E2-2003-128

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INTEGRAL NUCLEUS–NUCLEUS CROSS-SECTIONS

Submitted to «Kerntechnik»

The cross-sections of inelastic and elastic interactions of nuclei are necessary for solution of various theoretical and applied problems, in particular, for modelling transport of particle and ion beams in media by a design of electronuclear systems, radiation shielding, estimations of radiation damage to microelectronic devices and so on. Such cross-sections are especially important for the calculation of cosmic ray interactions with details of satellites where heavy nuclei of the iron group give a large contribution. However, experimental information about cross-sections is very poor [1]. Presently, the only way to get necessary data is an interpolation and extrapolation of the known experimental points by means of some analytical expressions with fitted parameters prompted by theoretical models.

Now there are several such approximations for some, as a rule, rather narrow energy and mass number intervals. The most universal one applicable for energies higher the Coulomb barrier (see Table I) and for various nucleus pairs is considered in [1, 2]:

$$\sigma(E/A_p) = 38.011(1. - B/E_c)(A_p^{1/3} + A_t^{1/3})+ \\ 1.85 * A_{pt} - C(E/A_p))^2, \quad (1)$$

where E is the laboratory projectile nucleus energy, $F_c = A_{pt}(E/A_p)$ is CMS nucleus energy, $A_{pt} = A_pA_t/(A_p + A_t)$, A_p and A_t are the projectile and target nuclei mass numbers, $B =$ is the Coulomb barrier.

This approximation is based on a model combining central (with small impact parameters) and peripheral collisions. The parameter C characterizing the transparency of the nucleus periphery is a smooth function of the energy $E/A_p = E_o/A_t$ where E_o is the energy in the opposite reference frame. This function is defined by comparison with almost all known experimental data up to now.

The precision of approximation of inelastic cross-section σ_{in} increases if $C(E_c/A_p)$ is chosen separately for deuterons (C_d) and for more heavy nuclei (C_h). Experimental information on the elastic cross-sections σ_{el} is much less, so C_{el} (see Table II) can be defined at once for all nuclei without separation of the light and heavy ones.

We see that the cross-section (1) is symmetrical with respect to laboratory and opposite coordinate systems.

One would try to improve the approximation and distinguish the cross-sections of isotopes [1, 2] by introducing into eq. (1) the additional term $\alpha(1 - Z_p/A_p - Z_t/A_t)$. However, the coefficient α is defined with too big error and Table II is calculated without this term.

Table I

Coulomb barrier for various pairs of nuclei

A_p	A_t	B,Mev	A_o	A_t	B,Mev
2	4	0.7	12	12	8
2	12	1.7	12	56	26
2	56	5.3	12	207	62
2	207	12	12	238	67
2	238	13	56	56	91
4	4	1.4	56	207	225
4	12.	3.5	56	238	246
4	56	11	207	207	585
4	207	24	207	238	640
4	238	26	238	238	702

In the most practicaly important cases the low boundary of approximation (1) $E_b = B(1 + A_p/A_t)$ is less than the thresholds at which the ions are considered as stopped ones due to the fast increasing influence of ionization processes and we do not need the cross-sections at $E \leq E_b$. However, for heavy projectiles which we encounter, for example, in collisions of the iron component of cosmic rays with matter the boundary E_b is large: $E_b \gg B$. For the estimations at the lower energies one must use the approximation which takes into account quantum effects in the vicinity of the Coulomb barrier B. We shall use the expression obtained in papers [3, 4]:

$$\sigma = \frac{\alpha}{E_c} ln\left[1 + exp\frac{E_c - B}{\beta}\right], \qquad (2)$$

where α is a normalization coefficient defined by sewing together with eq. (1) at some energy closer to B (we use $E_c = B/0.61$), $\beta = 1.095$ is a fitted constant. Approximation becomes somewhat more exact, if the energy dependence of nuclear radius $R_O(E) = R_o(1 - 0.021E/A_p)$ is introduced where $R_o = 2.1,\ 1.71,\ 1.47$ for deutron, $\alpha-$particle and all other nuclei.

The low boundary for this approximation is defined by the condition $d\sigma/dE = 0$ (at the lower energies σnt, in spite of the experiment, is increasing due to T_C in the denominator).

Table II

Energy dependency of the function $C(\lg(E/A_p, MeV))$

$\lg(E/A_p)$	C_d	C_h	C_{el}	, $lg(E/A_p)$	C_d	C_h	C_{el}
0.0	-1.791	0.12	0.80	2.301	1.091	2.1	2.55
0.477	-1.582	0.14	0.85	2.398	1.196	2.15	2.70
0.602	-1.172	0.165	0.89	2.477	1.279	2.2	2.80
0.699	-1.021	0.18	0.94	2.602	1.405	2.183	2.95
0.778	-0.888	0.2	0.97	2.699	1.498	2.15	3.05
0.845	-0.789	0.21	1.00	2.778	1.569	2.13	3.10
0.903	-0.707	0.22	1.02	2.845	1.627	2.1	3.12
0.954	-0.625	0.24	1.06	2.903	1.675	2.08	3.05
1.0	-0.556	0.26	1.09	2.954	1.715	2.06	3.01
1.176	-0.501	0.29	1.12	3.0	1.749	2.05	2.90
1.301	-0.278	0.36	1.19	3.176	1.869	2.0	2.85
1.398	-0.123	0.45	1.29	3.301	1.939	1.98	2.82
1.477	-0.003	0.54	1.39	3.398	1.984	1.98	2.82
1.602	0.096	0.64	1.47	3.477	2.016	1.98	2.80
1.699	0.252	0.8	1.61	3.602	2.054	1.98	2.79
1.778	0.472	1.2	1.85	3.699	2.072	1.98	2.78
1.845	0.554	1.35	1.94	3.778	2.081	1.98	2.77
1.903	0.625	1.5	2.03	3.845	2.083	1.98	2.76
1.954	0.687	1.6	2.10	3.903	2.084	1.98	2.80
2.0	0.743	1.7	2.16	3.954	2.085	1.98	2.82
2.176	0.949	1.95	2.40	4.0	2.085	1.98	2.83

Figs.1 and 2 illustrate the precision of the considered approximation for light nuclei. There are only few separate experimental data for heavy nuclei, the comparison with the calculated ones are shown in Table III. Taking into account the big experimental errors one can say that the calculated data are inside the corridor of these errors.

In Table III elastic cross-sections are compared. In average, uncertainties of experimental data and calculated values are much large than for σ_{in} in this case.

Table III

Experimental and calculated cross-sections of inelastic heavy nuclei interactions

Nuclei	E, MeV	σ_{in}, Exper.	σ_{in}, Caslc.
$^{12}C+^{54}Fe$	360	2195 ± 140	2344
	996	1795 ± 70	1925
$^{12}C+^{55.8}Fe$	2400	1643 ± 46	1638
	3000	1596 ± 46	1618
	3600	1588 ± 46	1631
$^{12}C+^{56}Fe$	360	2296 ± 160	2344
	996	1838 ± 70	1925
$^{12}C+^{57}Fe$	996	1797 ± 70	1941
$^{20}Ne+^{27}Al$	600	2146 ± 88	2161
	2000	1446 ± 120	1526
	6000	1328 ± 120	1438
	64000	1520 ± 70	1508
$^{20}Ne+^{64}Zn$	600	2875 ± 130	2873
	2000	2162*	2218
	6000	2407 ± 200	2133
$^{27}Al+^{40}Ca$	1188	2440 ± 190	2688
	2079	2030 ± 190	2304
$^{40}Ca+^{51}V$	1760	2940 ± 250	3343
$^{40}Ca+^{54}Fe$	1760	3130 ± 250	3391
$^{40}Ca+^{107}Ag$	1760	4000 ± 310	4279
$^{40}Ca+^{208}Pb$	1760	5000 ± 310	5337
	3080	5310 ± 500	5006
$^{238}U+^{238}U$	1766	1630 ± 110	2293

* Cross-section obtained by means of the optical analysis of the experimental data from the compilation [1]

Table IV

Experimental and calculated by eq. (1) cross-sections of elastic nucleus-nucleus interactions

Nuclei	E, MeV	σ_{el}, Exper.	σ_{el}, Caslc.
$d+^{27}Al$	160	594 ± 121	558
	650	287 ± 19	354.
	710	610 ± 120	344
$\alpha+^{12}C$	3480	266 ± 32	245
	8400	292 ± 33	291
	14400	325 ± 8	302
$\alpha+^{27}Al$	14400	579 ± 12	524
$^{12}C+^{12}C$	125	1230*	1039
	150	1230*	1060
	175	1230*	1074
	200	1230*	1077
	230	1250*	1076
	290	1300*	1055
	10440	317 ± 103	500

* Cross-section obtained by means of the optical analysis of the experimental data from the compilation [1]

In all cases when at the some energy there are two experimental values we present the averaged value

$$\sigma \pm \Delta\sigma = \frac{\sigma_1(\Delta\sigma_2)^2 + \sigma_2(\Delta\sigma_1)^2}{(\Delta\sigma_1)^2 + (\Delta\sigma_2)^2} \pm \frac{\Delta\sigma_1\Delta\sigma_2}{\sqrt{(\Delta\sigma_1)^2 + (\Delta\sigma_2)^2}}. \quad (3)$$

In addition, we present the FORTRAN code SIGION realizing the described above approximation procedure. An example of using of this code is shown. The code is desribed in detail, so user can introduce his own changes and improvements.

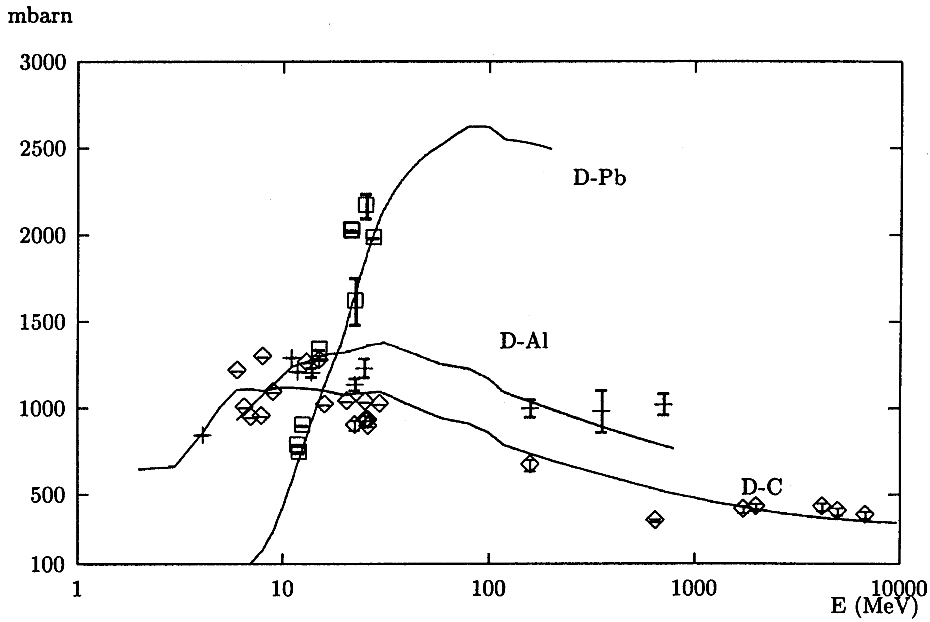

Fig. 1. Energy dependency of inelastic cross-sections. Deutron interactions with carbon, aluminum and lead. Experimental points are taken from compilation [1].

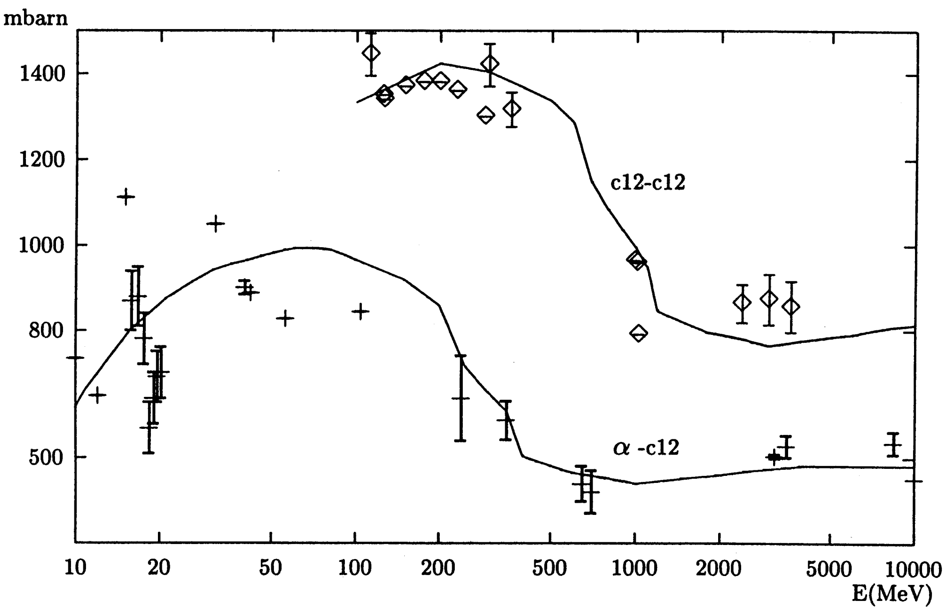

Fig. 2. Comparison of experimental and calculated inelastic cross-sections of $^{4}He+^{12}C$ and $^{12}C+^{12}C$ interactions. Experimental points from compilation [1].

```
C                              SIGION.FOR
C ************************************************
c *    Example of calculation of nucleus-nucleus    *
C  *                  cross-sections                *
c ************************************************
C  Calculated quantities:
C  Stot and Sin - total and inelastic cross-sections
C  (mbarn)
C ----------------------------------------------------
C  The used quantities:
C  IS=1 or 2  if Stot or Sin is calculated
C  A1, A2 - projectile and target mass numbers
C  Z1, Z2 - charge numbers of these nuclei
C  E - energy od the projectile in Lab system
C      (MeV/nucleon)
C  ----------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION E(5)
C  ----------------------------------------------------
C    Interaction of deutron with carbon is considered
             DATA E /100., 200., 400., 1000., 2000./
             A1=2.0
             Z1=1.0
             A2=12.0
             Z2=6.0
      WRITE(7,*)'     A1     A2     T,MeV   Sin,mb     Stot,mb'
      WRITE(7,*)'     -------------------------------------'
        DO  1  I=1,5
        T=(E(I)/A1)*(A1*A2/(A1+A2))
        Stot=SIGION(1,A1,Z1,A2,Z2,T)
        Sin =SIGION(2,A1,Z1,A2,Z2,T)
 1      WRITE(7,2)A1,A2,E(I),SIN,STOT
 2      FORMAT(2X,2F6.0, F9.1, 2F10.2)
        END
```

OUTPUT FILE

Results of the calculations

```
      A1     A2        E,MeV        Sin,mb       Stot,mb
```

```
-----------------------------------------------
    2.    12.     100.0       858.01      1299.45
    2.    12.     200.0       698.92      1013.80
    2.    12.     400.0       593.62       830.52
    2.    12.    1000.0       479.40       631.57
    2.    12.    2000.0       414.50       572.97
```

```
      REAL*8 FUNCTION SIGION(IS,A1,Z1,A2,Z2,T)
C *****************************************************
C * Calculation of nucleus-nucleus total and inelastic *
C *                   cross-sections                    *
C *****************************************************
C  The calculated quantities:
C  SIGION - total (IS=1) or inelastic (IS=2) cross-section
c          (mbarn)
C -----------------------------------------------------
C   The used quantities:
C   IS=1 or 2 - type of cross-section
C   A1 and Z1 - projectile mass and charge numbers
C   A2 and Z2 - target mass and charge numbers
C   T - projectile energy (MeV)
C   C(I) - phenomenological function for elastic
C          cross-section
C   CD(I),CL(I) - phenomenological functions for
C          deutron-nucleus and more heavy ion-nucleus
C          inelastic cross-sections
C    TL(I) - energies at which the functions C, CD, CL
C          are defined
C -----------------------------------------------------
C    The used subroutines:
C    BINT - linear interpolation
C    FHZ and FLE - calculation of high- and low-energy
C          cross-sections
C  ----------------------------------------------------
     IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /CX/ TL(42),C(42),CD(42),CL(42)
    *       /CCT/ A11,AP,AT,B,R0, CX,CT,TLIM
```

```
C ---------------------------------------------------
C Auxiliary quantities which will be used in the
C calculations
                A11=A1
                A12=A1*A2/(A1+A2)
                TP=T/A1
                AMP=A1*940.0
                AMT=A2*940.0
                RX=1.D0/3.D0
                AP=A1**RX
                AT=A2**RX
C               Parameter defining nucleus radii
                R0=1.4D0
                IF(A1 .LT. 2.1) THEN R0=2.1D0
                IF(ABS(A1-4.0) .LT. 0.1) R0=1.3D0

C         Coulomb barrier
          B=1.44D0*Z1*Z2/R0/(AP+AT)

C         CMS kinetic energy (energy of the particle with
C         the mass A12)
          TC=TP*A12

C         Energy at which sowing together of the low- and
C         high-energy approximations take place
          TLIM=B/0.61
C         (0.61 is a fitted constant)

            IF(TC .GE. TLIM) THEN
            TLOG=DLOG10(T/A1)
C               Calculation of the parameters
                IF(A1 .LT. 2.1) THEN
C               (inelastic deutron cross-section)
                CX=BINT(TLOG,TL,CD,42,1)
                GO TO 2
                ELSE
                CX=BINT(TLOG,TL,CL,42,1)
C               (inelastic heavy ion cross-sections)
```

```
                END IF
  2             IF(IS .EQ. 1) CT=BINT(TLOG,TL,C,42,1)
C               (elastic cross-cection)
C          High-energy  cross-sections
           SIGION=FHS(IS,TC)
           ELSE
C             low-energy cross-section
           SIGION=FLS(IS,TC)
           END IF
       RETURN
       END

        REAL*8 FUNCTION FHS(IS,EPC)
C  ************************************************
C  *  Calculation of high-energy nucleus-nucleus  *
C  *                cross-section                 *
C  ************************************************
C  The calculated quantities:
C  FHS - total (IS=1) or inelastic (IS=2)
C       cross-section (mbarn)
C--------------------------------------------------
C   The used quantities:
C  IS=1 or 2 - type of cross-section
C   EPC -  CMS kinetic energy (MeV)
C  Data in COMMON /CCT/
C --------------------------------------------------
       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON /CCT/ A11,AP,AT,B,R0,CX,CT,TLIM
C   --------------------------------------------------
          FHS=31.416*1.21*(1.-B/EPC)*(AP+AT+
     *         1.85*AP*AT/(AP+AT)-CX)**2
               IF(IS .EQ. 1) FHS=FHS+
                       31.416*1.21*(1.-B/EPC)*(AP+AT+
     *                 1.85*AP*AT/(AP+AT)-CT)**2
        RETURN
        END
```

```
      REAL*8 FUNCTION FLS (IS,EPC)
C  ********************************************
C * Calculation of low-energy nucleus-nucleus     *
C *                cross-section                  *
C ********************************************
C The calculated quantities:
C FLS - total (IS=1), inelastic (IS=2) cross-section
C      (mbarn)
C----------------------------------------------------
C  The used quantities:
C  IS=1 or 2 - type of cross-section
C  A1 mass number of projectile nucleus
C  EPC - CMS kinetic energy (MeV)
C  B and R0 - Coulomb barrier and parameter defining
C         nuclear radius
C  FC - auxiliary function
C ---------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /CCT/ A1,AP,AT,B,R0,CX,CT,TLIM
C   -------------------------------------------------
c              Parameter defining nuclear radius at
C              low-energy interactions
               IF(ABS(A1-4.0) .LT. 0.1) THEN
                R00=1.71
               ELSE
               R00=1.47
               END IF
               IF(A1 .LT. 2.1) R00=2.1
C              Linear energy dependence of radius
               R00=R00*(1.0-0.021*EPC/A1)
C              Renormalized Coulomb barrier
               BR=B*R0/R00
C     Check, is EPC lower the threshold energy or not
      F1=FC(EPC,BR)
      F2=FC(EPC-0.001,BR)
      IF(F2 .GT. F1) THEN
      FLS=0.0
      RETURN
```

```
      END IF

      REN=F1/FC(TLIM,BR)
      FLS=FHS(2,TLIM)*REN
      IF(IS .EQ. 1) FLS=FHS(1,TLIM)*REN
C     (rough estimation, without Coulomb scattering)
      IF(FLS .LE. 0.0) SIGION=1.0D-07
      RETURN
      END


      REAL*8 FUNCTION FC(E,B)
C   **********************************************
C   * Auxiliary function for calculation of  FLS  *
C   ***8******************************************
      IMPLICIT REAL*8 (A-H,O-Z)
C    ---------------------------
      X=(E-B)/1.095
      IF(X .GT. 5.0) THEN
      FC=X/E
      ELSE
      D=1.0+EXP(X)
      FC=DLOG(D)/E
      END IF
      RETURN
      END


      REAL*8 FUNCTION BINT(U,E,F,N,IS)
C ****************************************************
C * Linear or quadratic interpolation (extrapolation) *
C ****************************************************
C  The  calculated quantities:
C  Value of the function F(U)
C  -------------------------------------------------
C  Used quantities:
C  E(N) and F(N) - N-dimensional arrays of argument
C           and the extrapolated function values
```

```
C  IS=1 or 2  - linear or quadratic  interpolation
C  ---------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION E(N),F(N),X(3),Y(3)
C  ---------------------------------------------------
      IF(N .EQ. 1) THEN
      BINT=F(1)
      RETURN
      END IF
      IF(N .GT. 2) GO TO 2
C     Linear interpolation
             DO 1 I=1,2
             X(I)=E(I)
 1           Y(I)=F(I)
             GO TO 10
 2    IF(U .GT. E(1)) GO TO 4
C     Inter- and extrapolation to small values of E(I)
      DO 3 I=1,3
      X(I)=E(I)
 3    Y(I)=F(I)
      GO TO 9
 4    IF(U .LE. E(N-1)) GO TO 6
C     Inter- and extrapolation to large values of E(I)
      DO 5 I=1,3
      X(I)=E(N+I-3)
 5    Y(I)=F(N+I-3)
      GO TO 9
C     Interpolation without extrapolation
  6   IF(N .LE. 2) GO TO 9
      DO 8 J=2,N-1
      IF(U .GE. E(J)) GO TO 8
      DO 7 I=1,3
      X(I)=E(I+J-2)
 7    Y(I)=F(I+J-2)
      GO TO 9
 8    CONTINUE
 9     IF(IS.NE.2.OR.N.EQ.2)  GO TO 10
       BINT=Y(1)*(((U-X(2))*(U-X(3)))/
```

```
     *      ((X(1)-X(2))*(X(1)-X(3))))+
     *      Y(2)*(((U-X(1))*(U-X(3)))/
     *      ((X(2)-X(1))*(X(2)-X(3))))+
     *      Y(3)*(((U-X(1))*(U-X(2)))/
     *      ((X(3)-X(1))*(X(3)-X(2))))
 10    CONTINUE
       IF(IS.EQ.1.OR.N.EQ.2) BINT=
     * Y(1)+(U-X(1))*(Y(2)-Y(1))/(X(2)-X(1))
       RETURN
       END


                     BLOCK DATA CNN
C  ******************************************
C  * Nucleus-nucleus cross-section parameters  *
C  ******************************************
C    TL(I) - projectile energy (MeV/nucleon)
C    C(I) - parameters for elastic cross-section
C    CD(I),CL(I) - parameters for deutron-nucleus and
C                  more heavy nucleus-nucleus inelastic
C                  cross-sections
C ----------------------------------------------------
       IMPLICIT REAL*8 (A-H,O-Z)
        COMMON /CX/ TL(42),C(42),CD(42),CL(42)
C    ------------------------------------------
        DATA TL/ Data fron Table II /
        DATA CD/ Data fron Table II /
        DATA CL/ Data fron Table II /
        DATA C / Data fron Table II /
         END
```

Acknowledgement

One of the authors (HK) is grateful for BARC (Bombey) and ILTP (Delhi) for financial support.

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Received on July 2, 2003.

Барашенков В. С., Кумават Х. E2-2003-128

Интегральные сечения ядро-ядерных взаимодействий

Приведены аналитические выражения, аппроксимирующие экспериментальные данные по интегральным сериям упругих и неупругих взаимодействий тяжелых и легких ядер вплоть до энергий в несколько ГэВ/нуклон. Расчетные значения располагаются внутри коридора экспериментальных погрешностей или близко к нему. Также подробно приведен комментированный фортранный код для расчета сечений и соответствующий численный пример, поясняющий его использование.

Работа выполнена в Лаборатории информационных технологий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2003

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Integral Nucleus–Nucleus Cross-Sections

Expressions approximating the experimental integral cross-sections for elastic and inelastic interactions of light and heavy nuclei at the energies up to several GeV/nucleon are presented. The calculated cross-sections are inside the corridor of experimental errors or very close to it. Described in detail FORTRAN code and a numerical example of the cross-section approximation are also presented.

The investigation has been performed at the Laboratory of Information Technologies, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2003

Макет *Т. Е. Попеко*

Подписано в печать 17.07.2003.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 1,18. Уч.-изд. л. 1,37. Тираж 425 экз. Заказ № 54014.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@pds.jinr.ru
www.jinr.ru/publish/