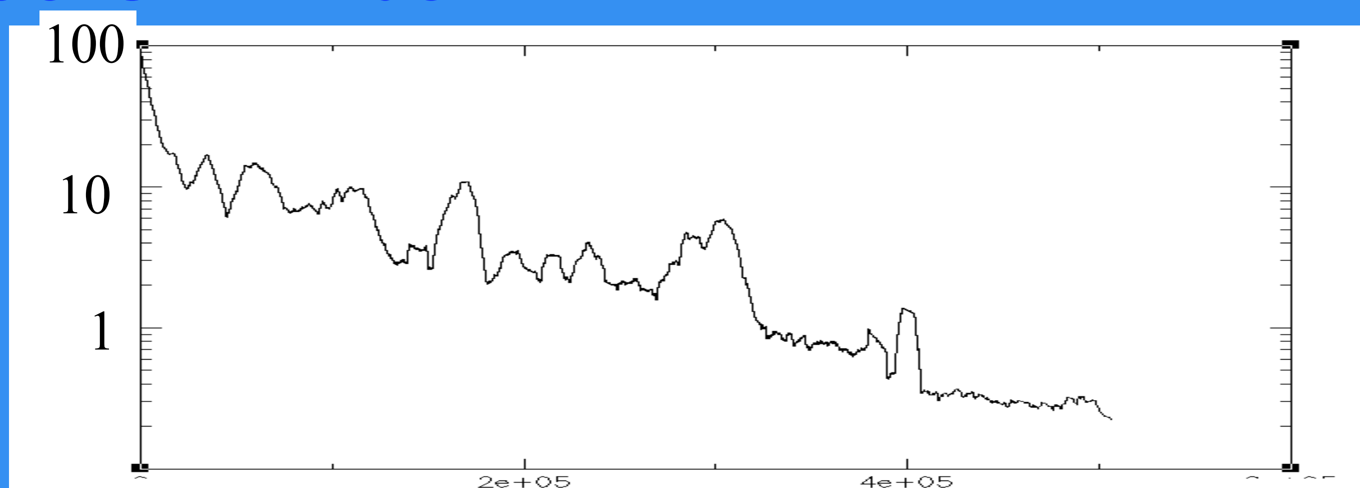
Intermittency:

q-ESS = quasi-Evolutionary Stable Strategy



# of transitions in window

Matt Hall



0 6×10^5

 # generations

1 generation

$= N(t) / p_{kill}$

Stability of the q-ESS:

Consider simple adiabatic approximation.

Stability of genotype S assuming: $n(S',t)$ independent of t for $S' \neq S$

Consider $$\frac{\partial n(S,t)}{\partial t} = [p_{off}(n(S,t),t) - p_{kill} - p_{mut}]\frac{n(S,t)}{N(t)}$$

Stationary solution $n_0(S)$ corresponds to $p_{off}(n_0(S)) - p_{kill} - p_{mut} = 0$

Fluctuation $\delta = n(S,t) - n_0(S)$

Fulfil $$\dot{\delta} = A\frac{n_0}{N_0}\delta$$

$$\text{with } A = -(1 - p_{mut})(p_{off})^2 e^{-H_0}\left(\frac{J}{N_0{}^2} + \mu\right) < 0$$

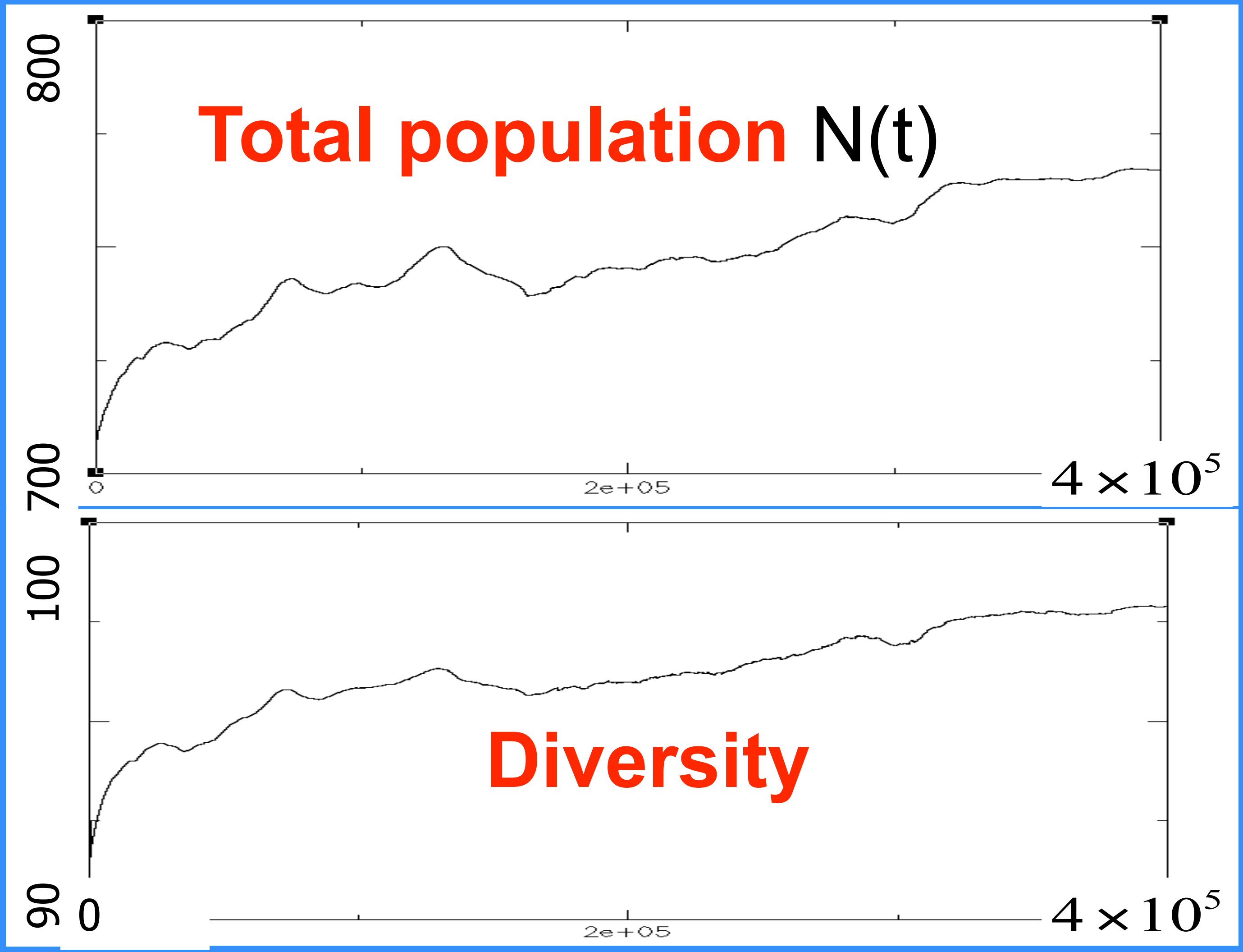
i.e. stability

Transitions between q-ESS caused by co-evolutionary collective fluctuations

$n(S',t)$ needs to be considered

dependent of t for $S' \neq S$

☻ Time dependence (averaged)



Time in Generations

Matt Hall

Origin of drift? Effect of mutation

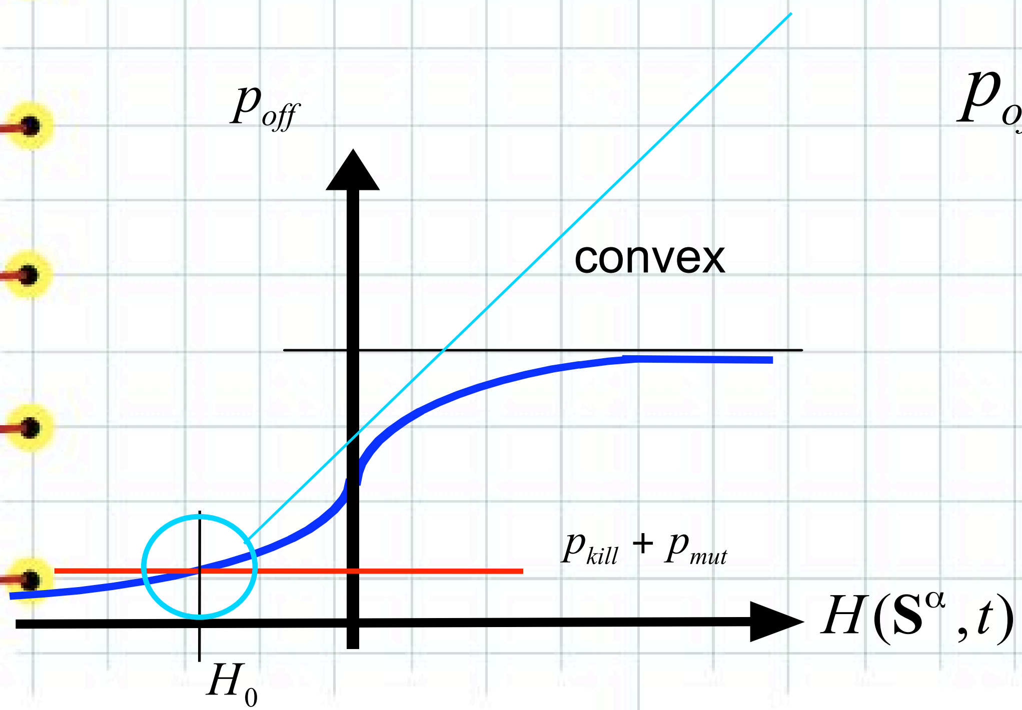
Let $H = \tilde{J} - \mu N$, then the effect of a mutation is

$$H \mapsto H + \delta \tilde{J}.$$

➡ Symmetric fluctuations $prob(\delta \tilde{J}) = prob(-\delta \tilde{J})$

leads to asymmetri

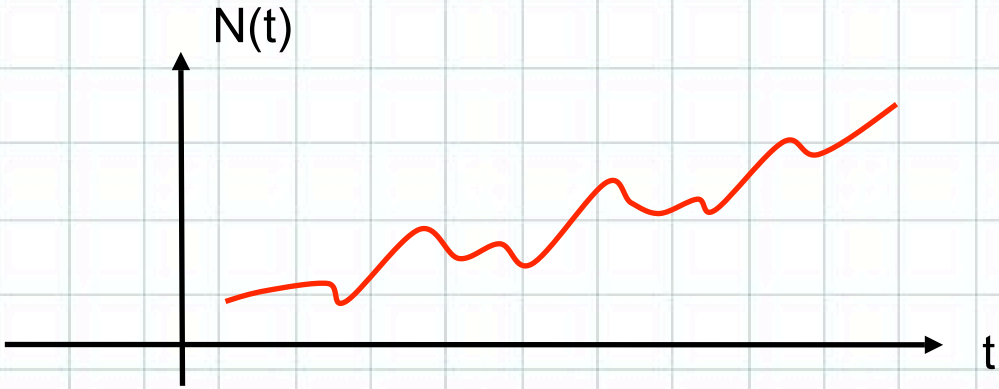
$$p_{off}(H_0 + \delta \tilde{J}) - p_{kill} > p_{kill} - p_{off}(H_0 - \delta \tilde{J})$$



$$p_{off}(H_0 + \delta \tilde{J}) - p_{kill} > p_{kill} - p_{off}(H_0 - \delta \tilde{J})$$

$$\Downarrow$$

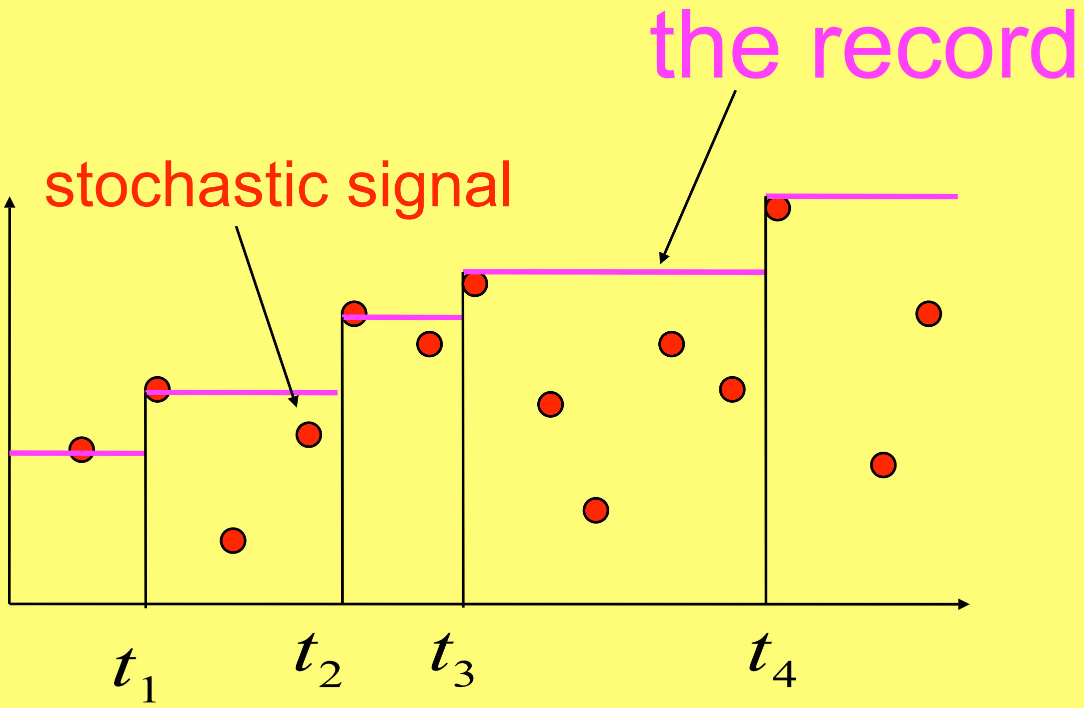
$$\delta N_+ > \delta N_-$$



Not the whole explanation: evolution not smooth.

Record dynamics

Record dynamics:



Paolo Sibani and Peter Littlewood (1992):

$$\tau = \ln(t_k) - \ln(t_{k-1}) = \ln(\frac{t_k}{t_{k-1}})$$ exponentially distributed

Record dynamics:

$$\tau = \ln(t_k) - \ln(t_{k-1}) = \ln\left(\frac{t_k}{t_{k-1}}\right)$$ exponentially distributed

- Poisson process in logarithmic time
- Mean and variance

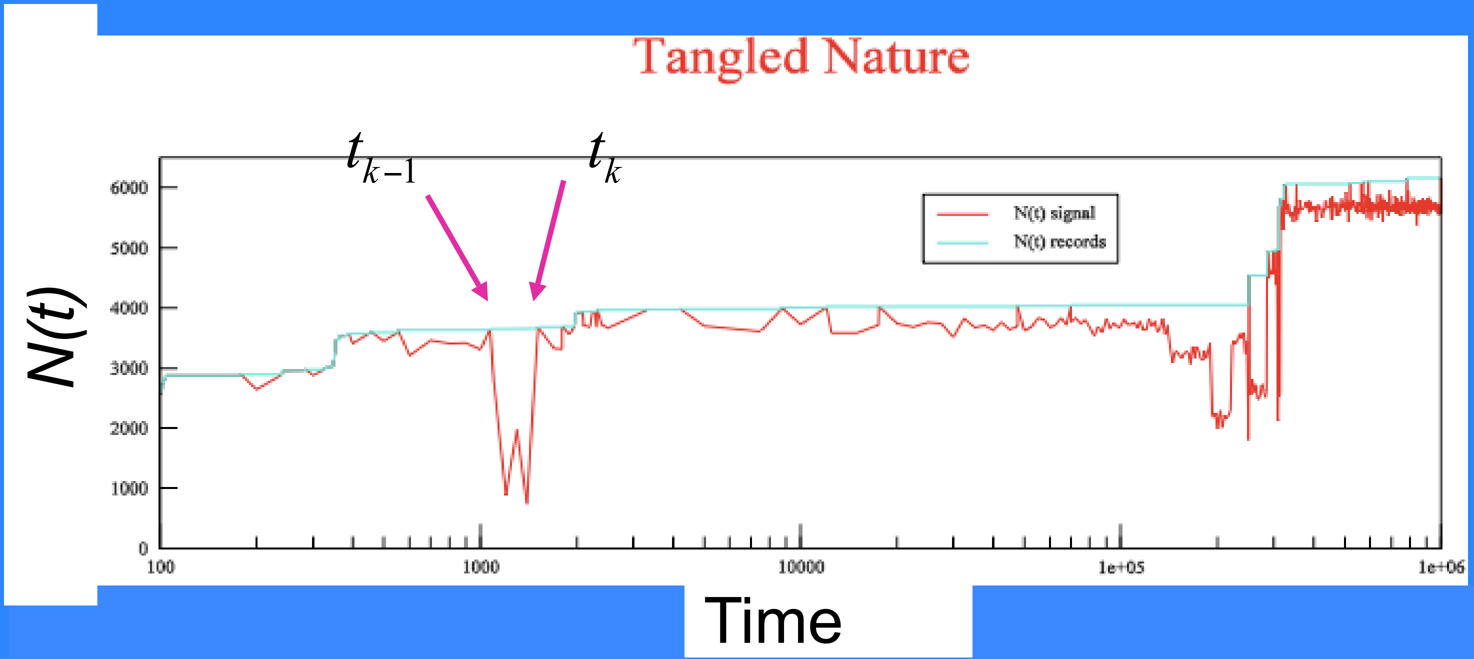
$$\langle Q \rangle \propto \ln(t) \quad \text{and} \quad \left\langle (Q - \langle Q \rangle)^2 \right\rangle \propto \ln(t)$$

- Rate of records constant as function of ln(*t*)
- Rate decreases $\propto 1/t$

Tangled Nature model:

Single realisation and record dynamics:

Extracting records from the population size

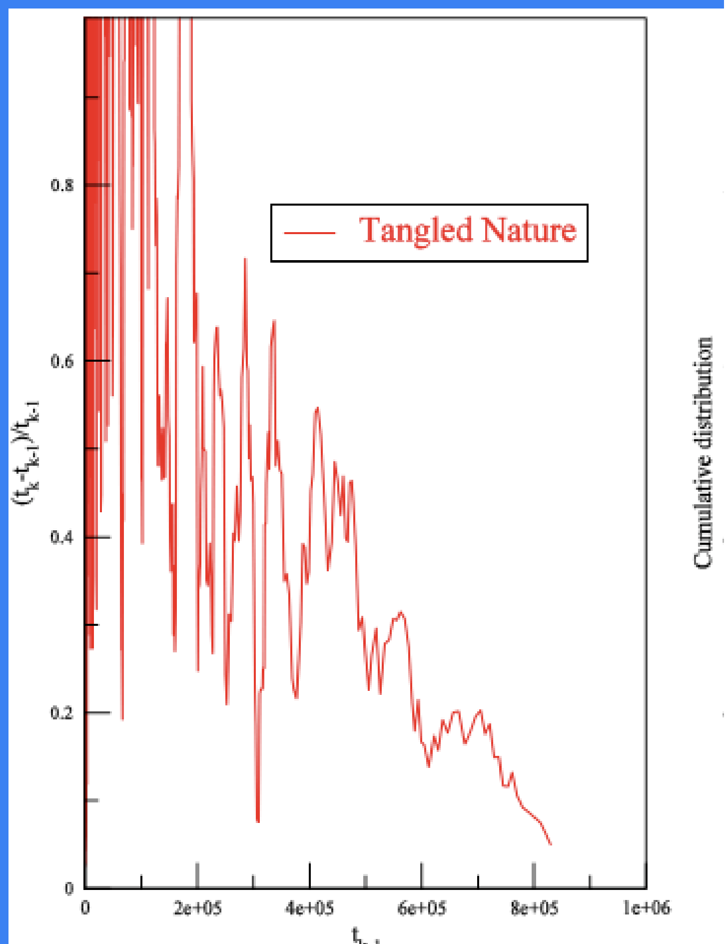


Paul Anderson

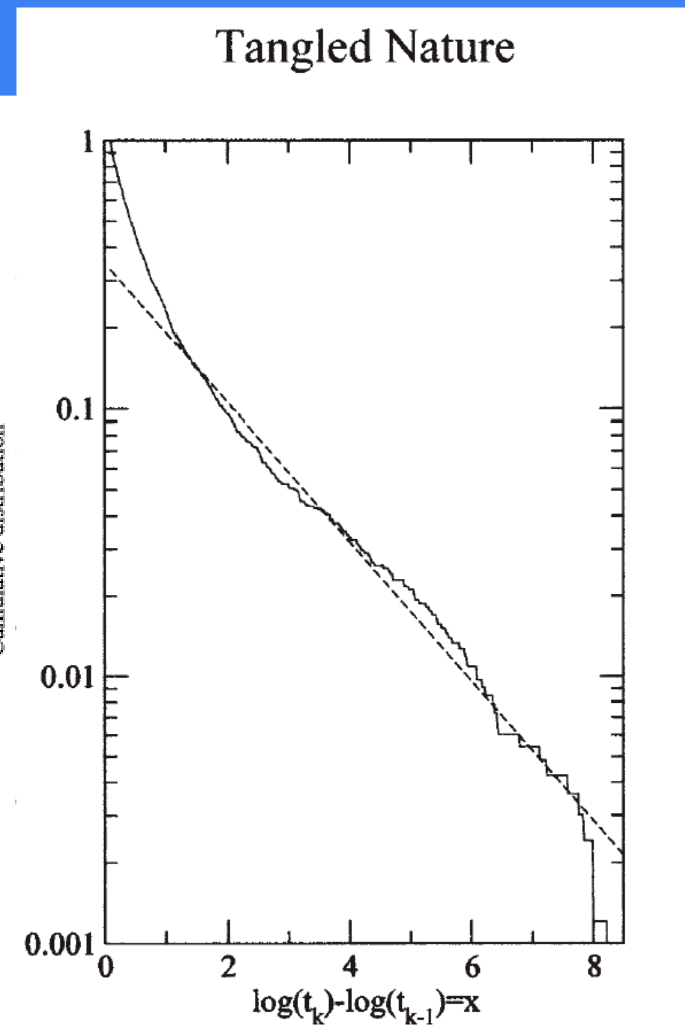
Record dynamics:

Ratio r remains non-zero

$$r = (t_k - t_{k-1}) / t_{k-1}$$



Cumulative Distribution



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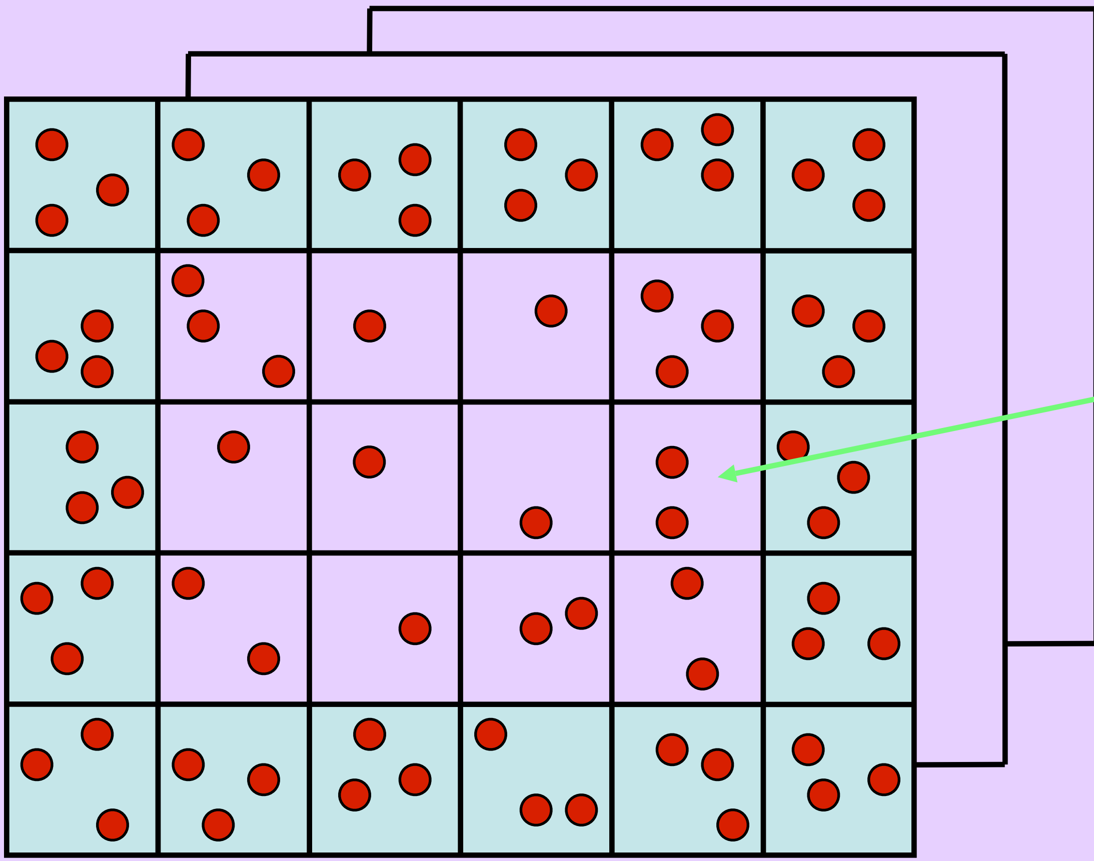
Second Model:

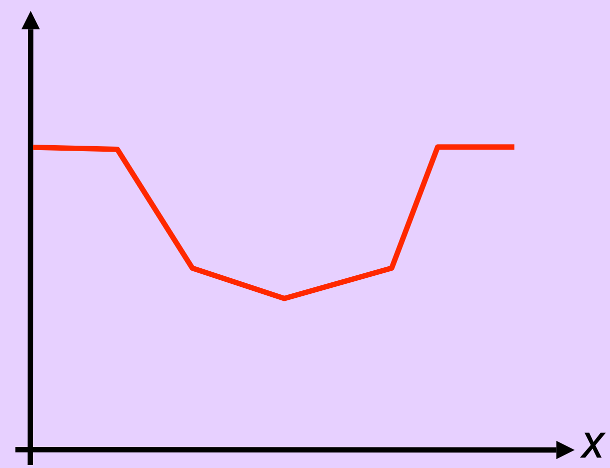
ROM

ROM

Monte Carlo Kawasaki dynamics on stack of coarse grained superconducting planes



$$n(x, y, z, t) = \mathrm{n_i}$$



ROM

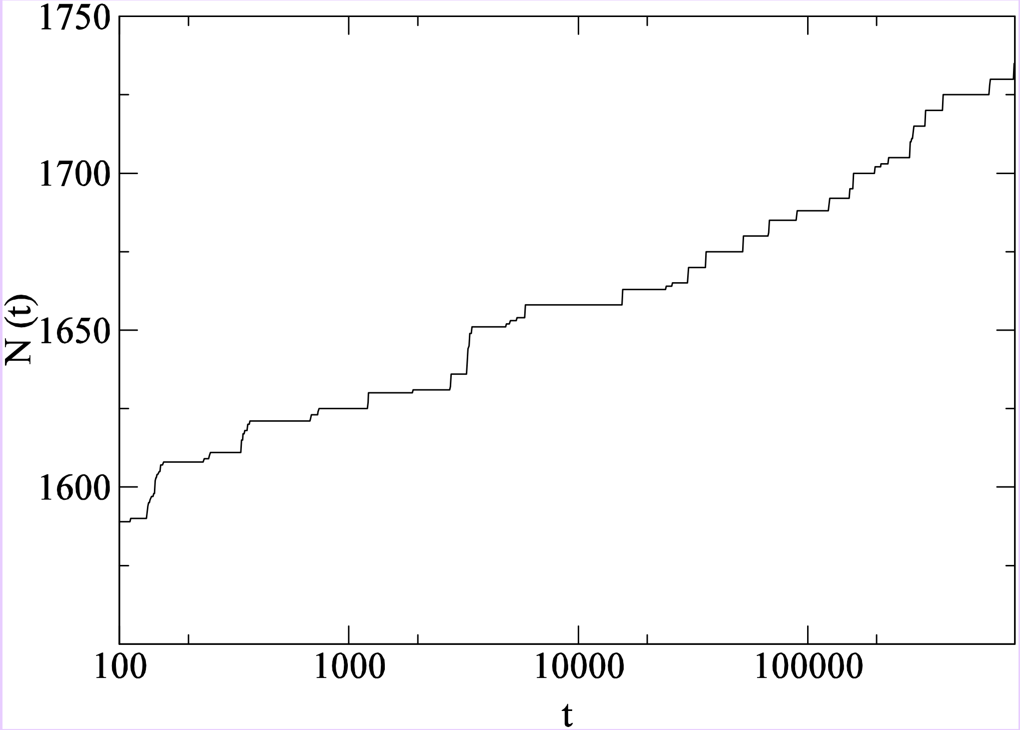
Hamiltonian

$$H = \sum_i n_i A_{ij} n_j - \sum_i A_{ii} n_i - \sum_i A_i^p n_i + \sum_{\langle ij \rangle_z} A_2 (n_i - n_j)^2$$

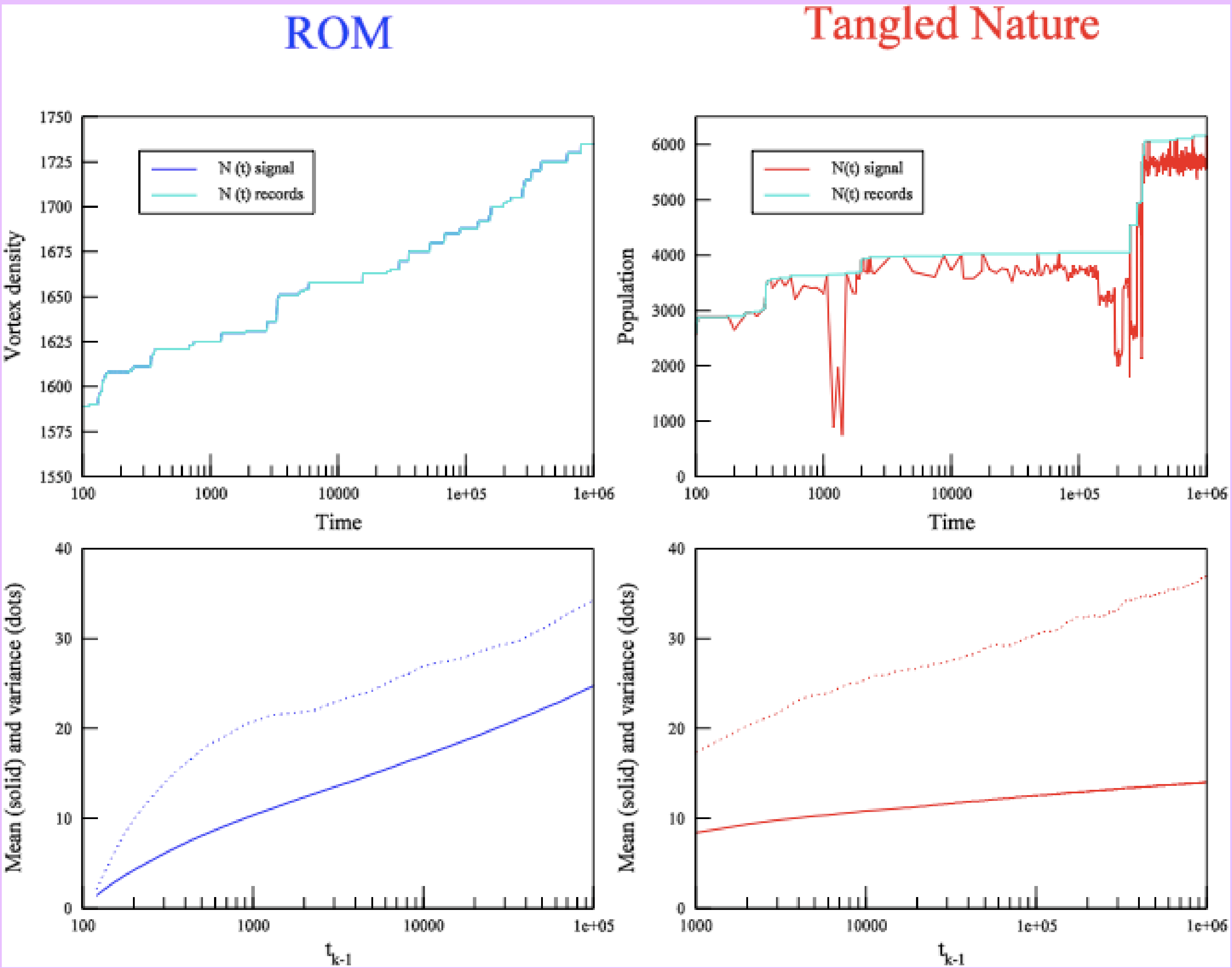
Here

$$0 \le n_i < N_{c2} = B_{c2} l_0^2 / \varphi_0$$

ROM: Temperature independent creep



Realisations of record dynamics

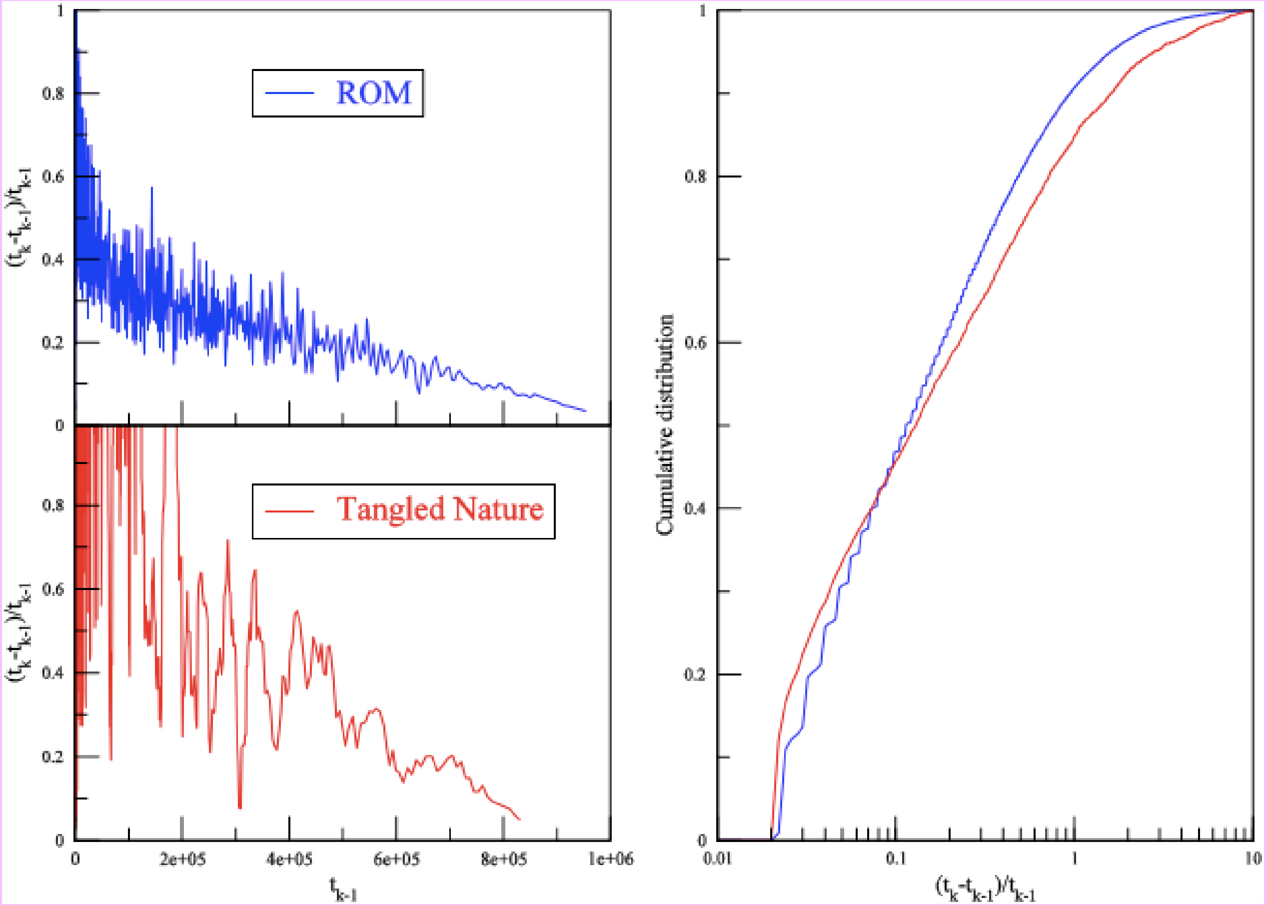


Manifestation of the decelerating activity.

$$\frac{t_k - t_{k-1}}{t_{k-1}} \approx \text{const}$$

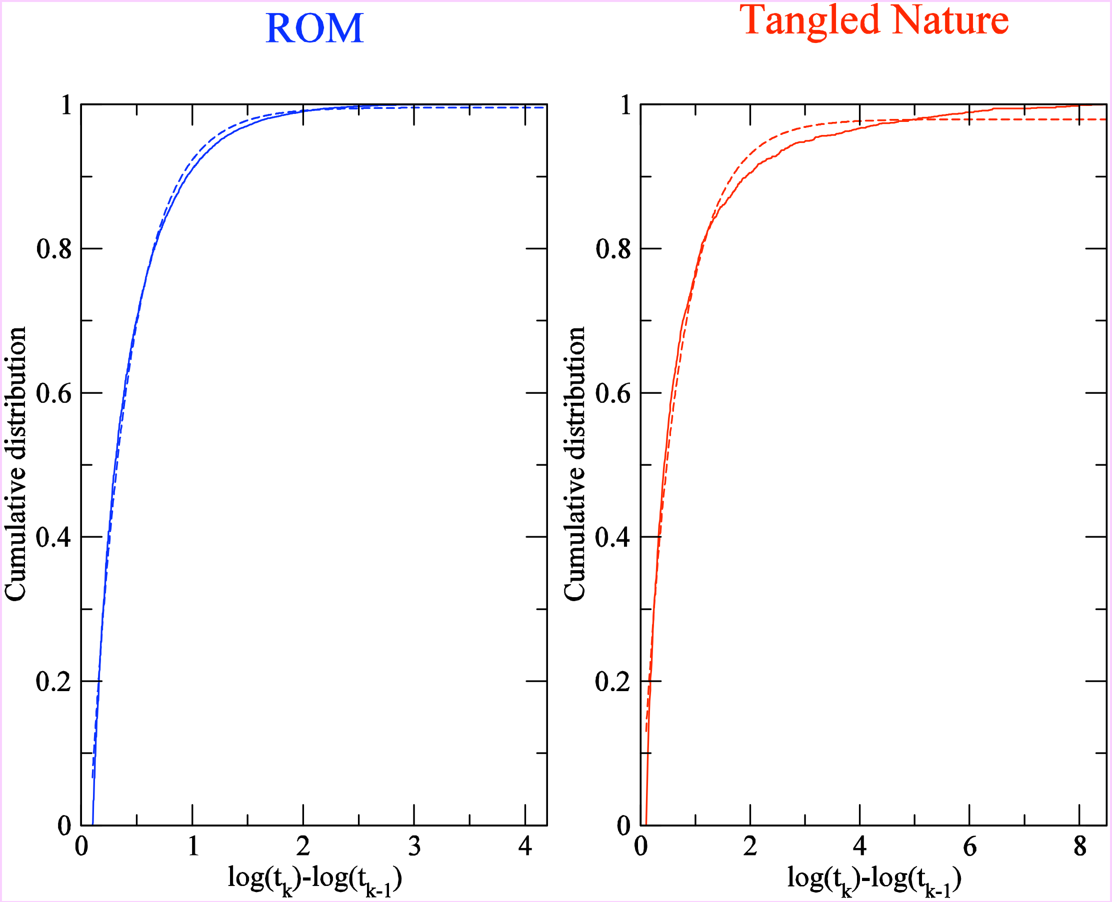
For stationary process

$$P\left(\frac{t_k - t_{k-1}}{t_{k-1}} = x\right) = \delta(x)$$

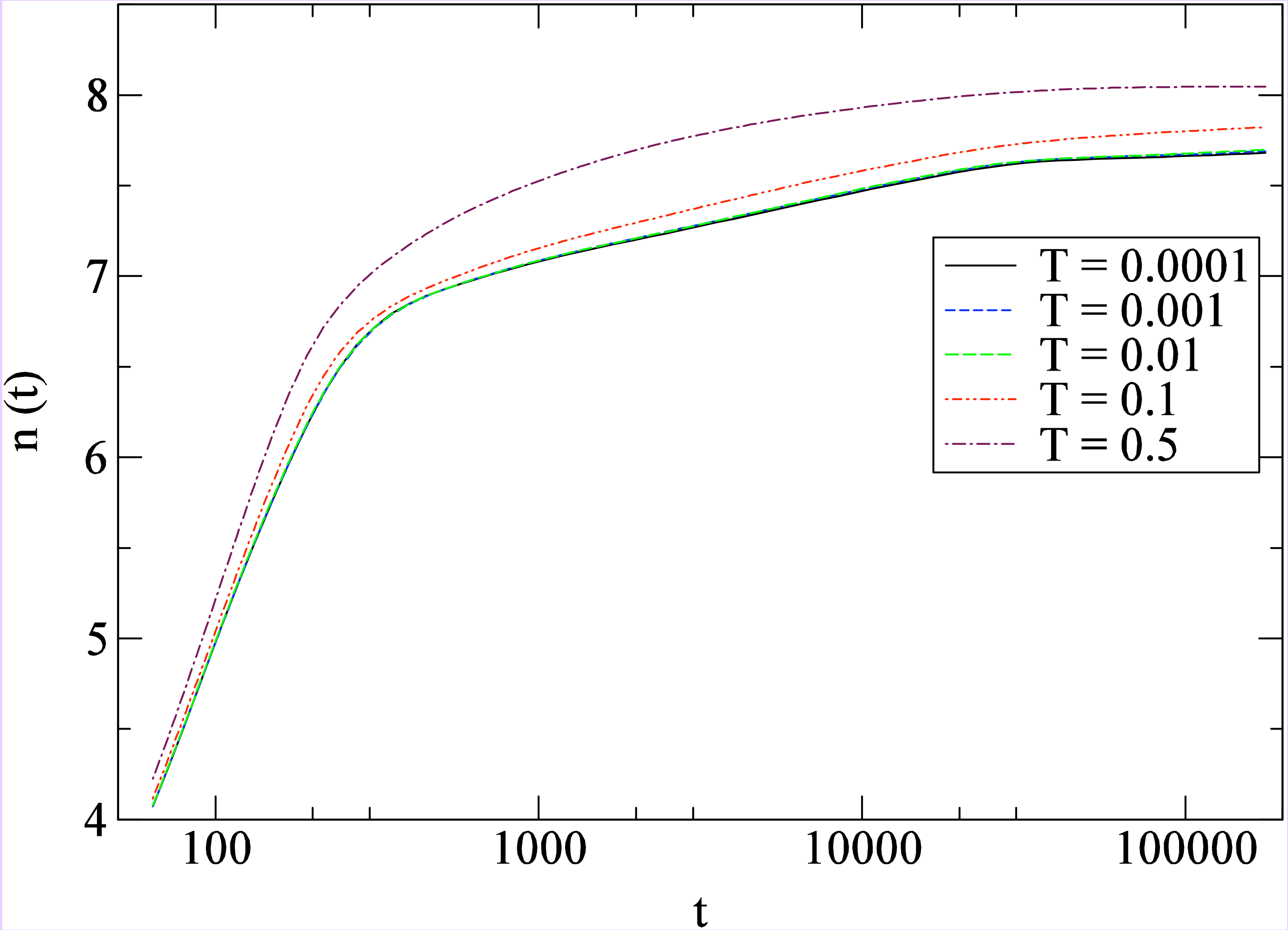


Further evidence

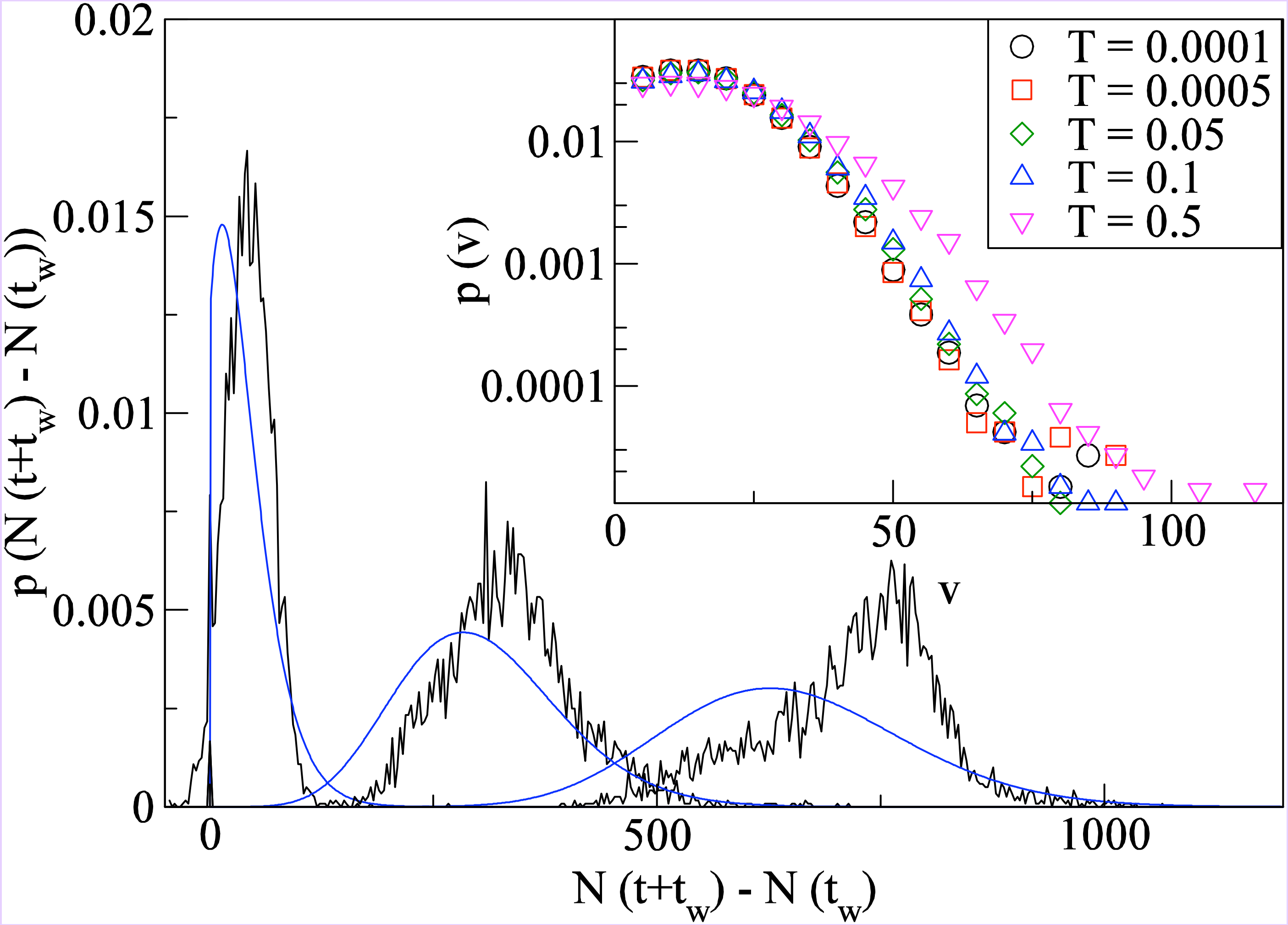
The cumulative distribution of the log waiting times. Comparison with exponential distribution.



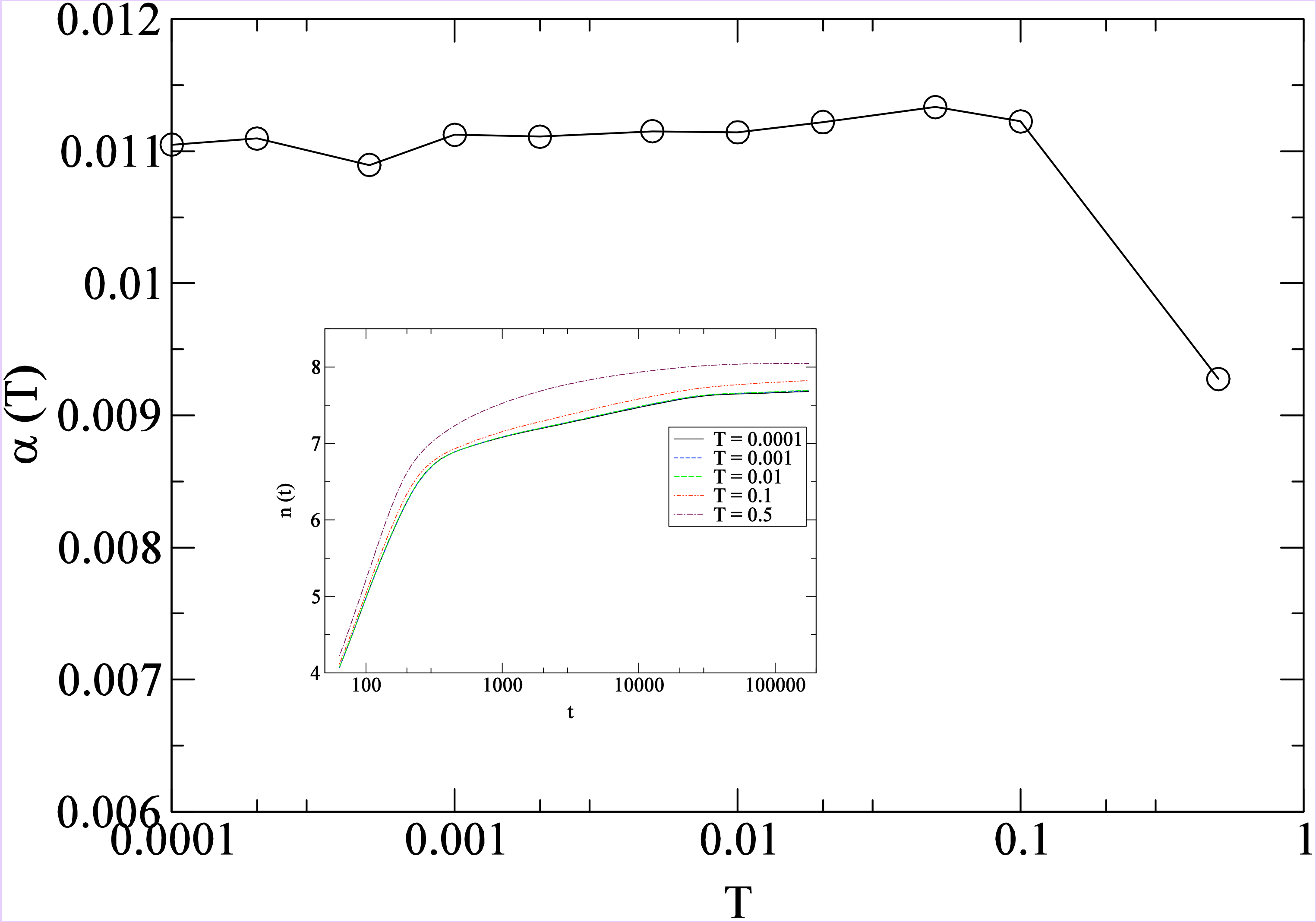
Number of vortices in the bulk as function of time



Quake statistics and the total number vortices entering.

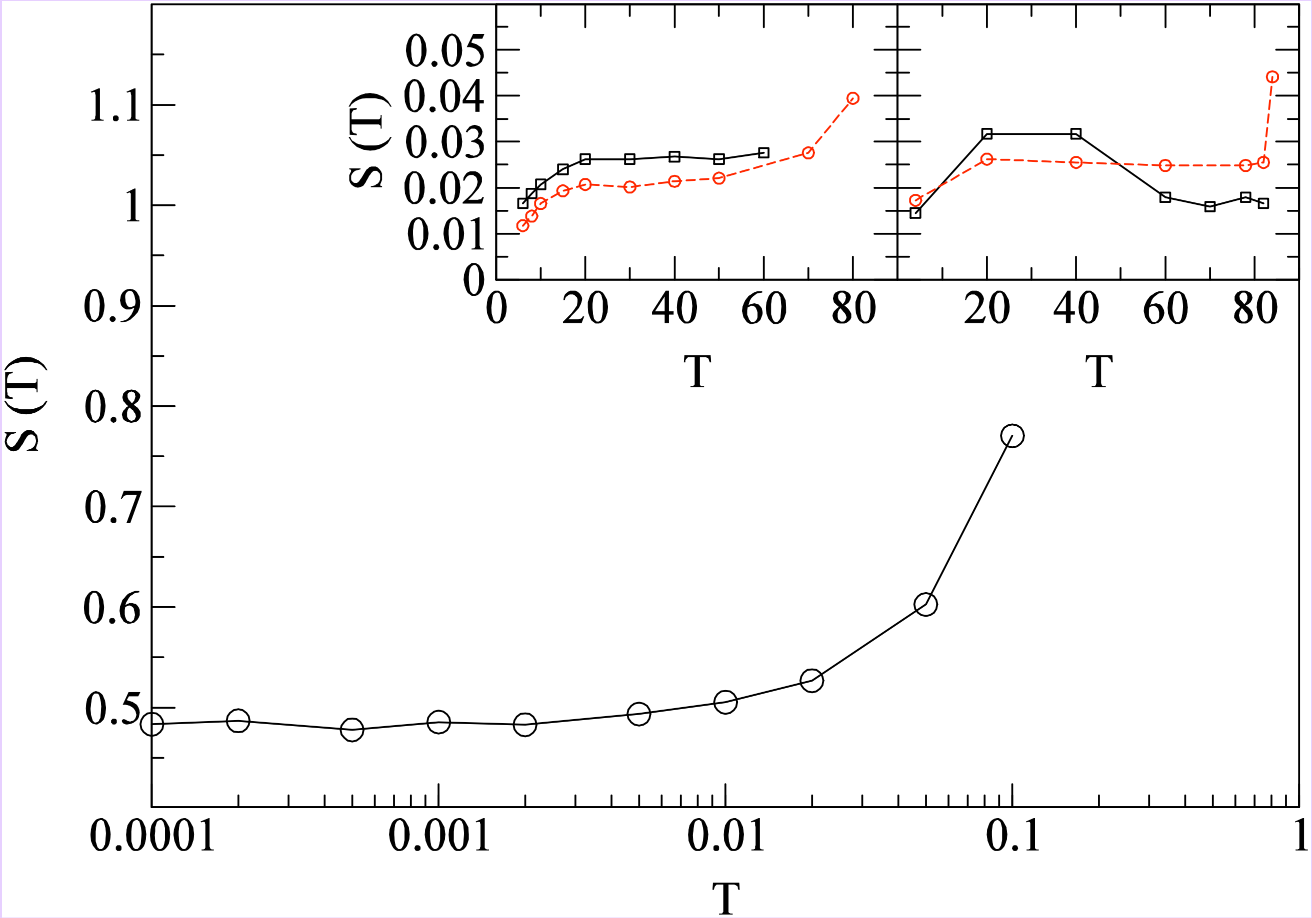


The temperature in-dependence of the quake rate.



The magnetic creep rate: $S = \frac{d\ln(M)}{d\ln(t)}$ where $M(t) = |N(t) - N_{ext}|$

comparison with experiment



Third Model:

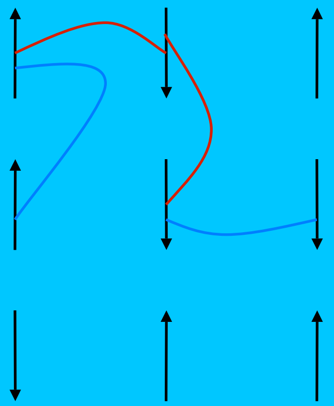
Spin Glass

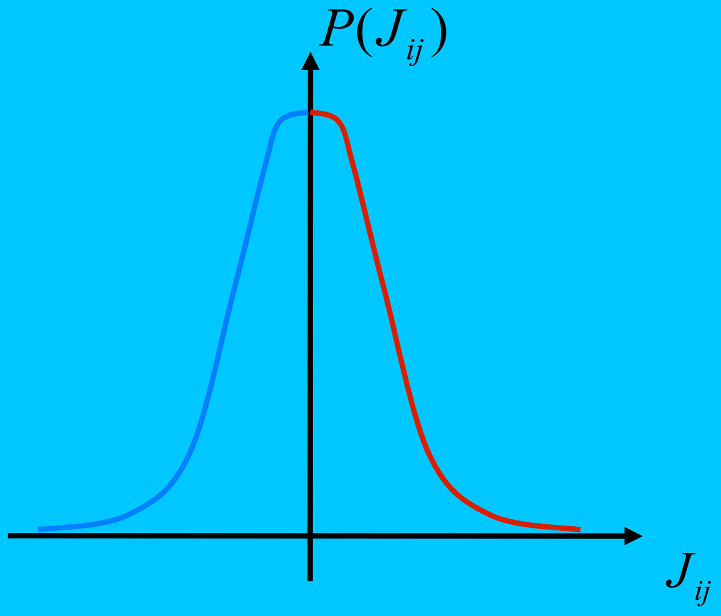
Spin glass

Microscopic magnetic moments – or spins – coupled together with random coupling constants.

The Hamiltonian:

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \text{ where } \mathbf{S}_i, \mathbf{S}_j = \pm 1$$



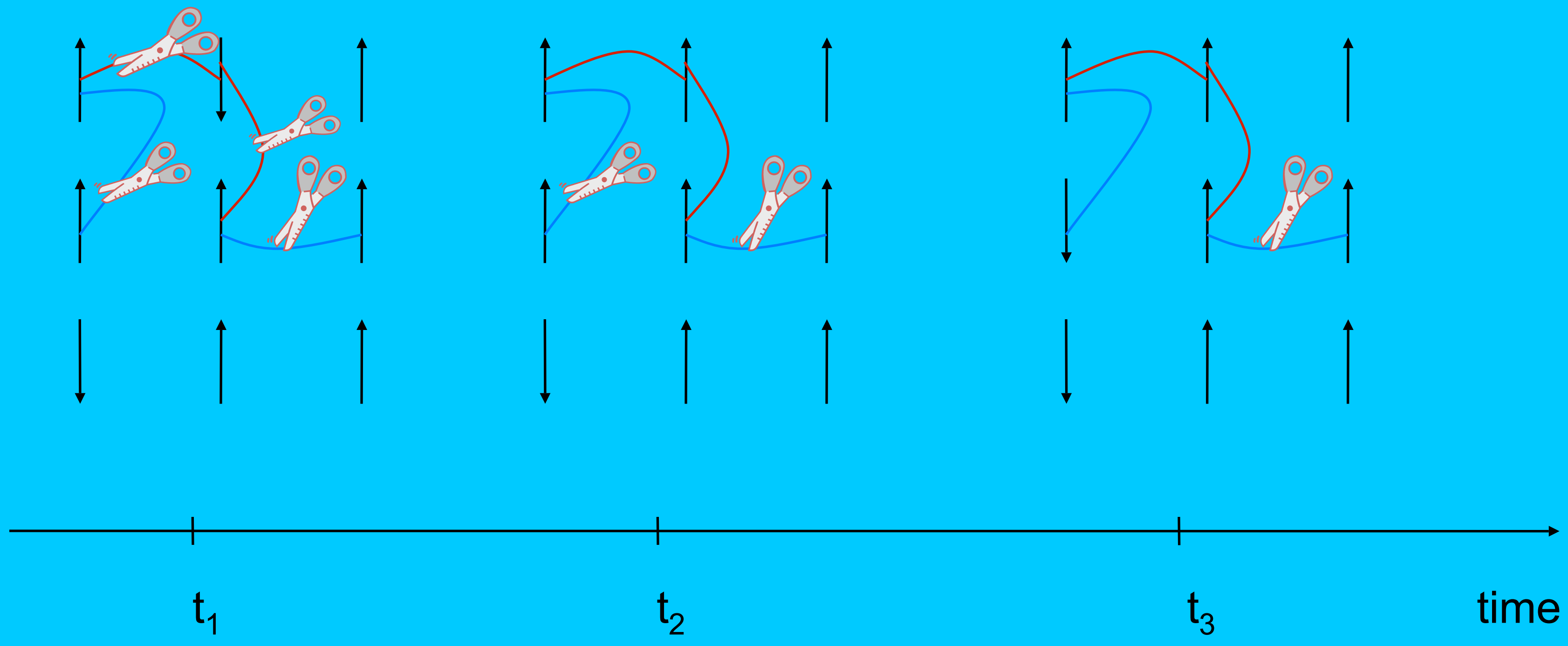


Spin glass

Quench from high temperature:

time < 0: T = high

time > 0: T = very low



Spin glass: heat transfer

Protocol: Quench from high temp. at time t= 0.

Measure heat transfer, *H*, between spin glass and reservoir during time interval

$$[t_w, t_w + \delta t]$$

- If $\delta t << t_w$ Gaussian p(H)
- If $\delta t \approx t_w$ exponential tail

Spin glass: heat transfer

$\delta t << t_w$ $\delta t \approx t_w$

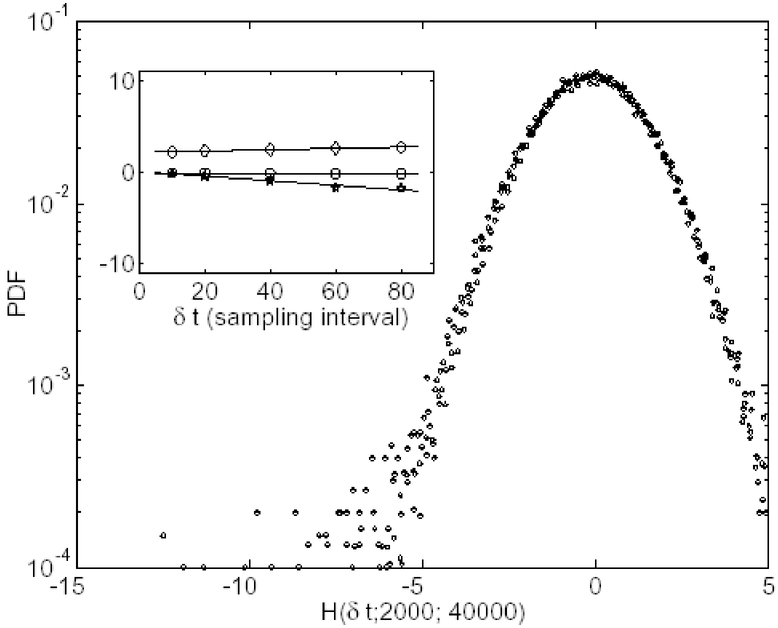


Fig. 1

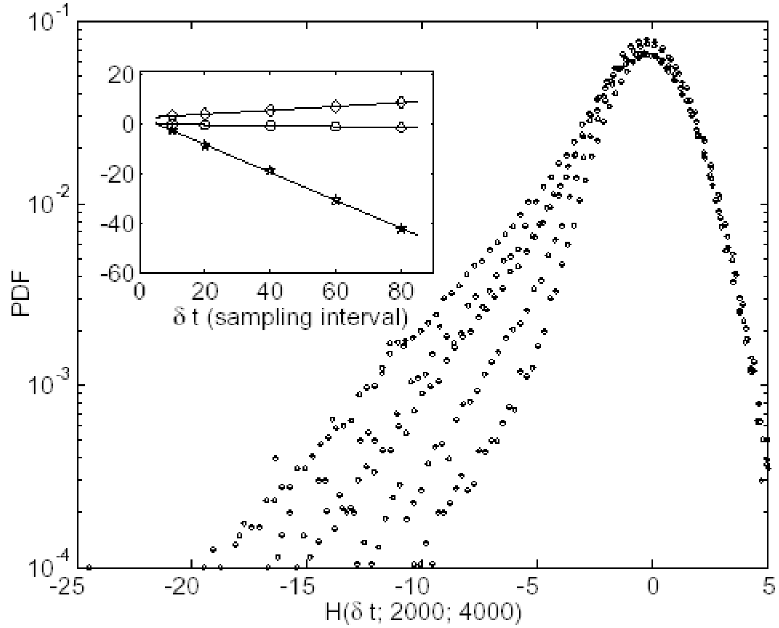


Fig. 2

Consequences of record dynamics.

Statistics of quake times independent of underlying "noise mechanism".

- Biology: same intermittent dynamics in micro as in macro evolution. Decreasing transition rate.
- Magnetic relaxation: temperature independent creep rate
- Spin glass: exponential tails

Conclusion/Summary

Considered spin-glasses, superconductors and biological evolution as typical complex systems.

Generic dynamics of complex systems:

- Non-stationary
- Intermittent record dynamics - quakes
- Rate of activity ~ $1/t$
- Stationary as function of log(t)

Collaborators: Paolo Sibani, Paul Anderson and Luis P Oliveira


Down load papers from:
www.ma.imperial.ac.uk/~hjjens
Collaborators: Paolo Sibani, Paul Anderson, Luis P Oliveira and Mario Nicodemi