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# TEMPERATURE DEPENDENCE OF SPREADING WIDTH OF GIANT DIPOLE RESONANCE

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# 1. Introduction

The present paper addresses the problem of a temperature dependence of the giant dipole resonance (GDR) width. Actually, our concern about is only with one part of the total GDR width - the spreading one.

GDR was found in a hot rotating nucleus formed in a collision of two heavy ions as early as 1981 [1]. As a result of quite sophisticated experiments performed during 20 years some integral characteristics of GDR were carefully studied. In particular, it is well proved that the energy of GDR and the exhaustion of the model independent Energy Weighted Sum Rule ($EWSR$) are quite stable against temperature increase. At the same time one observes a strongly increasing width of GDR with temperature of a nucleus $T$.

Several processes contribute to the GDR width at finite temperature [2–4]. Among them are quantum fluctuations which exist already in a cold nucleus: the Landau damping, the coupling with surface vibrations, the collisional damping (i.e. the coupling to incoherent two-particle–two-hole excitations) and the coupling to the single-particle continuum. At $T \neq 0$ the thermal fluctuations of a nuclear shape appear. Moreover, since a hot compound nucleus usually carries a large angular momentum, the rotation also affects the GDR width.

Extracting the GDR characteristics from the measured $\gamma$-spectra is not an absolutely unambiguous procedure. These spectra are in fact a weighted sum of the $\gamma$-ray yield emitted by many nuclei populated in the decay of the initial compound nucleus. The extracted GDR characteristics depend to some extent on assumptions about a shape of $E1$ strength function, and mass- and temperature- dependence of its parameters [5]. Also, the temperatures inferred from experimental excitation energy of a hot compound nucleus are sensitive to the level density parameter which is not known very accurately. The impressive example is the fate of a phenomenon of the so-called saturation of the GDR width at $T \geq 3.5-4$ MeV. After the appearance of new data and reanalysis of the previous ones [6, 7] the GDR width $\Gamma_{\rm GDR}$ was found permanently increasing up to $T \sim 3.2$ MeV. It was also established that the information about GDR at higher temperatures cannot be extracted reliably from the existing data.

Even a more ambiguous problem is the disentangling of different contributions to the experimental GDR width. Fortunately, due to the experiments with inelastically scattered $\alpha$-particles which yield a compound system with a small angular momentum [8] the effects of rotation and temperature on the GDR width were separated. However, in most cases conclusions can be made only by comparing the final results of theoretical calculations with the measured (extracted !) experimental value. Sometimes conclusions appear to be controversial. For example, the adiabatic coupling model [9] reasonably describes the experimental data on the GDR width in $^{120}$Sn and $^{208}$Pb supposing the intrinsic GDR width $\Gamma^{\downarrow}$ almost independent of temperature. According to studies [9], the main effect, which explains increasing of $\Gamma_{GDR}$, is the thermal nuclear shape fluctuations. On the other hand, according to [10], the behavior of the GDR parameters in the compound nucleus $^{86}$Mo cannot be explained by assuming $\Gamma^{\downarrow}$ be a constant.

Different theoretical approaches also predict a quite different $T$-dependence for the GDR width. The first calculations of a thermal behavior of $\Gamma^{\downarrow}$ were performed in [11]. At that time, it was already well known that the coupling of a single-particle motion with collective surface vibrations is the main mechanism of damping of giant resonances in cold nuclei. In [11], a temperature dependence of this coupling was studied with the Matsubara thermal Green's function technique and it was found that the GDR width was nearly constant when $T$ increased. The physical ground of these calculations was the Nuclear Field Theory [12] (NFT) treating a nucleus as a system of interacting quasiparticles and vibrations (RPA phonons). In more recent studies [13] the very weak dependence $\Gamma^{\downarrow}$ on $T$ was explained by the cancellation effect between self-energy and vertex contributions. However, several years ago in [14], where the problem was studied within the same formalism and under the same physical assumptions as in [11, 13], an increment of the spreading GDR width with $T$ was found.

The latter result qualitatively agrees with that of the approaches taking into account the coupling with incoherent 2p–2h excitations (the collisional damping) [4]. For example, a semiclassical theory based on exact solutions of the linearized Vlasov kinetic equation [15] predicted the increase in the GDR width with temperature although, according to [5], the increase was too slow. A phenomenological method of independent sources of dissipation [16] developed within the same semiclassical approach demonstrated the intrinsic GDR width quite stable against $T$ whereas the contribution of two-body dissipation increases with $T$. Also, the calculations performed within the quantum framework of the small amplitude limit of the extended time-dependent Hartree-Fock method with the non-Markovian collision term [17, 18] showed the increase in the intrinsic GDR width with temperature. However, the absolute value of the width is still uncertain ranging between 25% to 50% of the observed value [4].

Thus, the current situation with the temperature dependence of the GDR spreading width, as one can conclude from the above brief review, is not clear. That is why we present the results of calculations within one more approach. The approach was developed in [19–21] and is based on the two main ingredients: the Quasiparticle-Phonon Nuclear Model (QPM) [22–24] and the formalism of thermo field dynamics (TFD) [25, 26]. For a long time QPM was successfully used in theoretical investigations of damping of various giant resonances including two-phonon ones [27] in cold nuclei. The physical basis of QPM is very similar to that of the Nuclear Field Theory, and both the models have produced quite close results as applied to nuclear structure calculations at $T = 0$. In [19–21] the QPM was extended to finite temperatures by the use of the TFD formalism. Already at that formal stage interesting differences with [11] were noted. The main new scope of the present paper is numerical calculations of the $T$-dependence of $\Gamma^{\downarrow}$ in the TFD-QPM approach. Moreover, basing on the present results we discuss more carefully than before a relation of our approach to that of [11, 13, 14].

The paper is organized as follows. In Sect. 2, the extension of the Quasiparticle-Phonon Nuclear Model to finite temperatures is presented. In Sect. 3, the results of numerical calculations for $^{120}$Sn and $^{208}$Pb nuclei are presented. We discuss a physical background of our results and a comparison with other approaches in Sect. 4. A short conclusion is given in Sect. 5.

# 2. QPM at finite temperature

## 2.1 Thermal RPA

First attempts to apply the TFD formalism to nuclear structure problems were made in [26, 28] and [19–21]. Up to now the TFD formalism is not widely used in the nuclear structure studies. So it seems appropriate to outline how QPM can be extended to finite temperatures within the TFD thus repeating to some extent the results of [19–21].

The QPM Hamiltonian in a cold nucleus consists of phenomenological mean fields for protons and neutrons, pairing interaction of the BCS type and separable multipole particle - hole interactions with the isoscalar and isovector items

$$H = H_{\rm sp} + H_{\rm pair} + H_{\rm ph} \tag{1}$$

where

$$H_{\rm sp} = \sum_{jm\ \tau} (E_j - \lambda_\tau)\, c^+_{jm} c_{jm} \tag{2}$$

$$H_{\rm pair} = -\sum_{\tau} \frac{G_\tau}{4} \sum_{\substack{j_1 m_1 \\ j_2 m_2}}{}^{\tau} c^+_{j_1 m_1} c^+_{\overline{j_1 m_1}} c_{\overline{j_2 m_2}} c_{j_2 m_2} \tag{3}$$

$$H_{\rm ph} = -\frac{1}{2} \sum_{\lambda\mu} \sum_{\tau,\rho=\pm 1} \left(\kappa_0^{(\lambda)} + \rho\kappa_1^{(\lambda)}\right) M^+_{\lambda\mu}(\tau) M_{\lambda\mu}(\rho\tau)\ . \tag{4}$$

The operator $M^+_{\lambda\mu}(\tau)$ is the single-particle multipole operator

$$M^+_{\lambda\mu}(\tau) = \sum_{\substack{j_1 m_1 \\ j_2 m_2}}{}^{\tau} \langle j_1 m_1 | R(r) Y_{\lambda\mu}(\vec{r}/r) | j_2 m_2 \rangle c^+_{j_1 m_1} c_{j_2 m_2}$$

and $c^+_{jm}$, $c_{jm}$ are the creation and annihilation operators of particles with quantum numbers $n, l, j, m \equiv j, m$. The notation $\overline{jm}$ means the time-reversed state. The index $\tau$ is isotopic one. It takes two values, $\tau = n, p$. The symbol $\sum^\tau$ means that the summation is taken only over neutron or proton single - particle (hole) states and changing the sign of $\tau$ means changing $n \leftrightarrow p$. The parameters $G_n, G_p$ are constants of neutron-neutron and proton-proton BCS-pairing interactions and $\kappa_0^{(\lambda)}, \kappa_1^{(\lambda)}$ are coupling constants of isoscalar and isovector multipole - multipole (with multipolarity $\lambda$) interactions, respectively.

The first step in treating nuclear dynamics governed by the Hamiltonian (1) at finite temperature is formal doubling of the Hilbert space of a nucleus. To this aim, we introduce a fictitious (tilde-) system which is of exactly the same structure as the initial one. For any operator $A$ acting in the initial Hilbert space there exists its tilde counterpart $\widetilde{A}$ acting in the space of tilde states. The tilde-system is governed by the tilde Hamiltonian $\widetilde{H}$ which has the same structure as $H$, only the operators $c^+_{jm}$, $c_{jm}$ are substituted by their tilde-counterparts $\widetilde{c}^+_{jm}$ and $\widetilde{c}_{jm}$.

The thermal Hamiltonian of the QPM is by definition

$$\mathcal{H} = H - \widetilde{H}. \tag{5}$$

An excitation spectrum of a hot nucleus is obtained by diagonalization of $\mathcal{H}$. At the same time, the thermal behaviour of the nucleus is controlled by the thermal vacuum state $|0(T)\rangle$, which is the eigenstate of $\mathcal{H}$ with the zero eigenvalue.

To construct the thermal vacuum state $|0(T)\rangle$ we made two Bogoliubov transformations. The first one is the standard $(u, v)$ Bogoliubov transformation from the particle operators to the quasiparticle ones $\alpha^+_{jm}$ and $\alpha_{jm}$.

$$\begin{aligned} c^+_{jm} &= u_j \alpha^+_{jm} + (-1)^{j-m} v_j \alpha_{j-m} \\ c_{jm} &= u_j \alpha_{jm} + (-1)^{j-m} v_j \alpha^+_{j-m} \end{aligned} \tag{6}$$

The same transformation (with the same $u_j, v_j$ coefficients) is made with the tilde-operators thus producing tilde quasiparticle operators $\widetilde{\alpha}^+_{jm}$, $\widetilde{\alpha}_{jm}$. The second transformation is a unitary thermal Bogoliubov transformation [25] from ordinary and tilde quasiparticle operators to thermal quasiparticle operators $\beta, \beta^+, \widetilde{\beta}, \widetilde{\beta}^+$

$$\begin{aligned} \beta_{jm} &= x_j \alpha_{jm} - y_j \widetilde{\alpha}^+_{jm} \\ \widetilde{\beta}_{jm} &= x_j \widetilde{\alpha}_{jm} + y_j \alpha^+_{jm} , \end{aligned} \tag{7}$$

where

$$x_j^2 + y_j^2 = 1.$$

The coefficients of the Bogoliubov rotations (6) and (7) are determined simultaneously by minimization of the free energy $F^{(\tau)}$ (separately for neutron and proton subsystems)

$$F^{(\tau)} = \langle 0(T)| H^{(\tau)}_{\rm sp} + H^{(\tau)}_{\rm pair} |0(T)\rangle - TS^{(\tau)} - \lambda_\tau \langle 0(T)| \hat{N}^{(\tau)} |0(T)\rangle \tag{8}$$

where $\hat{N}^{(\tau)}$ is the operator of a number of neutrons (protons) in the nucleus

$$\hat{N}^{(\tau)} = \sum_{jm}^{(\tau)} c_{jm}^{+} c_{jm}$$

The entropy $S^{(\tau)}$ reads

$$S^{(\tau)} = -\sum_{j}^{\tau}(2j+1)\left[x_j^2 \ln x_j^2 + y_j^2 \ln y_j^2\right] \ . \tag{9}$$

Expectation values in (8) are taken with respect the thermal ground state $|0(T)\rangle$ which at this stage is supposed to be the vacuum state for the thermal quasiparticle operators

$$\beta_{jm}|0(T)\rangle = \widetilde{\beta}_{jm}|0(T)\rangle = 0 \ . \tag{10}$$

In terms of the operators $\alpha^{+}, \widetilde{\alpha}^{+}$ the vacuum $|0(T)\rangle$ is nothing but a coherent, or squeezed, state

$$|0(T)\rangle = \exp\left[\sum_{jm} \frac{y_j}{x_j} \alpha_{jm}^{+} \widetilde{\alpha}_{jm}^{+}\right] |0\rangle$$

where $|0\rangle$ is the direct product of the BCS vacuum and its tilde counterpart.

After variation of (8) over the coefficients $u_j, v_j, x_j, y_j$ we obtain the BCS-equations at finite temperature [28, 19].

$$N_\tau = \frac{1}{2}\sum_{j}^{\tau}(2j+1)\left(1 - \frac{(E_j - \lambda_\tau)(1-2n_j)}{\sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}}\right) \tag{11}$$

$$\frac{4}{G_\tau} = \sum_{j}^{\tau}(2j+1)\frac{1-2n_j}{\sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}} \ , \tag{12}$$

The expressions for the coefficients $u_j$, $v_j$ and the quasiparticle energy $\varepsilon_j$ are the following:

$$u_j^2 = \frac{1}{2}\left(1 + \frac{E_j - \lambda_\tau}{\varepsilon_j}\right), \quad v_j^2 = \frac{1}{2}\left(1 - \frac{E_j - \lambda_\tau}{\varepsilon_j}\right), \quad \varepsilon_j = \sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2} \tag{13}$$

And for the coefficients $x_j$, $y_j$ one gets

$$y_j^2 = n_j \ ; \qquad x_j^2 = 1 - n_j, \tag{14}$$

where $n_j$ is the Fermi-Dirac thermal occupation number for the quasiparticle with the energy $\varepsilon_j$

$$n_j = \frac{1}{1 + \exp\left(\varepsilon_j / T\right)}. \tag{15}$$

With the coefficients $u_j, v_j, x_j, y_j$ determined by (13–15) the part of the thermal Hamiltonian which consists of the single-particle and pairing terms and their tilde-counterparts takes the form

$$\mathcal{H}_{\rm TSQP} = \sum_{jm\tau} \varepsilon_j \left(\beta^+_{jm}\beta_{jm} - \widetilde{\beta}^+_{jm}\widetilde{\beta}_{jm}\right).$$

The Hamiltonian $\mathcal{H}_{\rm TSQP}$ describes a system of independent thermal quasiparticles with temperature dependent energies $\varepsilon_j$ (and $-\varepsilon_j$ for the tilde thermal quasiparticles). The ground state of this system is the thermal vacuum state $|0(T)\rangle$ defined by (10).

The term $\mathcal{H}_{\rm ph}$ is the interaction of thermal quasiparticles. After the transformations (6) and (7) the multipole operator $M^+_{\lambda\mu}(\tau)$ takes the form

$$M^+_{\lambda\mu}(\tau) = \frac{(-)^{\lambda-\mu}}{\sqrt{2\lambda+1}} \sum_{j_1j_2}^{\tau} f^{(\lambda)}_{j_1j_2} \left[A^+_\beta(j_1j_2;\lambda\mu) + (-)^{\lambda-\mu}A_\beta(j_1j_2;\lambda-\mu)\right] + B_\beta(j_1j_2;\lambda\mu) \quad (16)$$

The value $f^{(\lambda)}_{j_1j_2}$ is a reduced single-particle matrix element of the one-body multipole operator $M^+_{\lambda\mu}$. The operators $A^+_\beta(j_1j_2;\lambda\mu)$, and $B_\beta(j_1j_2;\lambda\mu)$ are defined as follows:

$$\begin{aligned}
A^+_\beta(j_1j_2;\lambda\mu) &= \frac{1}{2}u^{(+)}_{j_1j_2}\left(\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}\left[\beta^+_{j_1}\beta^+_{j_2}\right]_{\lambda\mu} - \sqrt{n_{j_1}}\sqrt{n_{j_2}}\left[\tilde{\beta}^+_{\overline{j_1}}\tilde{\beta}^+_{\overline{j_2}}\right]_{\lambda\mu}\right) \\
&- v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}\left[\beta^+_{j_1}\tilde{\beta}^+_{\overline{j_2}}\right]_{\lambda\mu} \\
B_\beta(j_1j_2;\lambda\mu) &= u^{(+)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}\left(\left[\beta^+_{j_1}\tilde{\beta}_{j_2}\right]_{\lambda\mu} + (-)^{\lambda-\mu}\left[\beta_{j_1}\tilde{\beta}^+_{j_2}\right]_{\lambda-\mu}\right) \\
&- v^{(-)}_{j_1j_2}\left(\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}\left[\beta^+_{j_1}\beta_{\overline{j_2}}\right]_{\lambda\mu} + \sqrt{n_{j_1}}\sqrt{n_{j_2}}\left[\tilde{\beta}_{j_1}\tilde{\beta}^+_{\overline{j_2}}\right]_{\lambda\mu}\right),
\end{aligned}$$

where

$$u^{(+)}_{j_1j_2} = u_{j_1}v_{j_2} + u_{j_2}v_{j_1}, \qquad v^{(-)}_{j_1j_2} = u_{j_1}u_{j_2} - v_{j_2}v_{j_1}.$$

The operator $A_\beta(j_1j_2;\lambda\mu)$ is the hermitian conjugate of $A^+_\beta(j_1j_2;\lambda\mu)$. The square brackets $[\ \ ]_{\lambda\mu}$ stand for the coupling of single-particle angular momenta $j_1, j_2$ to the sum angular momentum $\lambda$.

At the next step we take into account the RPA correlations due to interaction of thermal quasiparticles [26, 20]. To proceed, we introduce the following thermal phonon operator:

$$\begin{aligned}
Q^+_{\lambda\mu i} &= \frac{1}{2}\sum_{j_1j_2}\left(\psi^{\lambda i}_{j_1j_2}\left[\beta^+_{j_1}\beta^+_{j_2}\right]_{\lambda\mu} + 2\eta^{\lambda i}_{j_1j_2}\left[\beta^+_{j_1}\tilde{\beta}^+_{\overline{j_2}}\right]_{\lambda\mu} + \widetilde{\psi}^{\lambda i}_{j_1j_2}\left[\tilde{\beta}^+_{\overline{j_1}}\tilde{\beta}^+_{\overline{j_2}}\right]_{\lambda\mu}\right) \\
&- (-1)^{\lambda-\mu}\left(\phi^{\lambda i}_{j_1j_2}[\beta_{j_2}\beta_{j_1}]_{\lambda-\mu} + 2\zeta^{\lambda i}_{j_1j_2}[\tilde{\beta}_{\overline{j_2}}\beta_{j_1}]_{\lambda-\mu} + \tilde{\phi}^{\lambda i}_{j_1j_2}[\tilde{\beta}_{\overline{j_2}}\tilde{\beta}_{\overline{j_1}}]_{\lambda-\mu}\right) \qquad (17)
\end{aligned}$$

Further, we assume that these phonons are bosons and redefine the ground state of a hot nucleus. Hereafter it is a vacuum state for the thermal phonon operator $|\Psi_0(T)\rangle$, i.e.

$Q_{\lambda\mu}|\Psi_0(T)\rangle = 0$. Thus the function $|\Psi_0(T)\rangle$ is a temperature dependent wave function of the compound state. With an assumption on the bosonic nature of the phonon operator (17) the norm of a thermal one-phonon wave function is

$$\frac{1}{2}\sum_{j_1j_2}(\psi^{\lambda i}_{j_1j_2})^2 - (\phi^{\lambda i}_{j_1j_2})^2 + (\tilde{\psi}^{\lambda i}_{j_1j_2})^2 - (\tilde{\phi}^{\lambda i}_{j_1j_2})^2 + 2(\eta^{\lambda i}_{j_1j_2})^2 - 2(\zeta^{\lambda i}_{j_1j_2})^2 = 1 \tag{18}$$

Then the thermal RPA equations can be obtained by either applying the variational principle or the equation of motion method. Here we show only the secular equation for energies $\omega_{\lambda i}$ of thermal one-phonon states $|\lambda i\rangle$ and expressions for amplitudes of a thermal phonon wave function. The secular equation reads

$$[X_n(\omega) + X_p(\omega)]\left(\kappa_0^{(\lambda)} + \kappa_1^{(\lambda)}\right) - 4\kappa_0^{(\lambda)}\kappa_1^{(\lambda)}X_n(\omega)X_p(\omega) = 1, \tag{19}$$

where

$$X_\tau(\omega) = \frac{1}{2\lambda+1}\sum_{j_1j_2}^{\tau}(f^{(\lambda)}_{j_1j_2})^2\left[\frac{(u^{(+)}_{j_1j_2})^2(1-n_{j_1}-n_{j_2})(\varepsilon_{j_1}+\varepsilon_{j_2})}{(\varepsilon_{j_1}+\varepsilon_{j_2})^2-\omega^2} - \frac{(v^{(-)}_{j_1j_2})^2(n_{j_1}-n_{j_2})(\varepsilon_{j_1}-\varepsilon_{j_2})}{(\varepsilon_{j_1}-\varepsilon_{j_2})^2-\omega^2}\right]. \tag{20}$$

The amplitudes are

$$\psi^{\lambda i}_{j_1j_2} = \sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}u^{(+)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}}{(\varepsilon_{j_1}+\varepsilon_{j_2})-\omega_{\lambda i}};\quad \phi^{\lambda i}_{j_1j_2} = \sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}u^{(+)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}}{(\varepsilon_{j_1}+\varepsilon_{j_2})+\omega_{\lambda i}};$$

$$\eta^{\lambda i}_{j_1j_2} = -\sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}}{(\varepsilon_{j_1}-\varepsilon_{j_2})-\omega_{\lambda i}};\qquad \zeta^{\lambda i}_{j_1j_2} = -\sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}}{(\varepsilon_{j_1}-\varepsilon_{j_2})+\omega_{\lambda i}};$$

$$\tilde{\psi}^{\lambda i}_{j_1j_2} = \sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}u^{(+)}_{j_1j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}}{(\varepsilon_{j_1}+\varepsilon_{j_2})+\omega_{\lambda i}};\qquad \tilde{\phi}^{\lambda i}_{j_1j_2} = \sqrt{\frac{1}{2\mathcal{N}^{\lambda i}_\tau}}\frac{f^{(\lambda)}_{j_1j_2}u^{(+)}_{j_1j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}}{(\varepsilon_{j_1}+\varepsilon_{j_2})-\omega_{\lambda i}},$$

where the factor $\mathcal{N}^{\lambda i}_\tau$ is

$$\mathcal{N}^{\lambda i}_\tau = \frac{2\lambda+1}{2}\left[\frac{\partial}{\partial\omega}X^{\lambda i}_\tau(\omega)\Big|_{\omega=\omega_{\lambda i}} + \left(\frac{1-X^{\lambda i}_\tau(\omega_{\lambda i})(\kappa_0^{(\lambda)}+\kappa_1^{(\lambda)})}{X^{\lambda i}_{-\tau}(\omega_{\lambda i})(\kappa_0^{(\lambda)}-\kappa_1^{(\lambda)})}\right)^2\frac{\partial}{\partial\omega}X^{\lambda i}_{-\tau}(\omega)\Big|_{\omega=\omega_{\lambda i}}\right] \tag{21}$$

It is worthwhile to note that in contrast with RPA at $T = 0$ the solutions of (19) with negative energies have physical meaning (see also [14]). They correspond to the tilde-phonon states $\widetilde{Q}^+_{\lambda\mu i}|\Psi_0(T)\rangle$

$$\langle\Psi_0(T)|\left[\mathcal{H}, Q^+_{\lambda\mu i}\right]|\Psi_0(T)\rangle = -\langle\Psi_0(T)|\left[\mathcal{H}, \widetilde{Q}^+_{\lambda\mu i}\right]|\Psi_0(T)\rangle = \omega_{\lambda i}$$

Let us comment on the structure of a TRPA phonon. The components $\psi$ and $\phi$ are the same as in the standard quasiparticle RPA (QRPA) (see, e.g. [24]) and are only damped being heated by the factor $(1-n_j)$. The components $\tilde{\psi}$ and $\tilde{\phi}$ are totally due to the tilde part of the Fock space of a heated nucleus. They vanish in a cold nucleus. Note that the $\omega$ dependence of the forward and backward tilde amplitudes is just opposite to that of the ordinary amplitudes. It means that, e.g., while $\psi$ is of a pole character $\tilde{\psi}$ is not and instead the amplitude $\tilde{\phi}$ is a pole amplitude. The most interesting amplitudes are $\eta$ and $\zeta$. They could be specified as cross-over amplitudes containing both the ordinary and tilde thermal quasiparticles. Just due to them the poles $\varepsilon_{j_1}-\varepsilon_{j_2}$, which do not exist in QRPA at $T=0$, appear in (19). Note that these poles can appear at quite low energies, thus enriching a low-energy part of the phonon spectrum in comparison with QRPA at $T=0$. The amplitudes $\eta$ and $\zeta$ depend on the superfluid factor $v^{(-)}_{j_1j_2}$ which is enhanced when both the states $j_1$ and $j_2$ are of a particle or a hole type. In contrast, the four other amplitudes are proportional to the superfluid particle-hole factor $u^{(+)}_{j_1j_2}$.

In nuclei with pairing correlations the amplitudes $\eta,\zeta$ vanish when $T\to 0$. However, in normal nuclei the thermal phonon operator (17) consists of only two types of components $\eta$ and $\zeta$

$$Q^{+}_{\lambda\mu i}=\sum_{j_1j_2}\eta^{\lambda i}_{j_1j_2}\left[\beta^{+}_{j_1}\tilde{\beta}^{+}_{\overline{j_2}}\right]_{\lambda\mu}+(-1)^{\lambda-\mu}\zeta^{\lambda i}_{j_1j_2}\left[\beta_{j_1}\tilde{\beta}_{\overline{j_2}}\right]_{\lambda-\mu}.$$

The expressions for $\eta$ and $\zeta$ displayed above are valid in this case as well excepting that the value $v^{(-)}_{j_1j_2}$ equals to unity. The expression (20) also becomes simpler

$$X_\tau(\omega)=\frac{1}{2\lambda+1}\sum_{j_1j_2}^{\tau}\frac{(f^{(\lambda)}_{j_1j_2})^2(n_{j_1}-n_{j_2})(E_{j_1}-E_{j_2})}{(E_{j_1}-E_{j_2})^2-\omega^2}.$$

At the end of this subsection we display the expression for the matrix element $\Phi_{\lambda i}$ of the $E\lambda$-transition operator from the ground state of a hot nucleus to a thermal one-phonon state (i.e. for the transition $|\Psi_0(T)\rangle\to Q^{+}_{\lambda\mu i}|\Psi_0(T)\rangle$). It reads [20]

$$\begin{aligned}\Phi_{\lambda i} &= \sum_{j_1j_2}\langle j_1||\mathcal{M}(E\lambda)||j_2\rangle\left\{\frac{1}{2}u^{(+)}_{j_1j_2}\left[\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}\,(\psi^{\lambda i}_{j_1j_2}+\phi^{\lambda i}_{j_1j_2})\right.\right.\\ &- \left.\left.\sqrt{n_{j_1}}\sqrt{n_{j_2}}\left(\tilde{\psi}^{\lambda i}_{j_1j_2}+\tilde{\phi}^{\lambda i}_{j_1j_2}\right)\right]-v^{(-)}_{j_1j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}\,(\eta^{\lambda i}_{j_1j_2}+\zeta^{\lambda i}_{j_1j_2})\right\}, \qquad (22)\end{aligned}$$

where $\langle j_1||\mathcal{M}(E\lambda)||j_2\rangle$ is a reduced single-particle matrix element of the $E\lambda$ transition operator.

## 2.2 Interaction of thermal phonons

Now the thermal Hamiltonian reads in terms of the TRPA phonons and thermal quasi-particles[3]

$$
\begin{aligned}
\mathcal{H} &= \sum_{\lambda\mu i}\omega_{\lambda i}\left(Q^{+}_{\lambda\mu i}Q_{\lambda\mu i}-\widetilde{Q}^{+}_{\lambda\mu i}\widetilde{Q}_{\lambda\mu i}\right)- \\
&- \frac{1}{2\sqrt{2}}\sum_{\lambda\mu i}\sum_{\tau}\sum_{j_1j_2}{}^{\tau}\frac{f^{(\lambda)}_{j_1j_2}}{\sqrt{\mathcal{N}^{\lambda i}_{\tau}}}\{((-)^{\lambda-\mu}Q^{+}_{\lambda\mu i}+Q_{\lambda-\mu i})B_{\beta}(j_1j_2;\lambda-\mu)- \\
&- ((-)^{\lambda-\mu}\widetilde{Q}^{+}_{\lambda\mu i}+\widetilde{Q}_{\lambda-\mu i})\widetilde{B}_{\beta}(j_1j_2;\lambda-\mu)+h.c.\Big\}\ ,
\end{aligned}
\tag{23}
$$

The terms $\sim (Q^{+}+Q)\,B$ etc (hereafter we denote their sum by $\mathcal{H}_{\rm qph}$) couple a thermal one-phonon state with more complex thermal configurations, e.g., two-phonon ones. Due to this mixing the strength of a one-phonon state is fragmented over some energy interval. In other words, the term $\mathcal{H}_{\rm qph}$ produces a spreading width of a thermal one-phonon state. To describe the fragmentation of thermal phonons, we use again the variational method with a trial wave function of the form

$$
|\Psi_{\nu}(JM)\rangle=\left\{\sum_{i}R_i(J\nu)Q^{+}_{JMi}+\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}}P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\left[Q^{+}_{\lambda_1\mu_1 i_1}Q^{+}_{\lambda_2\mu_2 i_2}\right]_{JM}\right\}|\Psi_0(T)\rangle \tag{24}
$$

The equation for energies of states (24) is

$$
det\left|(\omega_{Ji}-\eta_{J\nu})\delta_{ii'}-\frac{1}{2}\sum_{\lambda_1 i_1\lambda_2 i_2}\frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}}\right|=0. \tag{25}
$$

The functions $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ are the coupling matrix elements between one- and two-phonon states. The expression for U is the following:

$$
\begin{aligned}
U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji,\tau) &= -\frac{1}{\sqrt{2}}\sqrt{2\lambda_1+1}\sqrt{2\lambda_2+1}\sum_{j_1j_2j_3}{}^{\tau}\left[(-)^{J}\Gamma^{\lambda_2 i_2}_{j_1j_2}\left\{\begin{matrix}\lambda_2 & \lambda_1 & J\\ j_3 & j_2 & j_1\end{matrix}\right\}\mathcal{K}^{\lambda_1 i_1 Ji}_{j_3j_2j_1}+\right. \\
&\left.+(-)^{\lambda_1-\lambda_2}\Gamma^{\lambda_1 i_1}_{j_1j_2}\left\{\begin{matrix}\lambda_1 & \lambda_2 & J\\ j_3 & j_2 & j_1\end{matrix}\right\}\mathcal{K}^{\lambda_2 i_2 Ji}_{j_3j_2j_1}+(-)^{J-\lambda_1}\Gamma^{Ji}_{j_1j_2}\left\{\begin{matrix}J & \lambda_1 & \lambda_2\\ j_3 & j_2 & j_1\end{matrix}\right\}\mathcal{L}^{\lambda_1 i_1\lambda_2 i_2}_{j_3j_2j_1}\right]\ ,
\end{aligned}
\tag{26}
$$

where $\Gamma^{\lambda i}_{j_1j_2}=f^{(\lambda)}_{j_1j_2}/\sqrt{\mathcal{N}^{\lambda i}}$ and the functions $\mathcal{K}^{\lambda_2 i_2 Ji}_{j_3j_2j_1}$ and $\mathcal{L}^{\lambda_1 i_1\lambda_2 i_2}_{j_3j_2j_1}$ are

$$
\begin{aligned}
\mathcal{K}^{\lambda_1 i_1 Ji}_{j_3j_2j_1} &= v^{(-)}_{j_1j_2}x_{j_1}x_{j_2}(-1)^{j_1+j_3+\lambda_1+J}\left(\psi^{\lambda_1 i_1}_{j_1j_3}\psi^{Ji}_{j_2j_3}+\phi^{\lambda_1 i_1}_{j_1j_3}\phi^{Ji}_{j_2j_3}+\eta^{\lambda_1 i_1}_{j_1j_3}\eta^{Ji}_{j_2j_3}+\zeta^{\lambda_1 i_1}_{j_1j_3}\zeta^{Ji}_{j_2j_3}\right)\\
&+ u^{(+)}_{j_1j_2}x_{j_1}y_{j_2}(-)^{j_1+j_2+\lambda_1}\left(\psi^{\lambda_1 i_1}_{j_1j_3}\eta^{Ji}_{j_3j_2}+\phi^{\lambda_1 i_1}_{j_1j_3}\zeta^{Ji}_{j_3j_2}+\eta^{\lambda_1 i_1}_{j_1j_3}\tilde{\psi}^{Ji}_{j_3j_2}+\zeta^{\lambda_1 i_1}_{j_1j_3}\tilde{\phi}^{Ji}_{j_3j_2}\right)\\
&- u^{(+)}_{j_1j_2}y_{j_1}x_{j_2}(-1)^{J}\left(\eta^{\lambda_1 i_1}_{j_3j_1}\psi^{Ji}_{j_2j_3}+\zeta^{\lambda_1 i_1}_{j_3j_1}\phi^{Ji}_{j_2j_3}+\tilde{\psi}^{\lambda_1 i_1}_{j_3j_1}\eta^{Ji}_{j_2j_3}+\tilde{\phi}^{\lambda_1 i_1}_{j_3j}\zeta^{Ji}_{j_2j_3}\right)\\
&- v^{(-)}_{j_1j_2}y_{j_1}y_{j_2}(-)^{j_2+j_3}\left(\eta^{\lambda_1 i_1}_{j_3j_1}\eta^{Ji}_{j_3j_2}+\zeta^{\lambda_1 i_1}_{j_3j_1}\zeta^{Ji}_{j_3j_2}+\tilde{\psi}^{\lambda_1 i_1}_{j_3j_1}\tilde{\psi}^{Ji}_{j_3j_2}+\tilde{\phi}^{\lambda_1 i_1}_{j_3j_1}\tilde{\phi}^{Ji}_{j_3j_2}\right)
\end{aligned}
$$

[3]Note that the term $B^{+}_{\beta}B_{\beta}$ and its tilde counterpart are omitted

$$
\begin{aligned}
\mathcal{L}^{\lambda_1 i_1 \lambda_2 i_2}_{j_3 j_2 j_1} &= v^{(-)}_{j_1 j_2} x_{j_1} x_{j_2} (-1)^{j_1+j_3+\lambda_1+\lambda_2} \left(\psi^{\lambda_1 i_1}_{j_1 j_3} \phi^{\lambda_2 i_2}_{j_2 j_3} + \phi^{\lambda_1 i_1}_{j_1 j_3} \psi^{\lambda_2 i_2}_{j_2 j_3} + \eta^{\lambda_1 i_1}_{j_1 j_3} \zeta^{\lambda_2 i_2}_{j_2 j_3} + \zeta^{\lambda_1 i_1}_{j_1 j_3} \eta^{\lambda_2 i_2}_{j_2 j_3}\right) \\
&+ u^{(+)}_{j_1 j_2} x_{j_1} y_{j_2} (-)^{j_1+j_2+\lambda_1} \left(\psi^{\lambda_1 i_1}_{j_1 j_3} \zeta^{\lambda_2 i_2}_{j_3 j_2} + \phi^{\lambda_1 i_1}_{j_1 j_3} \eta^{\lambda_2 i_2}_{j_3 j_2} + \eta^{\lambda_1 i_1}_{j_1 j_3} \tilde{\phi}^{\lambda_2 i_2}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_1 j_3} \tilde{\psi}^{\lambda_2 i_2}_{j_3 j_2}\right) \\
&- u^{(+)}_{j_1 j_2} y_{j_1} x_{j_2} (-)^{\lambda_2} \left(\zeta^{\lambda_1 i_1}_{j_3 j_1} \psi^{\lambda_2 i_2}_{j_2 j_1} + \eta^{\lambda_1 i_1}_{j_3 j_1} \phi^{\lambda_2 i_2}_{j_2 j_3} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1} \eta^{\lambda_2 i_2}_{j_2 j_3} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1} \zeta^{\lambda_2 i_2}_{j_2 j_3}\right) \\
&- v^{(-)}_{j_1 j_2} y_{j_1} y_{j_2} (-1)^{j_2+j_3} \left(\eta^{\lambda_1 i_1}_{j_3 j_1} \zeta^{\lambda_2 i_2}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_3 j_1} \eta^{\lambda_2 i_2}_{j_3 j_2} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1} \tilde{\phi}^{\lambda_2 i_2}_{j_3 j_2} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1} \tilde{\psi}^{\lambda_2 i_2}_{j_3 j_2}\right)
\end{aligned}
$$

Let us note that in case the pairing correlations vanish, expression (26) completely agrees with that from [14] (see eqs.(4.1)-(4.2) in the paper).

To calculate the $E1$-strength function in hot nucleus taking into account a fragmentation of thermal one-phonon dipole states, we explore the well-known strength function method [23, 24]. Avoiding to solve (25) we directly calculate the function

$$
b(E\lambda, \eta) = \sum_{\nu} \frac{1}{2\pi} \frac{\Delta}{(\eta - \eta_{\lambda\nu})^2 + \frac{\Delta^2}{4}} |\Phi(J\nu)|^2, \tag{27}
$$

where the coefficients $\Phi(J\nu)$ the are amplitudes of $E\lambda$-transitions from the ground state of a hot nucleus (a compound state) to states described by the wave functions (24). This amplitudes are superpositions of the matrix elements $\Phi_{Ji}$ (22) the with weight factors $R_i(J\nu)$ from (25)

$$
\Phi(J\nu) = \sum_i R_i(J\nu) \Phi_{Ji},
$$

and $\Delta$ is a smearing parameter.

# 3. Numerical results

We calculate the E1-strength distributions for $0 \leq T \leq 3$ MeV in $^{120}$Sn and $^{208}$Pb nuclei. All model parameters (mean field potentials, pairing constants, coupling constants of separable interactions etc) are fixed in accordance with the standard QPM procedure [22, 24], i.e., by the use of experimental data on the energies of low-lying vibrational states and giant resonances at $T = 0$. As a mean field the phenomenological Woods–Saxon potential is explored. The single-particle basis consists of all bound states and several quasibound ones with a relatively small escape widths.

Pairing correlations that exist only in the neutron system of the $^{120}$Sn nucleus are treated in the thermal BCS approximation. Since we do not make a particle projection, a neutron energy gap in this nucleus vanishes at $T \approx 1$ MeV.

Only multipole-multipole particle-hole interactions with $1 \leq \lambda \leq 7$ are included in the Hamiltonian. A radial form factor of the separable multipole interaction has the form $R(r) = dU/dr$, where $U$ is the central part of the Woods-Saxon potential. The coupling constant of the isoscalar dipole-dipole interaction is adjusted at every value of $T$ to make the energy of the spurious $1^-$-state zero in the TRPA calculations. The chemical potentials $\lambda_{n,p}$ are also adjusted at every $T$ value to keep the right average values of N, Z.

First, let us discuss the TRPA results. The E1-strength distributions in nuclei $^{120}$Sn and $^{208}$Pb at different temperatures are shown in Figs. 1 and 2, respectively. One can see that in TRPA we get qualitatively the same results as many other authors [17, 29]. When temperature increases, only some minor redistribution of the $E1$ strength between different one-phonon $1^-$ states takes place. The energy centroids and Landau widths of GDR in both the nuclei almost do not change with $T$.

In Figs. 3 and 4 the temperature dependencies of the dipole energy weighted sum rule ($EWSR$) in the same nuclei are shown. The model $EWSR$ is calculated by the following formula:

$$EWSR = \sum_{\tau} \sum_{j_1 \geq j_2} (f^{(1)}_{j_1 j_2})^2 \left[ (\varepsilon_{j_1} + \varepsilon_{j_2})(u^{(+)}_{j_1 j_2})^2 (1 - n_{j_1} - n_{j_2}) - (\varepsilon_{j_1} - \varepsilon_{j_2})(v^{(-)}_{j_1 j_2})^2 (n_{j_1} - n_{j_2}) \right]$$

The model independent value of $EWSR$ ($S_1$) is calculated in accordance with the standard expression

$$S_1 = \frac{9}{8\pi} \frac{e^2 \hbar^2}{m} \frac{NZ}{A} = 14.8 \frac{NZ}{A} \mathrm{e^2 fm^2 MeV}$$

Since the calculated values of $EWSR$ are quite close to the model independent ones, one can conclude that our single-particle basis is large enough to describe GDR in medium and heavy nuclei. An excess of our $EWSR$ over the model independent value $S_1$ in $^{120}$Sn at $T < 1$ Mev can be attributed to the effect of the BSC pairing. When the pairing correlations vanish, a deviation of $EWSR$ from $S_1$ as well as a steepness of the $T$-dependence of $EWSR$ in $^{120}$Sn become very similar to those in $^{208}$Pb. On the whole a difference between $S_1$ and $EWSR$ in the range $0 < T < 3$ MeV is less than 10% and a decrease in $EWSR$ with $T$ is less than 5% within the same temperature range.

Now we discuss the results when the interaction of thermal (TRPA) phonons is taken into account. We include in the one-phonon part of (24) 14 dipole phonons with the largest $B(E1)$ values at given $T$ from the energy range 0–30 MeV. These 14 $E1$ TRPA-phonons exhaust more than 80% of the model $EWSR$. The two-phonon part of (24) includes all possible two-phonon $1^-$ states from the energy range 0-30 MeV constructed by combining normal parity phonons of different energies with momenta $1 < \lambda < 7$. Some additional limitations to the two-phonon space will be discussed in Sect. 4. The smearing parameter $\Delta$ in the Lorentz weight function is taken to be equal to 1 MeV. We calculate the energy centroid $\overline{E}$ and spreading width $\Gamma^{\downarrow}$ of the E1-strength distributions with the following formulae:

$$\overline{E} = \sqrt{\frac{m_2}{m_0}}; \qquad \Gamma^{\downarrow} = \sqrt{\frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2}$$

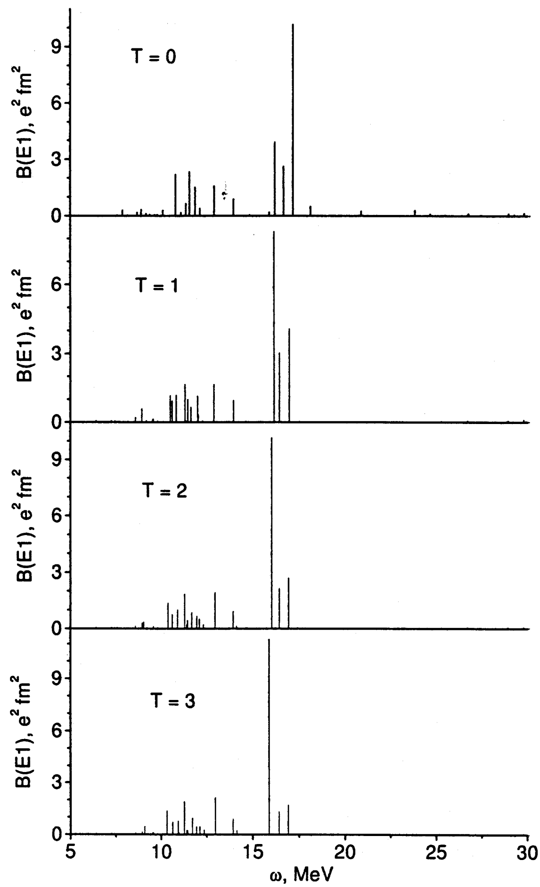


Figure 1: The TRPA results for the E1-strength distribution in $^{120}$Sn nucleus at different temperatures.

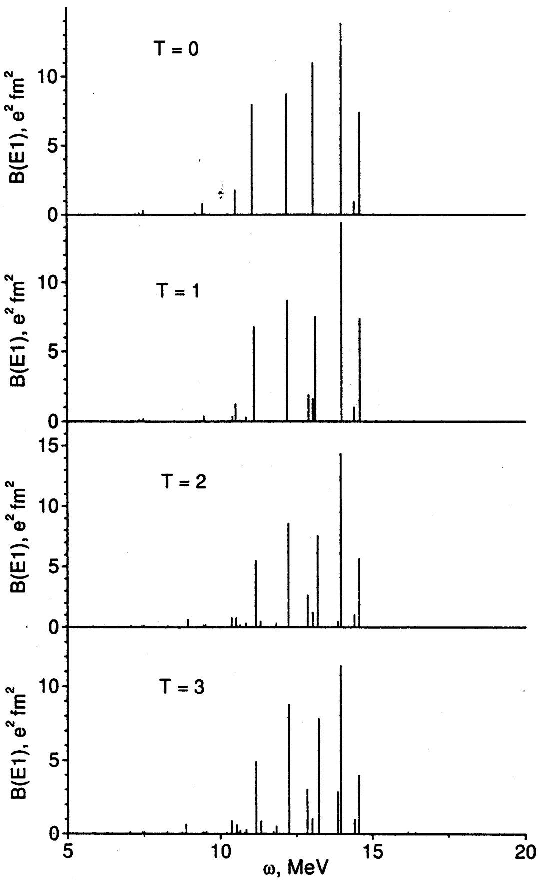


Figure 2: The TRPA results for the E1-strength distribution in $^{208}$Pb nucleus at different temperatures.

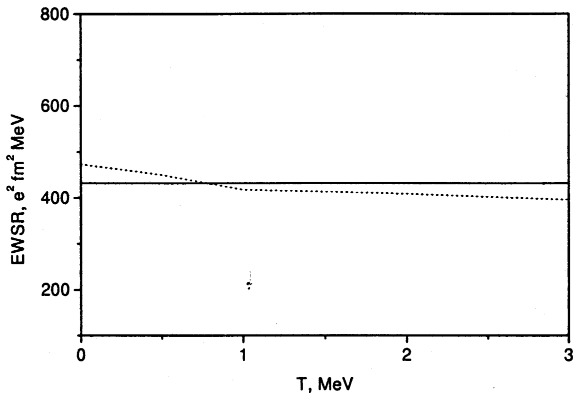


Figure 3: Temperature dependence of $EWSR$ in $^{120}$Sn (dashed line). Solid horizontal line – the value of the model independent energy weighted sum rule $S_1$.

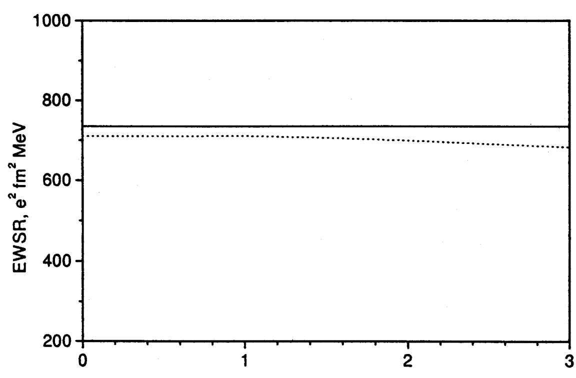


Figure 4: Temperature dependence of $EWSR$ in $^{208}$Pb (dashed line). Solid horizontal line – the value of the model independent energy weighted sum rule $S_1$.

where $m_k$ is the kth energy moment of the E1 strength function defined as follows:

$$m_k = \int_{E_{min}}^{E_{max}} \eta^k b(E1, \eta) d\eta.$$

Our results for the $T$-dependence of $\Gamma^{\downarrow}$ and the experimental data from [8, 30] in both the nuclei are shown in Figs. 5 and 6.

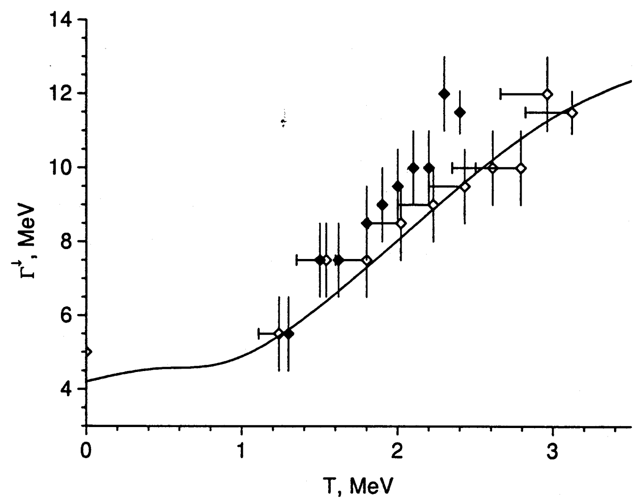


Figure 5: Temperature dependence of the GDR width $\Gamma^{\downarrow}$ in $^{120}$Sn. Open diamonds – experimental data from [8, 30]; full diamonds – revised experimental data from [6].

The most distinctive feature of the theoretical curves is that they show increase in the GDR spreading width with temperature. Moreover, the theoretical results agree quite well with the data of [8, 30]. In principle, it is not encouraging because $\Gamma^{\downarrow}$ is only a part of the total width $\Gamma(GDR)$ and apparently a difference has to exist between the experimental $\Gamma_{\mathrm{GDR}}$ and the calculated $\Gamma^{\downarrow}$. Fortunately, the difference appears when one takes the data of [8, 30] revised by D. Kusnezov et al. [6]. The revised data are also shown in Figs. 5 and 6. They are lying higher than the theoretical curves and seem to grow with temperature slightly faster. A reason of increasing $\Gamma^{\downarrow}$ will be discussed in the next section.

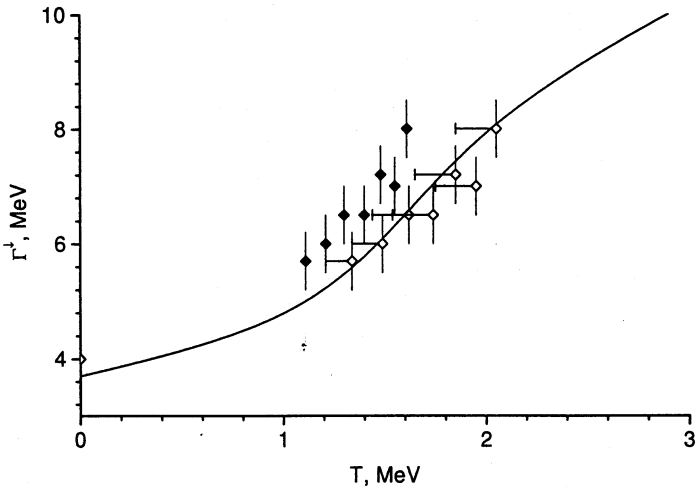


Figure 6: Temperature dependence of the GDR width $\Gamma^{\downarrow}$ in $^{208}$Pb. Open diamonds - experimental data from [8, 30]; full diamonds - revised experimental data from [6].

# 4. Discussion

Our results concerning $\Gamma^{\downarrow}(T)$ qualitatively agree with that of [14] and [15, 17, 18] but are in contradiction with those of [11, 13]. To understand why in our approach the value $\Gamma^{\downarrow}$ increases with temperature, we analyze the matrix elements of a phonon-phonon coupling $U^{\lambda_1 i_1}_{\lambda_2 i_2}(1^-_i)$ and found a strong effect of a few very low-lying thermal phonons appearing in the phonon spectrum only at $T \neq 0$ due to the nonvanishing thermal occupation factors. These states correspond to low-lying poles of the $(\varepsilon_{j_1} - \varepsilon_{j_2})$ type. Moreover, one amplitude $\eta_{j_1 j_2}$ dominates the phonon wave function, i.e., these phonons are noncollective and of the p-p or h-h type. It can be easily shown that the following expression for the amplitude $\eta_{j_1 j_2}$ of such a phonon is valid:

$$\eta^{\lambda i}_{j_1 j_2} \sim \sqrt{\frac{T}{\varepsilon_{j_1} - \varepsilon_{j_2}}} \approx \sqrt{\frac{T}{\omega_{\lambda i}}}$$

because for a non-collective phonon $\varepsilon_{j_1} - \varepsilon_{j_2} \approx \omega$. If one of the phonons $|\lambda_1 i_1\rangle$ or $|\lambda_2 i_2\rangle$ in the matrix element $U^{\lambda_1 i_1}_{\lambda_2 i_2}(1^-_i)$ (26) is of the afore-mentioned type, the value $U$ appears to be also proportional to $\sqrt{T/\omega}$ and $\Gamma^{\downarrow} \sim \sum U^2 \sim T/\omega$. Thus, a temperature dependence of $\Gamma^{\downarrow}$ arises. The appearance of a small value $\omega$ in a denominator explains a strong influence of these noncollective phonons on the $\Gamma^{\downarrow}$ value.

We conclude that the reason for the increment of $\Gamma^{\downarrow}$ with $T$ is the interaction of GDR with the noncollective p-p (or h-h) thermal phonons of the special type. On the whole this conclusion agrees with the results of [14] although in that work a special role of the low-lying p-p (h-h) phonons was not definitely pointed out. It seems that in [11] the noncollective thermal RPA excitations have been ignored (the same statement can be found in [14]). It follows from our consideration that if the thermal phonon space includes only those phonons which are of the p-h type at $T = 0$ $\Gamma^{\downarrow}$ will be quite stable against $T$.

Some questions concerning a dependence of our numerical results on parameters still remain. An appearance of low-lying p-p (h-h) states is dependent on the parameters of the mean field. It seems to us that the use of a phenomenological Saxon-Woods potential gives the upper limit for the role of these low-lying p-p (h-h) states because the density of single-particle states near the Fermi level is the largest one in this potential. We guess that, e.g., with the mean field calculating by the Hartree-Fock method with a density-dependent effective interaction like the Skyrme one, the influence of these phonons on $\Gamma^{\downarrow}$ will be weaker.

There is one more ingredient directly affecting the calculated value of $\Gamma^{\downarrow}$. It is obvious that the whole space of two-phonon states is overcomplete because thermal phonons are considered as bosons and any special projection of two- or four- fermion states into the bosonic ones is not made. This ovecompleteness is partially reduced due to the special limitation in constructing the two-phonon part of the trial wave function (24). Namely, only two-phonon configurations combining two collective or one collective - one noncollective phonons are included in the wave function. However, there is no clear cut separation between collective and noncollective states, especially because the "true" collective states like low-lying quadrupole and octupole phonons dissolve with increasing temperature. Therefore, in practice one needs a quantitative measure of "collectivity of a phonon". This measure can be introduced basing on a phonon structure. For example, a phonon is considered as a collective one if the largest two-fermion component in its wave functions (17) exhausts less than B% of the total norm. Evidently, B is a technical parameter. The larger is B the larger is the thermal two-phonon space or the number of two-phonon configurations taken into account in the calculations. Enlargement of the two-phonon space means strengthening of fragmentation or damping.

To estimate a possible effect of the two-phonon basis, we make the calculations with two different spaces of two-phonon states. In Fig. 7, we display the results of calculations of $\Gamma^{\downarrow}(T)$ in $^{208}$Pb for B = 50% and 60%. The width $\Gamma^{\downarrow}$ sizably decreases together with B. Moreover, a rate of the increase in $\Gamma^{\downarrow}$ with $T$ also becomes slower at a smaller value of B. Nevertheless, a general trend of the thermal behavior of the spreading GDR width is saved.

Note, the effect of a size of the two-phonon subspace is much weaker at $T = 0$. In

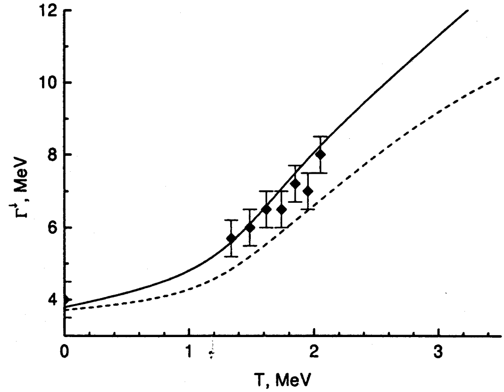


Figure 7: Temperature dependence of the GDR width $\Gamma_\downarrow$ in $^{208}$Pb calculated with different two-phonon model spaces: dashed line – B=50% ; solid line – B=60% . (see the text for comprehensive explanations)

[23], a consistent procedure taking into account the Pauli principle corrections in the two-phonon components was developed. The procedure is based on the exact (from the point of view of their fermionic structure) commutator of phonon operators. Its influence on the strength functions of multipole giant resonances in cold nuclei was studied in [31]. The corresponding corrections were found to be small because at $T = 0$ the main contribution to the spreading width of giant resonances is given by the coupling with the lowest collective quadrupole and octupole phonons and corresponding two-phonon configurations are weakly affected by the Pauli principle. However, with increasing of $T$ the low-lying collective vibrations dissolve and their contribution to the damping of giant resonances diminishes. At the same time, the role of the Pauli principle acting between large number of weakly collective and noncollective states treated as bosons becomes more and more important. The results presented in Fig. 7 reflect this process.

There is one more interesting and at the first glance principal difference between our approach and that of [11, 13, 14]. The difference has already been pointed out in [21] and now we would like to discuss it in more detail. In [11, 14], the GDR width depends on thermal occupation numbers of two types – the Fermi-Dirac and the Bose-Einstein occupation numbers. The appearance of Bose occupation factors is a consequence of treating all the phonons as quasibosons when the temperature dependent Green's function of a single phonon is introduced.

In the present paper, a reader can not find the thermal bosonic occupation numbers, and it seems there is no room for them. We start with the model Hamiltonian written in terms of nucleonic (i.e. fermionic) variables. The thermal occupation numbers appear in the game when we make the thermal Bogoliubov rotation (7) and thus produce the thermal Fock space. All further manipulations explore these "heated" fermions and the appearance of bosonic occupation numbers is quite questionable. Our thermal Hamiltonian in its final form (23) is the Hamiltonian of interacting phonons built from "heated" quasiparticles but the phonon system itself is not heated in the sense that there is no thermal smearing of phonons over their energy levels. This corresponds to a transparent phenomenological picture: when one heats a nucleus putting there a good piece of energy, a nucleonic motion is changed and due to this the properties of a nuclear surface are changed. As a consequence of the latter the properties of surface vibrations are changed. However, one cannot heat nuclear surface vibrations themselves.

In [14], the authors start just with the Hamiltonian of the interacting TRPA phonons implying, as an obvious fact, that the phonon system has the same temperature $T$ as the underlying fermions forming the thermal phonons. In our opinion this is an additional assumption which has to be justified. Similarly, in [13] from the beginning a nucleus is treated as a system of phonons and quasiparticles. But since phonons and quasiparticles are considered as some "initial" ingredients, the structure of phonons has to be as it is in a cold nucleus and cannot be changed by heating the system. Thus, they cannot satisfy the thermal RPA equation.

The point is that quasiparticles and phonons are not independent variables in a nucleus. The phonon is a coherent superposition of bifermionic excitations. So, starting with the model Hamiltonian given in terms of nucleonic degrees of freedom one has to make a mapping of pure fermionic states to a subspace consisting of ideal "quasiparticle" and "bosonic" elementary modes.

In this regard, Hatsuda [26] discussed already two ways to consider a hot nucleus. The first is to make a mapping of the initial Hamiltonian and the initial pure fermionic Fock space of a cold system (nucleus) and only after this to thermalize a system in question. For the approach presented here it means that degrees of freedom should be doubled for the quasiparticle-phonon image of the Hamiltonian (1)–(4) (see, e.g., [23, 24]). Then one gets the thermal Hamiltonian with both the types of thermal occupation numbers and, consequently, the GDR width also should depend on them. However, Hatsuda [26] has also shown taking the Lipkin model as an example that "thermalizing" of the bosonic image of the initial fermionic Hamiltonian one cannot derive in the leading order the TRPA equations for these bosons.

The second way is just the way of the present paper: while heating we treat a nucleus as a system of fermions and only after this we project or transform the original nucleonic degrees of freedom to more convenient ones (bosonic or bosonic + fermionic).

We would like to stress that the problem how to treat a thermalized nucleus in terms of quasiparticles and phonons is not so trivial as it may seem at the first glance. It is in intimate correspondence with a proper choice of physically important degrees of freedom and their consistent mapping which has to comply with the particle statistic requirements. Some aspects of the problem were discussed also in [32].

As concerns the effect of the thermal phonon occupation numbers on the $T$-dependence of $\Gamma^{\downarrow}$, it is not significant, we guess. At least this is not the crucial point for increasing $\Gamma^{\downarrow}$ with temperature.

## 5. Conclusions

A temperature dependence of the spreading GDR width has been studied within the Quasiparticle – Phonon Model extended to finite temperature within the TFD formalism. According to the results of numerical calculations, $\Gamma^{\downarrow}$ increases with $T$ in the temperature range $0 < T \leq 3$ MeV. In our opinion, this is the main result of the paper. A reasonable agreement of the theoretical value of $\Gamma^{\downarrow}$ with the experimental data is of less importance because the former is determined up to a factor of 1.5.

Our results agree qualitatively with those of [14, 15, 17, 18]. Moreover, the matrix element of a thermal phonon interaction $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ coincides with the corresponding vertex of [14]. It seems that in [11, 13] the influence of thermal p-p and h-h phonons was missed.

We also drew attention to the problem of a proper choice of relevant nuclear degrees of freedom to describe giant resonances in a hot nucleus. To our knowledge, this aspect of a giant resonance theory was overlooked before.

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Зависимость spreading-ширины гигантского дипольного резонанса от температуры

Зависимость spreading-ширины гигантского дипольного резонанса $\Gamma^{\downarrow}$ от температуры исследована в рамках квазичастично-фононной модели ядра (КФМ), обобщенной на ненулевые температуры с помощью формализма термополевой динамики (ТПД). Численные расчеты проведены для компаунд-ядер $^{120}$Sn и $^{208}$Pb. Величина $\Gamma^{\downarrow}$ быстро растет с ростом температуры. Обсуждаются причины этого эффекта. Подход КФМ–ТПД сравнивается с другими подходами, предложенными ранее.

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Temperature Dependence of Spreading Width of Giant Dipole Resonance

The Quasiparticle-Phonon Nuclear Model extended to finite temperature within the framework of Thermo Field Dynamics is applied to calculate a temperature dependence of the spreading width $\Gamma^{\downarrow}$ of a giant dipole resonance. Numerical calculations are made for $^{120}$Sn and $^{208}$Pb nuclei. It is found that $\Gamma^{\downarrow}$ increases with $T$. The reason of this effect is discussed as well as a relation of the present approach to other ones, existing in the literature.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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