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Abstract

Many major sports leagues are characterized by a combination of cross-subsidization mechanisms like revenue-sharing arrangements and payroll restrictions. Up to now, the effects of these policy tools have only been analyzed separately. This article provides a theoretical model of a team sports league and analyzes the combined effect of salary restrictions (caps and floors) and revenue sharing. It shows that the effect on club profits, player salaries, and competitive balance crucially depends on the mix of these policy tools. Moreover, the invariance proposition does not hold even under Walrasian-conjectures if revenue sharing is combined with either a salary cap or a salary floor.

Keywords: Team sports leagues, invariance proposition, competitive balance, revenue sharing, salary cap, salary floor

JEL classification: C72; L11; L83

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1 Introduction

Invariance principles are the golden eggs of economics. Franco Modigliani, Merton Miller, and Ronald Coase were awarded Nobel prizes for their formulations of important invariance principles. A predecessor of the famous Coase theorem is Rottenberg's invariance proposition. According to Rottenberg (1956), the distribution of playing talent between clubs in professional sports leagues does not depend on the allocation of property rights to players' services. El-Hodiri and Quirk (1971), Fort and Quirk (1995), and Vrooman (1995) extend this invariance proposition to gate revenue sharing. Based on their models, they claim that revenue sharing does not change the level of competitive balance within a league. This form of invariance proposition has become one of the most heavily disputed issues in sports economics because its centerpieces, revenue sharing and the uncertainty of outcome hypothesis, represent two of the most important idiosyncrasies in the professional team sports industry.

According to the uncertainty of outcome hypothesis, fans prefer to attend games with uncertain outcomes and enjoy close championship races. Unlike Toyota, which benefits from weak competitors in the automobile industry, Real Madrid and the New York Yankees need strong competitors to maximize their revenues. In sports, a weak team produces a negative externality on its stronger competitors. Revenue-sharing arrangements have been introduced as a measure to improve the competitive balance by (partially) internalizing this externality. If the invariance proposition held, revenue sharing would be worthless.

Current revenue-sharing schemes vary widely among professional sports leagues all over the world. In the United States, the most prominent is possibly that operated by the National Football League (NFL), where the visiting club secures 40% of the locally earned television and gate receipt revenue. In 1876, Major League Baseball (MLB) introduced a 50-50 split of gate receipts that was reduced over time. Since 2003, all the clubs in the American League have put 34% of their locally generated revenue (gate, concession, television, etc.) into a central pool, which is then divided equally among all the clubs. The National Basketball Association (NBA) and the National Hockey League (NHL) also operate with a pool-sharing arrangement. Moreover, in the Australian Football League (AFL), gate receipts were at one time split evenly between the home and the visiting team. This 50-50 split was finally abolished in 2000.

Other measures to increase competitive balance are salary caps and floors. A salary cap (floor) puts an upper (lower) bound on a club's payroll. Since most leagues compute their salary caps and floors on the basis of the revenues of the preceding season, caps and floors can be treated as fixed limits.

The NBA was the first league to introduce a salary cap for the 1984-1985 season. For the 2008-2009 season the (soft) salary cap is fixed at US$ 58.7 million. Today, salary caps are in effect in professional team sports leagues all over the world. In the NHL, for example, each team had to spend between US$ 34.3 million and 50.3 million on player salaries in the 2007-08 season. In the NFL, the salary cap in 2009 is approximately US$ 128 million per team, whereas the salary floor was 87.6% of the salary cap, which is equivalent to US$ 112.1 million. The AFL also operates with a combined salary cap and floor: for 2009, the salary cap was fixed at A$ 7.69 million, the floor at 7.12 million. Another Australian league, the National Rugby League (NRL), has implemented a salary cap and floor system which forced each team to spend between A$ 3.96 million and 4.4 million in 2009. In Europe, salary caps are in effect in the Guiness Premiership in rugby union and the Super League in rugby league.[1]

In most industries, payroll caps would be regarded as an exploitation of market power and would be prohibited by anti-trust authorities. In professional team sports, however, salary cap (and floor) arrangements are usually granted anti-trust exemption whenever they are the result of collective bargaining agreements between representatives of club owners and players.

In the sports economic literature, the invariance proposition with regard to revenue sharing has been derived under two major assumptions: First, club owners are modeled as profit maximizers (rather than win maximizers). Second, talent supply is regarded as fixed. There is wide agreement that the invariance proposition does not hold in leagues with either win-maximizing owners or flexible talent supply (Atkinson et al. 1988; Falconieri et al., 2004; Késenne 2000b, 2005, 2007; Szymanski, 2003). There is disagreement, however, over whether the invariance proposition holds in a league with profit-maximizing owners and a fixed talent supply. The models of El-Hodiri and Quirk (1971), Fort and Quirk (1995), and Vrooman (1995) show that the invariance proposition does hold with respect to revenue sharing, whereas the model of Szymanski and Késenne (2004) con-

[1]The data in the paragraph is taken from the collective bargaining agreements of the respective leagues.

cludes that gate revenue sharing results in a more uneven distribution of talent between large- and small-market clubs and therefore contradicts the invariance proposition. Since all of these models use the same assumptions, namely, a fixed supply of talent and profit-maximizing club owners, the contradiction results from methodological differences. El-Hodiri and Quirk, Fort and Quirk, and Vrooman use "Walrasian conjectures," whereas Szymanski and Késenne employ "Nash conjectures."

This paper contributes to the literature in three dimensions: (i) Our article is the first to analyze the joint effect of salary restrictions and revenue sharing on club profits, player salaries, and competitive balance. The existing literature analyzes the effects of revenue-sharing arrangements (e.g., Marburger, 1997; Késenne, 2000a; Dietl and Lang, 2008) and payroll restrictions (e.g., Késenne, 2000b; Vrooman, 2008; Dietl et al., 2009) separately despite the fact that revenue sharing arrangements and salary restrictions are used simultaneously in many leagues such as the NHL, NFL and NBA. (ii) We show that the invariance proposition does not hold even in a standard "Fort and Quirk" style (FQ-style) model if one considers the combined effect of salary restrictions (cap and floor) and revenue-sharing agreements.[2] (iii) This article is the first to provide a theoretical analysis of salary floors in a sports league.

Our analysis shows that in leagues with a binding salary cap for large clubs but no binding salary floor for small clubs, revenue sharing will decrease the competitive balance and increase the profits of the small clubs as well as aggregate profits. The effect on the profits of the large clubs is ambiguous. In this case, a salary cap also results in a more balanced league and decreases the cost per unit of talent. The effect of a stricter salary cap on the profits of small clubs is positive, whereas the effects on the profits of the large clubs and on aggregate profits are ambiguous.

Moreover, in leagues with a binding salary floor for the small clubs but no binding salary cap for the large clubs, revenue sharing will increase the competitive balance. In addition, revenue sharing will decrease (increase) the profits of large (small) clubs. Implementation of a higher salary floor will produce a more balanced league, but will increase the cost per unit of talent. Furthermore, a salary floor will result in lower profits for all clubs.

Finally, our analysis shows that revenue sharing decreases the cost per unit of talent

[2]Note that there is wide agreement in the literature that the invariance proposition holds in a FQ-style model (El-Hodiri and Quirk, 1971; Fort and Quirk, 1995; Vrooman, 1995, 2000, 2007).

in all regimes except when either the salary cap or the salary floor is binding for all clubs.

The remainder of the article is organized as follows. In the next section we present our model setup with the main assumptions. In Subsection 2.1, we consider Regime A which represents the benchmark case without a (binding) salary cap/salary floor. In Subsection 2.2, we consider Regime B where the salary cap is only binding for the large-market club and the salary floor is not binding for the small-market club. In Subsection 2.3, we analyze Regime C where the salary floor is only binding for the small-market club and the salary cap is not binding for the large-market club. Subsection 2.4, represents Regime D where either the salary cap or the salary floor is binding for both clubs. Section 3 provides a discussion that addresses the issue of Walrasian vs. Nash conjectures. Finally, Section 4 concludes.

2 The Model

We model the investment behavior of two profit-maximizing clubs in a standard FQ-style league, i.e., a closed league with a fixed supply of talent. Each club $i = 1, 2$ invests independently in playing talent t_i in order to maximize its own profits. Our league features a pool revenue-sharing arrangement, and salary payments (payroll) are restricted by both a salary cap (upper limit) and a salary floor (lower limit).

The revenue of club i, R_i, depends on its market size, m_i, as well as its own win percentage, w_i, and competitive balance $w_i w_j$ in the league, where w_j denotes the win percentage of the other club j.[3] We assume that the revenue function has the following properties: $\exists\ w_i' \in [0, 1]$ such that if $w_i \geq w_i'$ then $\frac{\partial R_i}{\partial w_i} < 0$, otherwise $\frac{\partial R_i}{\partial w_i} > 0$, and $\frac{\partial^2 R_i}{\partial w_i^2} < 0$ everywhere.[4]

The win percentage w_i of club i is characterized by the contest-success function (CSF), which maps the vector (t_1, t_2) of talent onto probabilities for each club. We apply the logit approach, which is the most widely used functional form of a CSF in sporting contests.[5]

[3] For an analysis of competitive balance in the North American Major Leagues, see, e.g., Fort and Lee (2007).

[4] See Szymanski and Késenne (2004, p. 168). Note that the assumption of concavity for the revenue function, however, rules out important convexities that might exist in the real worlds, e.g., the non-linear incentives associated with playoffs or championships. We are grateful to an anonymous referee for this point.

[5] The logit CSF was generally introduced by Tullock (1980) and subsequently axiomatized by Skaperdas (1996) and Clark and Riis (1998). An alternative functional form would be the probit CSF (e.g., Lazear and Rosen, 1981; Dixit, 1987) and the difference-form CSF (e.g., Hirshleifer, 1989).

The win percentage of club $i = 1, 2$ is then given by

$$w_i(t_i, t_j) = \frac{t_i^\gamma}{t_i^\gamma + t_j^\gamma}, \tag{1}$$

with $i, j = 1, 2,\ i \neq j$. For the sake of tractability, we set the "discriminatory power" parameter γ in the following to one.[6] Given that the win percentages must sum up to one, we obtain the adding-up constraint: $w_j = 1 - w_i$. Since we consider a standard FQ-style model, we assume a fixed supply of talent given by $s > 0$ and adopt the so-called "Walrasian conjectures" $\frac{dt_i}{dt_j} = -1$. These conjectures indicate that clubs internalize that, due to the fixed amount of talent, a one-unit increase of talent hired at one club implies a one-unit reduction of talent at the other club.[7] We compute the derivative of (1) as

$$\frac{\partial w_i}{\partial t_i} = \frac{t_i + t_j - t_i(1 + \frac{\partial t_j}{\partial t_i})}{(t_i + t_j)^2} = \frac{1}{t_i + t_j},$$

with $i, j = 1, 2,\ i \neq j$. Note that competitive balance $w_i w_j$ attains its maximum of $1/4$ for a completely balanced league in which both clubs invest the same amount in talent such that $w_i = w_j = 1/2$. A less balanced league is then characterized by a lower value than $1/4$.

Next, we specify the revenue function for club i as follows:[8]

$$R_i = m_i \left[\beta w_i + w_i w_j\right] = m_i \left[(\beta + 1) w_i - w_i^2\right]. \tag{2}$$

The parameter $\beta > 0$ represents the weights fans put on own team winning relative to competitive balance. Note that club i's revenues initially increase with winning until the maximum is reached for w_i' with $w_i' \equiv \frac{\beta+1}{2}$. By increasing the win percentage above w_i', club i's revenues start to decrease because excessive dominance by one team is detrimental to the competition. This reflects the uncertainty of outcome hypothesis: the lower the

[6]See Dietl et al. (2008) and Fort and Winfree (2009) for a more detailed analysis of the role of the discriminatory power parameter.

[7]Note that in a league with a fixed supply of talent it is standard to apply Walrasian conjectures (El-Hodiri and Quirk, 1971, Fort and Quirk, 1995, and Vrooman 1995, 2007, 2008), whereas Szymanski (2004) proposes to use the "Nash conjectures" $dt_i/dt_j = 0$. In a first step, we follow the standard approach and apply Walrasian conjectures. See Section 3 for a discussion that addresses the issue of Walrasian vs. Nash conjectures.

[8]This specification of the revenue function satisfies the properties from above and is widely used in the sports economic literature: see, e.g., Hoehn and Szymanski (1999), Szymanski (2003), Szymanski and Késenne (2004), Késenne (2007) and Vrooman (2007, 2008).

value of β, i.e., the higher the fans' preference for competitive balance, the lower the threshold value and the sooner revenues start to decrease due to dominance by one team. Since the qualitative results do not depend on β, we set $\beta \equiv 1$ in the subsequent analysis for simplicity.

Moreover, without loss of generality, we assume that club 1 is the large market club with a higher drawing potential than the small market club 2 such that $m_1 > m_2$. For notational simplicity and without loss of generality, we normalize m_2 to unity and write m instead of m_1 with $m > 1$.

We introduce revenue sharing in our league and assume that club revenues are shared according to a pool-sharing agreement. In a simplified pool-sharing agreement, each club contributes a certain percentage $(1-\alpha)$ of its pre-shared revenues in a pool that is managed by the league and equally distributed among the clubs.[9] In its simplest version, the post-sharing revenues of club i can be written as

$$\widehat{R}_i = \alpha R_i + \frac{(1-\alpha)}{2}(R_i + R_j),$$

with $\alpha \in (0,1]$ and $i,j = 1,2$, $i \neq j$. The limiting case of $\alpha = 1$ describes a league without revenue sharing, whereas $\alpha = 0$ describes a league with full revenue sharing. Marginal post-sharing revenues are derived as

$$\frac{\partial \widehat{R}_i}{\partial t_i} = \alpha \frac{\partial R_i}{\partial w_i}\frac{\partial w_i}{\partial t_i} + \frac{(1-\alpha)}{2}\left[\frac{\partial R_i}{\partial w_i}\frac{\partial w_i}{\partial t_i} + \frac{\partial R_j}{\partial w_j}\frac{\partial w_j}{\partial t_i}\right] = \left[\alpha \frac{\partial R_i}{\partial w_i} + \frac{(1-\alpha)}{2}\left(\frac{\partial R_i}{\partial w_i} - \frac{\partial R_j}{\partial w_j}\right)\right]\frac{\partial w_i}{\partial t_i}$$

with $\frac{\partial w_i}{\partial t_i} = -\frac{\partial w_j}{\partial t_i}$ and $i,j = 1,2$, $i \neq j$.

Moreover, as is standard in the literature, we assume constant marginal costs c of talent such that the salary payments (payroll) of club i, denoted by x_i, are given by $x_i = c \cdot t_i$.[10]

The profit function of club $i = 1,2$ is then given by post-sharing revenues minus salary

[9] Note that the results are robust also for a gate revenue-sharing agreement where club i obtains share α of its own revenues R_i and from the away match share $(1-\alpha)$ of club j's revenues R_j. In this case, the after-sharing revenues of club i are given by $\widehat{R}_i = \alpha R_i + (1-\alpha)R_j$ (for an analysis, see, e.g., Dietl and Lang, 2008).

[10] For the sake of simplicity, we do not take into account non-labor costs and normalize the fixed capital cost to zero. See Vrooman (1995) for a more general cost function where clubs have different marginal costs or Késenne (2007) for a cost function with a fixed capital cost. Idson and Kahane (2000) analyze the effect of team attributes on player salaries.

payments

$$\pi_i(t_i, t_j) = \widehat{R}_i(t_i, t_j) - c \cdot t_i,$$

with $i, j = 1, 2$, $i \neq j$.

As mentioned above, we introduce both an upper limit (salary cap) and a lower limit (salary floor) for each club's payroll. The sizes of the salary cap and salary floor, which are the same for each club, are based on total league revenues in the previous season, divided by the number of clubs in the league. We therefore assume that the salary cap and the salary floor are exogenously given in the current season as it is the case, e.g., in the NHL and NFL.[11]

Each club invests independently in playing talent such that its own profits are maximized subject to the salary cap and salary floor constraints. That is, salary payments $x_i = c \cdot t_i$ must be at least as high as $floor > 0$, given by the salary floor, but must not exceed $cap > 0$, given by the salary cap. The maximization problem for club $i = 1, 2$ is given by

$$\max_{t_i \geq 0} \left\{ \alpha R_i(t_i, t_j) + \frac{(1-\alpha)}{2}(R_i(t_i, t_j) + R_j(t_i, t_j)) - c \cdot t_i \right\}$$

$$\text{subject to } floor \leq c \cdot t_i \leq cap.$$

The corresponding first-order conditions are derived as[12]

$$\begin{aligned} \frac{\partial \widehat{R}_i}{\partial t_i} - c - \lambda_{i1} c + \lambda_{i2} c \leq 0, \quad cap - ct_i \geq 0, \quad ct_i - floor \geq 0, \\ t_i \left(\frac{\partial \widehat{R}_i}{\partial t_i} - c - \lambda_{i1} c + \lambda_{i2} c \right) = 0, \quad \lambda_{i1}(cap - ct_i) = 0, \quad \lambda_{i2}(ct_i - floor) = 0, \end{aligned} \tag{3}$$

where $\lambda_{ij} \geq 0$ are Lagrange multipliers. The equilibrium in talent (t_1^*, t_2^*) is characterized by (3) and the market-clearing condition $t_1^* + t_2^* = s$ due to the fixed supply of talent.

We must distinguish different regimes depending on whether the salary cap and/or salary floor is binding or not.

[11] See, e.g., Késenne (2000*b*) and Dietl, Lang and Rathke (2009).

[12] It can easily be verified that the second-order conditions for a maximum are satisfied.

2.1 Regime A: neither salary cap nor salary floor is binding

In this section, we assume that the salary cap and salary floor are ineffective for both clubs; i.e., we consider the benchmark case that no (binding) salary cap/floor exists. In Regime A, the equilibrium allocation of talent and the cost per unit of talent are computed from (3) as[13]

$$\begin{aligned} \left(t_1^A, t_2^A\right) &= \left(\frac{m}{m+1}s, \frac{1}{m+1}s\right) = \left(w_1^A s, w_2^A s\right), \\ c^A &= \frac{2\alpha m}{s(m+1)}. \end{aligned} \tag{4}$$

We derive that the large club demands more talent in equilibrium than does the small club, because the marginal revenue of talent is higher for the large club. Furthermore, the equilibrium win percentages in Regime A, given by $(w_1^A, w_2^A) = \left(\frac{m}{m+1}, \frac{1}{m+1}\right)$, also maximize aggregate club revenues $\widehat{R}_1 + \widehat{R}_2 = m\left(2w_1 - w_1^2\right) + \left(2w_2 - w_2^2\right)$.[14] The equilibrium salary payments in Regime A, denoted $\left(x_1^A, x_2^A\right)$, are computed as

$$\left(x_1^A, x_2^A\right) = \left(\frac{2\alpha m^2}{(m+1)^2}, \frac{2\alpha m}{(m+1)^2}\right).$$

Thus, we are in Regime A if $floor < x_2^A$ and $cap > x_1^A$.

In the following proposition, we summarize the effect of changing the revenue-sharing parameter α in Regime A:

Proposition 1

(i) The invariance proposition holds in Regime A: more revenue sharing has no effect on the distribution of talent.

(ii) More revenue sharing decreases the cost per unit of talent in Regime A.

(iii) In Regime A, more revenue sharing increases the profits of the small club and aggregate club profits. The profits of the large club only increase if the difference between both clubs in terms of market size is not too large, i.e., if $m < m' \approx 2.83$.

[13] Note that the demand for talent before calculating the equilibrium cost per talent is given by $\left(t_1^A(c), t_2^A(c)\right) = \left(\frac{m(1+\alpha)s - cs^2}{m(1+\alpha)+(1-\alpha)}, \frac{(1+\alpha)s - cs^2}{(1+\alpha)+m(1-\alpha)}\right)$.

[14] Note that for $w_2 = (1 - w_1)$, aggregate club revenue is computed as $\widehat{R}_1 + \widehat{R}_2 = 1 + 2mw_1 - (1 + m)w_1^2$ with the first-order condition given by $2m - 2(1+m)w_1 = 0$. Thus, the revenue-maximizing win percentages are $(w_1^*, w_2^*) = (m/(m+1), 1/(m+1))$. For a comparison of the noncooperative outcome and the socially optimal outcome, see, e.g., Cyrenne (2001), Whitney (2005) and Dietl, Lang and Werner (2009).

Proof. See Appendix A.1. ■

In accordance with the literature, we derive that the well-known "invariance proposition" with respect to revenue sharing holds in our FQ-style model when neither the salary cap nor the salary floor is binding.[15] That is, revenue sharing has no effect on the win percentages and thus does not change the league's competitive balance in Regime A.

To illustrate this result, Figure 1 depicts the downward-sloping marginal post-sharing revenue curves as functions of the win percentages for the two clubs. The two topmost lines indicate the case of no revenue sharing, i.e., $\alpha = 1$. When revenues start to be shared, the marginal revenue curves shift down for both clubs. Instead of receiving all the additional revenues from an extra unit of talent, the clubs receive only $(1+\alpha)/2$ of the additional revenues. This results in a downward shift of both marginal revenue curves, where the shift is more pronounced for the large club.

Moreover, increasing the win percentage of club i is tantamount to reducing the win percentage of club j. As a result, club j's contribution to the shared pool is shrinking. It follows that club i loses $(1-\alpha)/2$ of club j's reduced revenues when increasing its win percentage. Note that the contribution to the pool increases with the degree of revenue sharing. Since the large club's contribution to the pool is always greater than the small club's contribution, it follows that the small club loses more through a higher degree of revenue sharing. As a consequence, more revenue sharing implies that marginal revenues are decreasing faster for the small club, whereas the marginal revenue curve of the large club is getting flatter. Overall, even though the intercept of the large club shifts down more than the intercept of the small club, the two curves still intersect at the same pair of win percentages (w_1^A, w_2^A) for all values of α because the changing slopes offset the change of the intercepts.

Moreover, the proposition shows that a higher degree of revenue sharing, i.e., a lower value of α, lowers the equilibrium cost per unit of talent. As argued above, marginal revenues decrease for both clubs and with it talent demand $t_i^A(c)$. Hence, the market-clearing cost per unit of talent c^A set by the "Walrasian auctioneer" also has to be lower.

Even though revenue sharing leaves the distribution of talent unchanged and therefore also the pre-shared revenues of both clubs, it has implications for club profits. A higher degree of revenue sharing will increase the profits of the small club in Regime A, because

[15] See, e.g., El-Hodiri and Quirk (1971), Fort and Quirk (1995), and Vrooman (1995).

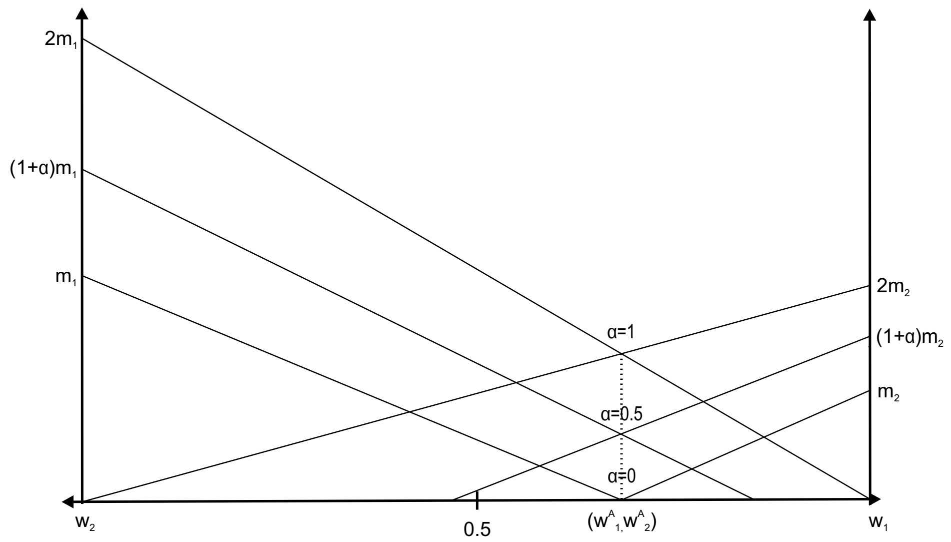

Figure 1: Effect of revenue sharing on marginal revenues

revenue sharing lowers the cost per unit of talent and redistributes some of the money to the small club. As a result, the small club's post-sharing revenues $\widehat{R}_2$ and profits increase through revenue sharing.

Despite the fact that salary payments x_i^A will decrease for both clubs, revenue sharing decreases the profits of the large club if the difference between both clubs in terms of market size is too large, i.e., $\frac{\partial \pi_1^A}{\partial \alpha} > 0 \Leftrightarrow m > m' \approx \grave{O}2.83$. Note that the large club's post-sharing revenues $\widehat{R}_1$ decline as a result of the redistribution to the small club. If the market size is greater than m', the lower costs cannot compensate for the lower revenues.[16]

On aggregate, however, club profits increase because aggregate revenues $R_1^A + R_2^A$ are independent of α and thus remain constant but costs decline through revenue sharing. Due to the contest structure, the maximum level of aggregate club profits would be attained in a league with full revenue sharing, i.e., for $\alpha = 0$, because in this case both clubs would fully internalize the externality they impose on the other club when hiring an additional unit of talent.[17]

[16]Note that the large club's salary payments x_1^A are an increasing function in the market size m.

[17]However, we assume that players have a certain reservation wage $c^w > 0$ such that $\alpha = 0$ is not a feasible solution.

2.2 Regime B: salary cap is binding for large club, but salary floor is not binding for small club

In this section, we assume that the salary cap is only binding for the large-market club and that the salary floor is not binding for the small-market club. In Regime B, the equilibrium allocation of talent and the cost per unit of talent are computed as[18]

$$\begin{aligned} \left(t_1^B, t_2^B\right) &= \left(\frac{2cap}{(\alpha-1)m+\phi^B}s, \left(1-\frac{2cap}{(\alpha-1)m+\phi^B}\right)s\right) = \left(w_1^B s, w_2^B s\right), \\ c^B &= \frac{(\alpha-1)m+\phi^B}{2s}, \end{aligned} \tag{5}$$

with $\phi^B \equiv \sqrt{(\alpha-1)^2m^2+4cap(1+\alpha+m(1-\alpha))}$.[19] The equilibrium salary payments in Regime B are computed as

$$\left(x_1^B, x_2^B\right) = \left(cap, \frac{1}{2}((\alpha-1)m+\phi^B-2cap)\right).$$

Thus, we are in Regime B if $cap \in \left(\underline{cap}, \overline{cap}\right) = \left(\frac{1+\alpha-m(1-\alpha)}{4}, x_1^A\right)$ with a sufficiently low salary floor.[20] The condition for cap guarantees that the salary cap is only binding for the large club.[21] Moreover, the condition $cap \in \left(\underline{cap}, \overline{cap}\right)$ implicitly defines the interval of feasible revenue-sharing parameters α for Regime B with $\alpha \in (\underline{\alpha}^B, \overline{\alpha}^B) = \left(\frac{cap(1+m)^2}{2m^2}, \frac{4cap+m-1}{1+m}\right)$.[22]

2.2.1 The effect of a salary cap in Regime B

In this subsection, we analyze the effect of changing the salary cap parameter given that the league has set a certain degree α' of revenue sharing. We derive the following results:

[18]The demand for talent before calculating the equilibrium cost per talent is given by $\left(t_1^B(c), t_2^B(c)\right) = \left(\frac{cap}{c}, \frac{(1+\alpha)s-cs^2}{(1+\alpha)+m(1-\alpha)}\right)$.

[19]Note that $(\alpha-1)^2m^2+4cap(1+\alpha+m(1-\alpha))>0$ since $cap \in \left(\underline{cap}, \overline{cap}\right)$.

[20]Note that $\underline{cap}$ is less than zero if the difference between both clubs is too big: i.e., $\underline{cap}<0 \Leftrightarrow m > \frac{1+\alpha}{1-\alpha}$.

[21]If $cap > \overline{cap}$, then the salary cap is not binding for any club and we are in Regime A, while if $cap < \underline{cap}$, then the salary cap is binding for both clubs and we are in Regime D.

[22]Suppose that the league has set a certain $cap' \in \left(\underline{cap}, \overline{cap}\right)$. Decreasing (increasing) the revenue-sharing parameter α induces both $\underline{cap}$ and $\overline{cap}$ to decrease (increase). If α decreases below $\underline{\alpha}$, then $cap' > \overline{cap} = x_1^A$, and we would be in Regime A because the cap would not be binding anymore. If α increases above $\overline{\alpha}^B$, then $cap' < \underline{cap}$ and we would be in Regime D.

Proposition 2

In Regime B, a more restrictive salary cap increases competitive balance and decreases the cost per unit of talent.

Proof. See Appendix A.2. ■

The salary cap forces the large club to cut back on expenses, lowering the overall demand for talent, and thus the market-clearing cost per unit of talent c^B set by the Walrasian auctioneer is lower. As a consequence, the small club will hire a greater amount of talent.

Hence, a more restrictive salary cap (i.e., a lower value of *cap*) induces a reallocation of talent from the large to the small club. That is, the large club decreases its level of talent by the same amount by which the small club increases its level of talent, i.e., $0 < \frac{\partial t_1^B}{\partial cap} = -\frac{\partial t_2^B}{\partial cap} > 0$. As a consequence, a more restrictive salary cap increases the win percentage w_2^B of the small club and decreases the win percentage w_1^B of the large club in Regime B. Since the large club is the dominant team, competitive balance increases and thus a salary cap produces a more balanced league. It follows that the pre-shared revenues R_1^B of the large club decrease and that the pre-shared revenues R_2^B of the small club increase through a more restrictive salary cap. Aggregate club revenues $R_1^B + R_2^B$, however, will decline because the league departs from the revenue-maximizing win percentages (w_1^A, w_2^A). Thus, the post-sharing revenues $\widehat{R}_1$ of the large club decline, and the post-sharing revenues $\widehat{R}_2$ of the small club increases (see also Figure 1).

The second part of the proposition states that the cost per unit of talent will be lower in equilibrium through the introduction of a salary cap, i.e., $\frac{\partial c^B}{\partial cap} > 0$. It is therefore clear that a more restrictive salary cap helps the large club to control costs, because the large club decreases its salary payments, i.e., $\frac{\partial x_1^B}{\partial cap} > 0$. But will a salary cap also help the small club to lower costs? We derive that the effect of a more restrictive salary club on the small club's salary payments is ambiguous because

$$\frac{\partial x_2^B}{\partial cap} = \left(\frac{(1+\alpha+m(1-\alpha))}{\phi^B} - 1 \right) \begin{cases} > 0 \text{ if } cap \in (\underline{cap}, \widetilde{cap}), \\ = 0 \text{ if } cap = \widetilde{cap}, \\ < 0 \text{ if } cap \in (\widetilde{cap}, \overline{cap}), \end{cases}$$

with $\widetilde{cap} = \frac{1+2m+a(2+a(1-2m))}{4(1+a+m(1-a))}$.[23] That is, if the salary cap is not too restrictive, i.e., $cap \in (\widetilde{cap}, \overline{cap})$, the increase in the level of talent offsets for the decrease in the cost per unit of talent such that salary payments x_2^B of the small club increase. If, however, the salary cap is relatively restrictive, i.e., $cap \in (\underline{cap}, \widetilde{cap})$, the decrease in the cost per unit of talent outweighs the increase in the level of talent, and salary payments x_2^B decrease. Moreover, we derive that a salary cap always decreases aggregate salary payments, i.e., $\frac{\partial(x_1^B+x_2^B)}{\partial cap} > 0$.[24] That is, the increase in the small club's salary payments never offsets the decrease in the large club's salary payments.

In the next proposition, we analyze how changes in the salary cap affect club profits:

Proposition 3

In Regime B, a more restrictive salary cap increases the profits of the large club and aggregate club profits until the maximum is reached for $cap = cap^*$ *and* $cap = cap^{**}$*, respectively, whereas the profits of the small club will always increase through a more restrictive salary cap.*

Proof. See Appendix A.3. ■

Figure 2 illustrates the proposition's results. A more restrictive salary cap increases aggregate club profits π^B until the maximum is reached for $cap = cap^{**}$. Intuitively, a salary cap has two effects on club profits. On the one hand, a more restrictive salary cap lowers aggregate club revenues because the league departs from the revenue-maximizing win percentages from Regime A. On the other hand, it lowers the cost per unit of talent. Suppose that the league has set a relatively loose salary cap. By implementing a more restrictive salary cap, the marginal (positive) effect of lower aggregate club costs $x_1^B + x_2^B$ outweighs the marginal (negative) effect of lower aggregate club revenues $R_1^B + R_2^B$ such that aggregate club profits increase. Both effects balance each other out for $cap = cap^{**}$. By implementing a more restrictive salary cap than cap^{**}, the lower club costs cannot compensate for the lower aggregate club revenues, and therefore aggregate club profits will decrease.[25]

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[23]Note that depending on the parameters (α, m), the threshold $\widetilde{cap}$ can be bigger than $\overline{cap}$. In this case, the salary payments of the small club always decrease through a tighter salary cap.

[24]To see this note that $x_1^B + x_2^B = c^B(t_1^B + t_2^B) = c^B s$ and $\frac{\partial c^B}{\partial cap} > 0$.

[25]Note the equilibrium cost per talent $c^B(cap)$ is a convex function in cap, i.e., $\frac{\partial^2 c^B(cap)}{\partial cap^2} > 0$. Thus, tightening the salary cap for high values of cap decreases the aggregate salary payments more than for low values of cap.

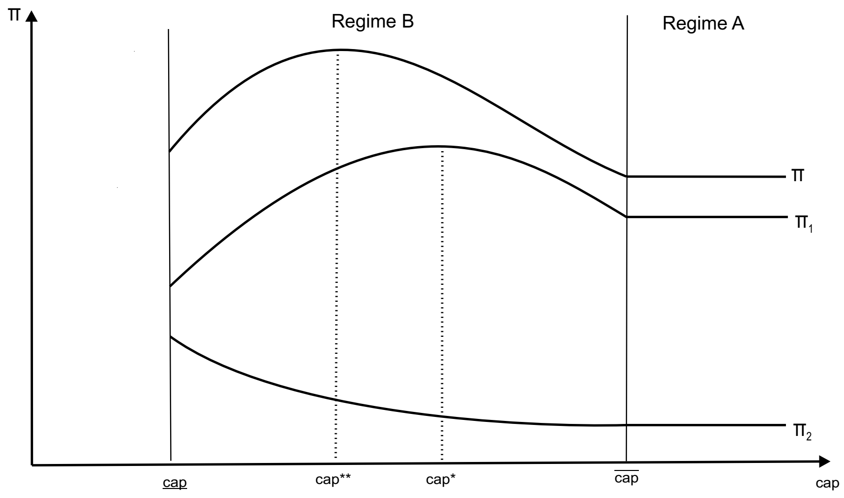

Figure 2: Effect of a salary cap on club profits

For a relatively loose salary cap, the profits of both clubs will increase through the introduction of a salary cap. The small club, however, will always benefit, independent of the size of the salary cap, whereas the large club has an interest in the salary cap not being too restrictive. Formally, a more restrictive salary cap increases the profits of the large club π_1^B until the maximum is reached for $cap = cap^*$. The intuition is as follows. Remember that a more restrictive salary cap will increase (decrease) the small (large) club's post-sharing revenues. For the small club, even in the case that a more restrictive salary cap increases the club's costs (i.e., for $cap \in (\widetilde{cap}, \overline{cap})$), the higher revenues offset for the higher costs and the profits of the small club will increase. For the large club the reasoning is similar to that for aggregate profits above. The lower costs can only outweigh the lower club revenues if the salary cap is not set to be too restrictive, i.e., if $cap > cap^*$. Otherwise, the profits of the large club will decrease through a more restrictive salary cap and can even be lower than in Regime A.

Moreover, note that the salary cap that maximizes the profits of the large club is less restrictive than the salary cap that maximizes aggregate club profits, i.e., $cap^* > cap^{**}$. If $cap < cap^*$, the profits of the large club already start to decrease, but the additional profits of the small club exceed the losses of the large club, and aggregate profits thus still increase until $cap = cap^{**}$.

2.2.2 The effect of revenue sharing in Regime B

In this subsection, we analyze the effect of changing the revenue-sharing parameter α in Regime B, given that the league has set a certain $cap' \in \left(\underline{cap}, \overline{cap}\right)$.

The effect of revenue sharing on the allocation of talent and the cost per unit of talent is derived in the following proposition:

Proposition 4

(i) The invariance proposition does not hold in Regime B: more revenue sharing decreases competitive balance.

(ii) More revenue sharing decreases the cost per unit of talent in Regime B.

Proof. See Appendix A.4. ∎

The proposition shows that the invariance proposition with respect to revenue sharing does not hold when a revenue-sharing arrangement is combined with a (binding) salary cap. A higher degree of revenue sharing (i.e., a lower value of α) induces a reallocation of talent from the small to the large club.

According to Section 2.1, more revenue sharing inevitably decreases marginal revenue as it dilutes investment incentives. Thus, while the large team is constrained, the talent demand by the small club and thereby also overall talent demand decreases. It follows that the market-clearing cost c^B per unit of talent has to decrease in order to clear the labor market. Since the salary payments of the large club are bound by the salary cap, the equilibrium amount of talent hired by the large club, $t_1^B = cap/c^B$, increases as the unit price of talent decreases. In equilibrium, the small club's level of talent decreases by the same amount by which the large club's level of talent increases, i.e., $0 > \frac{\partial t_1^B}{\partial \alpha} = -\frac{\partial t_2^B}{\partial \alpha} < 0$. As a consequence, revenue sharing increases the win percentage w_1^B of the large club and decreases the win percentage w_2^B of the small club, producing a more unbalanced league. Further note that the salary payments of the small club and aggregate salary payments in the league decrease.

Moreover, the pre-shared revenues R_1^B (R_2^B) of the large (small) club increase (decrease) through a higher degree of revenue sharing. In the aggregate, club revenues $R_1^B + R_2^B$ in Regime B will increase through more revenue sharing because the league approaches the revenue-maximizing win percentages (w_1^A, w_2^A). Thus, revenue sharing counteracts the salary cap's positive effect on competitive balance in the league.[26]

The effect of revenue sharing on club profits is analyzed in the following proposition:

Proposition 5

In Regime B, the introduction of revenue sharing increases the profits of both clubs and thus also aggregate club profits.

Proof. See Appendix A.5. ■

The proposition shows that both the small and the large club benefit from the introduction of a revenue-sharing arrangement in Regime B. On the one hand, revenue sharing increases aggregate club revenues $R_1^B + R_2^B$ and the large club's pre-shared revenues R_1^B, but it decreases the small club's pre-shared revenues R_2^B. On the other hand, revenue sharing decreases the costs of the small club due to its lower salary payments but does not change the costs of the large club because this club's salary payments are bound by the salary cap. When revenues start to be shared, the large club's profits increase due to its higher revenues. If, however, the degree of revenue sharing is getting too high, then profits of the large club might decrease again due to its lower post-shared revenues $\widehat{R}_1^B$. Even though the pre-shared revenues R_2^B of the small club decrease, this club always benefits from the introduction of revenue sharing due to its lower costs and higher post-shared revenues $\widehat{R}_2^B$. Finally, aggregate club profits always increase through a higher degree of revenue sharing because aggregate revenues increase and costs decrease.

What would happen if in addition to a binding salary cap (for the large club), a binding salary floor (for the small club) was also introduced? The salary floor would have an effect opposite to that of the salary cap. The salary floor would artificially boost the demand of the small club. This would increase the cost per unit of talent and reallocate talent from the large to the small club. Aggregate revenues would deteriorate as the distribution of win percentages would move further away from the optimal allocation. As a consequence, profits of the large club would shrink as revenues decrease and costs rise. For the small club, a binding salary floor would also have a negative effect on profits. Since the small-market club maximizes profits, marginal revenue equals marginal cost in

[26] See also Vrooman (2007, 2008).

equilibrium in Regime B. Forcing the small club in this situation to increase its salary payments implies lower profits.[27]

2.3 Regime C: salary cap is not binding for large club, but salary floor is binding for small club

In this section, we assume that the salary floor is only binding for the small-market club and the salary cap is not binding for the large-market club. In Regime C, the equilibrium allocation of talent and the cost per unit of talent are computed as[28]

$$\begin{aligned}\left(t_1^C, t_2^C\right) &= \left(\left(1 - \frac{2floor}{(\alpha-1)+\phi^C}\right)s, \frac{2floor}{(\alpha-1)+\phi^C}s\right) = (w_1^C s, w_2^C s),\\ c^C &= \frac{(\alpha-1)+\phi^C}{2s},\end{aligned} \tag{6}$$

with $\phi^C \equiv \sqrt{(\alpha-1)^2 + 4floor(1-\alpha+m(1+\alpha))}$.[29] The equilibrium salary payments are computed as

$$\left(x_1^C, x_2^C\right) = \left(\frac{1}{2}((\alpha-1)+\phi^C - 2floor), floor\right).$$

Thus, we are in Regime C if $floor \in \left(\underline{floor}, \overline{floor}\right) = \left(x_2^A, \frac{\alpha-1+m(1+\alpha)}{4}\right)$ with a sufficiently loose salary cap. The condition for $floor$ guarantees that the salary floor is only binding for the small club. Moreover, the condition $floor \in \left(\underline{floor}, \overline{floor}\right)$ implicitly defines the interval of feasible revenue-sharing parameters α for Regime C with $\alpha \in (\underline{\alpha}^C, \overline{\alpha}^C) = \left(\frac{1+4floor-m}{1+m}, \frac{floor(1+m)^2}{2m}\right)$.

2.3.1 The effect of a salary floor in Regime C

In this subsection, we analyze the effect of changing the salary floor parameter given that the league has set a certain degree α'' of revenue sharing. We derive the following results:

Proposition 6

In Regime C, a more restrictive salary floor increases both competitive balance and the

[27] We are grateful to an anonymous referee for this point.

[28] Note that the demand for talent before calculating the equilibrium cost per talent is given by $\left(t_1^C(c), t_2^C(c)\right) = \left(\frac{m(1+\alpha)s-cs^2}{m(1+\alpha)+(1-\alpha)}, \frac{floor}{c}\right)$.

[29] Note that $(\alpha-1)^2 + 4floor(1-\alpha+m(1+\alpha)) > 0$ since $floor \in \left(\underline{floor}, \overline{floor}\right)$.

cost per unit of talent.

Proof. See Appendix A.6. ■

The reasoning for this result is similar to that for Regime B. The salary floor forces the small club to enhance expenses thereby raising the overall demand for talent and thus the market clearing cost per unit of talent. Despite this, the small club hires a larger amount of talent.

Hence, implementing a more restrictive salary floor induces a reallocation of talent from the large club to the small club, i.e., $0 < -\frac{\partial t_1^C}{\partial floor} = \frac{\partial t_2^C}{\partial floor} > 0$.[30] A higher value of $floor$ decreases the win percentage w_1^C of the large club and increases the win percentage w_2^C of the small club. As a result, competitive balance increases in Regime C. Moreover, the large club's pre-shared revenues R_1^C will decrease, and the small club's pre-shared revenues R_2^C will increase. Aggregate club revenues $R_1^C + R_2^C$, however, will decrease because the league departs from the revenue-maximizing win percentages from Regime A.

Moreover, a more restrictive salary floor will increase the salary payments for both clubs in equilibrium, i.e., $\frac{\partial x_i^C}{\partial floor} > 0$, $i = 1, 2$. This is obvious for the small club, as price and the level of talent increase. For the large club, the decrease in the level of talent cannot compensate for the increase in cost per unit of talent. As a result, salary payments will also increase for the large club.

The effect of a salary floor on club profits is stated in the following proposition:

Proposition 7

In Regime C, a more restrictive salary floor decreases the profits of both clubs and thus also aggregate club profits.

Proof. See Appendix A.7. ■

It is clear that the profits of the large club will decrease because this club's revenues decrease and its costs increase in Regime C. However, the effect of a more restrictive salary floor on the profits of the small club is also negative. Note that in Regime A, the condition that marginal revenue equals marginal cost holds for the small club. Moreover, a more restrictive salary floor yields a higher win percentage for the small club and thus induces a decrease in the marginal revenue of the small club. Additionally, cost per unit

[30] Note that a more restrictive salary floor is characterized by a higher level of $floor$.

of talent increases. All together this implies that additional revenues cannot compensate for the higher costs.

2.3.2 The effect of revenue sharing in Regime C

In this subsection, we analyze the effect of changing the revenue-sharing parameter α in Regime C given that the league has fixed a certain $floor' \in \left(\underline{floor}, \overline{floor}\right)$.

We analyze the effect of revenue sharing on the allocation of talent and the cost per unit of talent in the following proposition:

Proposition 8

(i) The invariance proposition does not hold in Regime C: more revenue sharing increases competitive balance.

(ii) More revenue sharing decreases the cost per unit of talent in Regime C.

Proof. See Appendix A.8. ■

In Regime C, the invariance proposition does not hold when revenue sharing is combined with a (binding) salary floor. In contrast to Regime B, a higher degree of revenue sharing induces a reallocation of talent from the large to the small club and thus produces a more balanced league in Regime C.

As noted above, revenue sharing always decreases marginal revenue and thus the talent demand for the large club, while the small club is constrained. Analogously to Regime B, this implies that the market-clearing cost c^C per unit of talent decreases. Since the salary payments of the small club are bound by the salary floor, equilibrium amount of talent hired by the small club, $t_2^C = floor/c^C$, increases. Thus, the large club decreases its talent level by the same amount by which the small club increases its talent level, i.e., $0 > -\frac{\partial t_1^C}{\partial \alpha} = \frac{\partial t_2^C}{\partial \alpha} < 0$.

As a result, more revenue sharing increases the win percentage w_2^C of the small club and decreases the win percentage w_1^C of the large club producing a more balanced league. Thus, the pre-shared revenues R_2^C (R_1^C) of the small (large) club increase (decrease) through more revenue sharing. Moreover, the salary payments of the large club and aggregate salary payments in the league decrease. Further note that the league departs from the revenue-maximizing win percentages from Regime A such that aggregate club revenues $R_1^C + R_2^C$ decline through revenue sharing.

Both mechanisms – a salary floor and a revenue-sharing arrangement – contribute to producing a more balanced competition. However, the revenue-sharing arrangement achieves this goal with lower costs (salary payments), because it lowers the costs of the large club, whereas a salary floor increases the costs of both clubs.

The effect of revenue sharing on club profits is analyzed in the following proposition:

Proposition 9

In Regime C, the introduction of revenue sharing increases the small club's profits and aggregate club profits, while the large club's profits decrease.

Proof. See Appendix A.9. ■

The proposition shows that only the small club benefits from the introduction of a revenue-sharing arrangement in Regime C. That is, the positive effect of revenue sharing through lower costs and higher pre-shared revenues R_2^C for the small club compensates for the lower aggregate revenues $R_1^C + R_2^C$. For the large club, however, the effect is different, because the lower costs cannot compensate for lower (pre-shared and aggregate) revenues, and thus profits decrease. Even though aggregate revenues decrease through the introduction of revenue sharing, aggregate club profits increase because the lower costs compensate for the lower revenues.

2.4 Regime D: either salary cap or salary floor is binding for both clubs

In this section, we assume that either the salary cap or the salary floor is binding for both clubs. For notation's sake, we write $\lambda \in \{floor, cap\}$.[31] In Regime D, the equilibrium allocation of talent and the cost per unit of talent are computed as:

$$\begin{aligned} \left(t_1^D, t_2^D\right) &= \left(\frac{s}{2}, \frac{s}{2}\right) = (w_1^D s, w_2^D s), \\ c^D &= \frac{2 \cdot \lambda}{s}. \end{aligned} \tag{7}$$

Note that in Regime D, the league is perfectly balanced such that both clubs have an equal win percentage given by $w_1^D = w_2^D = 0.5$. The equilibrium salary payments are given by $\left(x_1^D, x_2^D\right) = (\lambda, \lambda)$ with $\lambda \in \{floor, cap\}$, depending on whether we consider a

[31] Note that we consider both cases at the same time because the analyses are very similar.

binding salary floor or salary cap for both clubs. Thus, we are in Regime D if either $floor > \overline{floor}$ or $cap < \underline{cap}$. In the first case, the salary floor is binding for both clubs, and in the second case, the salary cap is binding for both clubs.[32]

From (7), we derive that a change in the salary cap (salary floor) does not change the distribution of talent in Regime D. However, by implementing a more restrictive salary cap, the cost per unit of talent c^D decreases, whereas c^D increases through a more restrictive salary floor.

A salary cap is therefore beneficial for club profits because it lowers the costs of both clubs and club revenues remain unchanged. The opposite is true for a more restrictive salary floor, because it raises clubs' costs and leaves clubs' revenues unchanged.

Moreover, we see that talent demand and the cost per unit of talent are independent of the revenue-sharing parameter α if the salary floor (cap) is binding for both clubs, i.e., for $\lambda \in \{floor, cap\}$. Thus, the invariance principle holds in Regime D because revenue sharing has no effect on the distribution of talent and thus does not affect pre-shared club revenues. Moreover, the cost per unit of talent c^D is also unaffected by revenue sharing.

As in Regime A, revenue sharing redistributes revenues from the large to the small club. As a consequence, the profits of the large club decrease and the profits of the small club increase through a higher degree of revenue sharing. Aggregate club profits, however, are not affected by revenue sharing in Regime D.

3 Discussion

In this paper, we have analyzed the combined effect of salary restrictions and revenue sharing in sports leagues by using a standard textbook model (see, e.g., Késenne, 2007). The invariance proposition with regard to revenue sharing was originally derived in this framework featuring a fixed supply of talent and Walrasian conjectures (see El-Hodiri and Quirk, 1971; Fort and Quirk, 1995; Vrooman, 1995, 2007, 2008).[33] Szymanski (2004), however, has argued that Nash conjectures should be applied because these conjectures are standard in the mainstream IO literature. Under Walrasian conjectures, clubs internalize that, due to the fixed amount of talent, a one-unit increase of talent hired at one

[32]Note that the salary cap has to be sufficiently large (first case) and the salary floor has to be sufficiently small (second case).

[33]Remember that the term dt_i/dt_j is the "conjectural variation", i.e., the rate of change in club i's choice variable anticipated by club j in response to its own change.

club necessarily leads to a one-unit reduction of talent at the other clubs. Under Nash conjectures, on the other hand, clubs choose best responses to given choices of the other clubs. Which behavioral assumption is appropriate continues to be a heavily disputed issue in the sports economics literature and no consensus has emerged so far. As an example, Eckard (2006) claims that Walrasian conjectures should always be applied when talent supply is fixed, while Szymanski (2006) disagrees with this claim.

Several articles in the classical IO literature argue that a zero conjectural variation (i.e., Nash conjecture) is not always meaningful. For example, according to Kamien and Schwartz (1983), "the assumption of zero conjectural variation is suspect, since it leads to a logical inconsistency even if the equilibrium is attained through a simultaneous rather than a sequential process." Moreover, the authors state that the "assumption of zero conjectural variation is naive, and experience usually shows it to be inappropriate."[34] In this context, a conjectural variation is consistent if it is equivalent to the optimal response of the other firms at the equilibrium defined by that conjecture. Unfortunately, neither Nash conjectures nor Walrasian conjectures are consistent in the context of our model.

Since the choice of the conjectural variation can potentially influence the results, we verified whether the results of our paper are sensitive with respect to the conjectural variation applied. Fortunately, the main conclusions drawn from our analysis do not change when using the Nash approach with one exception. This exception concerns the result regarding the invariance proposition in Regime A. As it is well known in the literature,[35] under Nash conjectures, revenue sharing alters the distribution of talent in Regime A and thus the invariance proposition does no longer hold in Regime A. In particular, more revenue sharing dilutes investment incentives of both clubs. Since this effect is more pronounced for the small than for the large club, the large club ends up with a larger share of the (fixed) talent supply. As a result of this dulling effect, the level of competitive balance decreases. Apart from this exception, the main results in Regimes B, C and D remain qualitatively the same under Nash conjectures.[36]

[34] Also see Perry (1982), Dixit (1986), and Friedman and Mezzetti (2002) who derive consistent conjectural variations different from zero.

[35] See, e.g., Szymanski and Késenne (2004), Késenne (2005), and Dietl and Lang (2008).

[36] Unfortunately, under Nash conjectures, we were not able to derive all results analytically and thus we had to rely on numerical simulations to analyze certain issues (e.g., comparative statics).

4 Conclusion

Many major sports leagues (e.g., NHL, NFL and NBA) are characterized by a combination of cross-subsidization mechanisms like revenue-sharing arrangements and payroll restrictions. Up to now, the effects of these policy tools have never been studied jointly but only separately.

In this article, we have analyzed the combined effect of salary restrictions (salary cap and floor) and revenue-sharing agreements on club profits, player salaries, and competitive balance. For our analysis, we used a standard FQ-style model with Walrasian conjectures. This model setup resembles nicely the closed US Major Leagues which are characterized through a fixed supply of talent and a combination of revenue-sharing arrangements and salary restrictions.

Tables 1 and 2 summarize the main findings of our paper.

Table 1: Effect of a more restrictive salary floor/cap

	CB	Large club (profits)	Small club (profits)	Salaries
Regime A	no effect	no effect	no effect	no effect
Regime B	increase	increase (if $cap > cap^*$)	increase	decrease
Regime C	increase	decrease	decrease	increase
Regime D	no effect	decrease/increase	decrease/increase	decrease/increase

Our analysis shows that in the well-known case of a league without a binding salary cap or floor (Regime A), the famous invariance proposition holds. Although revenue sharing has no effect on the distribution of talent it has implications for the distribution of benefits between clubs and players. Revenue sharing inevitably lowers the market-clearing cost per unit of talent and increases the profits of the small clubs and aggregate club profits. The effect on the profits of the large club is ambiguous and depends on the difference between the clubs in terms of market size (see Regime A in Table 2). This means that revenue sharing can be used to redistribute rents from clubs to players and vice versa.

Table 2: Effect of revenue sharing

	CB	Large club (profits)	Small club (profits)	Salaries
Regime A	no effect	decrease (if $m > m'$)	increase	decrease
Regime B	decrease	increase	increase	decrease
Regime C	increase	decrease	increase	decrease
Regime D	no effect	decrease	increase	no effect

The invariance proposition, however, does not hold even under Walrasian conjectures if revenue sharing is combined with either a salary cap or a salary floor (Table 2). Introducing a salary cap has the intended effect of increasing competitive balance and increasing the profits of the small club. A salary cap therefore effectively supports the small clubs. The increased competitive balance, however, is detrimental to aggregate league revenues, because talent is removed from its most productive use. In this situation, adding a revenue-sharing arrangement helps to reallocate talent back to its most productive use. Additionally, increased revenue sharing lowers costs and increases profits. Therefore, far from being invariant, revenue sharing is a very effective tool for cross-subsidization.

Introducing a salary floor is beneficial to players but achieves this by departing from the productive allocation of talent and lowering the profits of the clubs. In this case, revenue sharing will worsen the misallocation (Tables 1 and 2). We conclude that the mixture of revenue sharing and salary caps is preferable.

Moreover, the analysis has shown that both a salary cap and a salary floor contribute to improving competitive balance in the league. From the perspective of a league planner, however, a fully balanced league is not desired, i.e., a certain degree of imbalance is favorable. In our model, the allocation of talent that maximizes aggregate league revenues, is characterized by an allocation of talent where the large club is the dominant team that has a higher win percentage than the small club. According to our analysis, this league optimal degree of imbalance, which increases in the difference between clubs, is already achieved in a league with revenue sharing that has implemented neither a salary cap nor a salary floor (Regime A).[37] Every intervention to improve competitive balance like salary caps and salary floors combined with revenue-sharing arrangements, is counter-productive, because it will result in an unproductive allocation of talent.

Finally, this paper has shown that, for team sports leagues like the US Major Leagues, it is crucial to analyze the effect of a combination of policy tools and not the effect of these tools separately. Our results have important policy implications for theses leagues, because league authorities should take into account that changes in one policy tool strongly influences the working of the others. This allows league authorities to pursue various objectives at the same time (e.g., competitive balance and redistribution to small clubs) by using a suitable combination of the policy tools.

An interesting avenue for further research in this area is to analyze to which extend the results carry over to other settings like open leagues in which the supply of talent is elastic or leagues with an endogeneously determined salary cap/floor.

[37] Remember that due to the invariance principle, revenue sharing has no effect on the revenue-maximizing allocation of talent.

A Appendix: Proofs

A.1 Proof of Proposition 1

The proof of Part (i) and (ii) is straightforward by inspection of (4) which represents the allocation of talent and the cost per unit of talent in equilibrium.

In order to prove Part (iii), we compute the equilibrium post-sharing revenues of club $i = 1, 2$ in Regime A as follows:

$$\begin{aligned}\widehat{R}_1^A &= \frac{1 + m + m(1+\alpha)(1+m)^2 - \alpha(3m+1)}{2(1+m)^2} \\ \widehat{R}_2^A &= \frac{(1+m)(1+m+m^2) - \alpha(m-1)(1+m(3+m))}{2(1+m)^2}\end{aligned}$$

We derive the derivatives with respect to α as: $\frac{\partial \widehat{R}_1^A}{\partial \alpha} = \frac{m(1+m)^2-(3m+1)}{2(1+m)^2} > 0$ and $\frac{\partial \widehat{R}_2^A}{\partial \alpha} = -\frac{(m-1)(1+m(3+m)}{2(1+m)^2} < 0 \ \forall \alpha \in (1, 0)$. Thus post-sharing revenues of the large (small) club decrease (increase) through a higher degree of revenue sharing, i.e., a lower value of the parameter α.

The equilibrium profits of club $i = 1, 2$ in Regime A are then given by $\pi_1^A = \widehat{R}_1^A - x_1^A$ and $\pi_2^A = \widehat{R}_2^A - x_2^A$ with the corresponding derivatives

$$\frac{\partial \pi_1^A}{\partial \alpha} = \frac{m(-2 + m(m-2)) - 1}{2(1+m)^2} \text{ and } \frac{\partial \pi_2^A}{\partial \alpha} = \frac{-m(1+m)^2 + (1-m)}{2(1+m)^2}.$$

It follows that $\frac{\partial \pi_1^A}{\partial \alpha} > 0 \Leftrightarrow m^3 - 2m(1+m) - 1 > 0$. Thus, $\frac{\partial \pi_1^A}{\partial \alpha} > 0 \Leftrightarrow m > m' \approx 2.83$. Moreover, $\frac{\partial \pi_2^A}{\partial \alpha} < 0 \ \forall \alpha \in (1, 0)$ and $m > 1$. Thus revenue sharing always increases the profits of the small club π_2^A whereas the profits of the large club π_1^A only increase if the difference between both clubs in terms of market size is not too large, i.e., if $m < m'$. It is obvious that aggregate club profits increase through revenue sharing because aggregate revenues are independent of α whereas the clubs' costs (aggregate salary payments) decrease. This completes the proof of the proposition.

A.2 Proof of Proposition 2

First of all, remember that we are in Regime B, i.e., $cap \in \left(\underline{cap}, \overline{cap}\right) = \left(\frac{1+\alpha-m(1-\alpha)}{4}, x_1^A\right)$. In order to prove that a more restrictive salary cap produces a more balanced league by increasing the win percentages of the small club and decreasing the win percentage of the large club, we derive the equilibrium win percentages in Regime B as

$$w_1^B = \frac{t_1^B}{t_1^B + t_2^B} = \frac{2cap}{m(\alpha - 1) + \phi^B} \text{ and } w_2^B = 1 - w_1^B \tag{8}$$

with $\phi^B = \sqrt{(\alpha - 1)^2 m^2 + 4cap(1 + \alpha + m(1 - \alpha))}$. The corresponding derivatives are given by $\frac{\partial w_1^B}{\partial cap} = \frac{1}{\phi^B} > 0$ and $\frac{\partial w_2^B}{\partial cap} = -\frac{1}{\phi^B} < 0$. It follows that a more restrictive salary cap, i.e., a lower value of cap, produces a more balanced league by increasing competitive balance. Remember that club 1 is the dominant team which has a higher win percentage than club 2.

The derivative of the equilibrium cost per unit of talent $c^B = \frac{(\alpha-1)m+\phi^B}{2s}$ in Regime B with respect to cap is given by $\frac{\partial c^B}{\partial cap} = \frac{1+\alpha+m(1-\alpha)}{\phi^B s} > 0$. This completes the proof of the proposition.

A.3 Proof of Proposition 3

In order to prove the claim, without loss of generality, we normalize the supply of talent to unity, i.e., we set $s = 1$. Moreover, we consider a league without revenue sharing, i.e., we set $\alpha = 1$.[38] In this case, the maximum of aggregate club profits π^B and the profits of the large club π_1^B are given by

$$\begin{aligned}
\max_{cap>0} \pi^B \quad &: \quad \frac{\partial \pi^B}{\partial cap} = \frac{\sqrt{2}(ms - 1) - \sqrt{cap}(1 + m)s}{2\sqrt{cap}s} = 0 \Leftrightarrow cap^{**} = \frac{2(ms - 1)^2}{s^2(m + 1)^2} \\
\max_{cap>0} \pi_1^B \quad &: \quad \frac{\partial \pi_1^B}{\partial cap} = -1 + m\left(\frac{1}{\sqrt{2cap}} - \frac{1}{2}\right) = 0 \Leftrightarrow cap^* = \frac{2m^2}{(2 + m)^2}
\end{aligned}$$

We derive that $cap^* > cap^{**}$. Furthermore, the derivative of the small club's profits π_2^B with respect to cap is computed as

$$\frac{\partial \pi_2^B}{\partial cap} = \frac{1}{2} - \frac{1}{\sqrt{2cap}} < 0 \ \forall cap \in \left(\underline{cap}, \overline{cap}\right).$$

This completes the proof of the proposition.

[38] It can be shown that the result holds true for all $\alpha \in [0, 1]$.

A.4 Proof of Proposition 4

Ad (i) In order to prove that the invariance proposition does not hold in Regime B, we compute the derivative of equilibrium allocation of talent (t_1^B, t_2^B) with respect to the revenue sharing parameter as follows:

$$\frac{\partial t_1^B}{\partial \alpha} = -\frac{2cap\left(m + \frac{-2cap(m-1)+m^2(\alpha-1)}{\phi^B}\right)}{((\alpha-1)m+\phi^B)^2} = -\frac{\partial t_2^B}{\partial \alpha},$$

with $\phi^B = \sqrt{(\alpha-1)^2 m^2 + 4cap(1+\alpha+m(1-\alpha))}$. We deduce that $\frac{\partial t_1^B}{\partial \alpha} < 0$ and $\frac{\partial t_1^B}{\partial \alpha} > 0$, because $\alpha \in (\underline{\alpha}^B, \overline{\alpha}^B) = \left(\frac{cap(1+m)^2}{2m^2}, \frac{4cap+m-1}{1+m}\right)$.[39] Thus, revenue sharing changes the allocation of talent in Regime B because it induces the large club to increase its level of talent and the small club to decrease its level of talent. As a consequence the large (small) club's win percentage w_1^B (w_2^B) increases (decreases). Since the large club is the dominant team, competitive balance decreases as a result of more revenue sharing.

Ad (ii) In order to prove that revenue sharing decreases the cost per unit of talent c^B in Regime B, we derive the derivative of c^B with respect to α as

$$\frac{\partial c^B}{\partial \alpha} = \frac{1}{2s}\left(m + \frac{-2cap(m-1)+(\alpha-1)m^2}{\phi^B}\right).$$

We deduce that $\frac{\partial c^B}{\partial \alpha} > 0$, because $\alpha \in (\underline{\alpha}^B, \overline{\alpha}^B)$. Thus, more revenue sharing (i.e., a lower value of α) decreases c^B. This completes the proof of the proposition.

A.5 Proof of Proposition 5

In order to prove that the introduction of revenue sharing increases the profits of the large club, we evaluate the derivative of the large club's profit function π_1^B with respect to α at $\alpha = 1$ as:[40]

$$\frac{\partial \pi_1^B}{\partial \alpha}|_{\alpha=1} = \frac{1}{4}\left[cap(1-m^2) + m\sqrt{2cap}(2m+1) - 2(1+m^2)\right].$$

[39] Remember that we are in Regime B since $cap \in \left(\underline{cap}, \overline{cap}\right)$. This determines implicitely the corresponding interval of feasible α.

[40] Note that $\alpha = 1$ is always in the interval of feasible α in Regime B because $\overline{\alpha}^B \geq 1$ for $\alpha = 1$ and $cap \in \left(\underline{cap}, \overline{cap}\right)$.

One can show that

$$\frac{\partial \pi_1^B}{\partial \alpha}|_{\alpha=1} < 0 \Leftrightarrow cap < \frac{2 - m(2m+1)\sqrt{4m^3+m^2+4} + m^2(1+2m(2+m))}{(m^2-1)^2}.$$

The last inequality is fulfilled for all $cap \in \left(\underline{cap}, \overline{cap}\right)$ if m is not too large. We proceed analogously for the small club: we evaluate the derivative of the small club's profit function π_2^B with respect to α at $\alpha = 1$ as:

$$\frac{\partial \pi_2^B}{\partial \alpha}|_{\alpha=1} = \frac{1}{4}\left[2 - \sqrt{2cap} - 2m\right].$$

One can show that $\frac{\partial \pi_2^B}{\partial \alpha}|_{\alpha=1} < 0$ for all $cap \in \left(\underline{cap}, \overline{cap}\right)$. This completes the proof of the proposition.

A.6 Proof of Proposition 6

First of all, remember that we are in Regime C, i.e., $floor \in \left(\underline{floor}, \overline{floor}\right) = \left(x_2^A, \frac{\alpha-1+m(1+\alpha)}{4}\right)$. In order to prove that a more restrictive salary floor produces a more balanced league by increasing the win percentages of the small club and decreasing the win percentage of the large club, we derive the equilibrium win percentages in Regime C as

$$w_1^C = \frac{t_1^C}{t_1^C + t_2^C} = \frac{(\alpha-1) - 2floor + \phi^C}{(\alpha-1) + \phi^C} \text{ and } w_2^C = 1 - w_1^C, \tag{9}$$

with $\phi^C \equiv \sqrt{(\alpha-1)^2 + 4floor(1-\alpha+m(1+\alpha))}$. The corresponding derivatives are given by $\frac{\partial w_1^C}{\partial floor} = -\frac{1}{\phi^C} < 0$ and $\frac{\partial w_2^C}{\partial floor} = \frac{1}{\phi^C} > 0$. It follows that a more restrictive salary floor produces a more balanced league by increasing competitive balance. Remember that club 1 is the dominant team which has a higher win percentage than club 2.

The derivative of the equilibrium cost per unit of talent $c^C = \frac{(\alpha-1)m+\phi^B}{2s}$ in Regime C with respect to $floor$ is given by $\frac{\partial c^C}{\partial floor} = \frac{1+\alpha(m-1)+m}{\phi^C s} > 0$. This completes the proof of the proposition.

A.7 Proof of Proposition 7

It is straightforward to prove that the profits of the large club π_1^B decrease through a more restrictive salary floor: On the one hand, revenues (pre-shared and aggregate revenues) decrease and on the other hand costs (salary payments) increase for the large club. As a consequence, profits decrease. A similar argument holds true to show that aggregate club profits π^B decrease.

In order to prove that also profits of the small club decrease we derive the derivative of π_2^B with respect to $floor$ as

$$\frac{\partial \pi_2^B}{\partial floor} = \frac{a^2(m-1) - (1+m)(3\phi^C - 1) + a(\phi^C - m(\phi^C - 6))}{2\phi^C(1+\alpha(m-1)+m)}$$

with $\phi^C \equiv \sqrt{(\alpha-1)^2 + 4floor(1-\alpha+m(1+\alpha))}$. We derive that $\frac{\partial \pi_2^B}{\partial floor} < 0$ for all $floor \in \left(\underline{floor}, \overline{floor}\right)$. This completes the proof of the proposition.

A.8 Proof of Proposition 8

Ad (i) In order to prove that the invariance proposition does not hold in Regime C, we compute the derivative of the equilibrium allocation of talent (t_1^C, t_2^C) with respect to the revenue sharing parameter α as follows

$$\frac{\partial t_1^C}{\partial \alpha} = -\frac{2floor\left(\alpha - 1 + 2floor(m-1) + \phi^C\right)}{\phi^C(\alpha - 1 + \phi^C)^2} = -\frac{\partial t_2^C}{\partial \alpha},$$

with $\phi^C \equiv \sqrt{(\alpha-1)^2 + 4floor(1-\alpha+m(1+\alpha))}$. We deduce that $\frac{\partial t_1^C}{\partial \alpha} > 0$ and $\frac{\partial t_1^C}{\partial \alpha} < 0$, because $\alpha \in (\underline{\alpha}^C, \overline{\alpha}^C) = \left(\frac{1+4floor-m}{1+m}, \frac{floor(1+m)^2}{2m}\right)$. Thus, revenue sharing changes the allocation of talent in Regime C, because it induces the large (small) club to decrease (increase) its level of talent. As a consequence the large (small) club's win percentage w_1^C (w_2^C) decreases (increases). Since the large club is the dominant team, competitive balance increases as a result of revenue sharing.

Ad (ii) In order to prove that revenue sharing decreases the cost per unit of talent c^C in Regime C, we derive the derivative of c^C with respect to α as

$$\frac{\partial c^C}{\partial \alpha} = \frac{1}{2s}\left(1 + \frac{\alpha - 1 + 2floor(m-1)}{\phi^C}\right)$$

We deduce that $\frac{\partial c^C}{\partial \alpha} > 0$, because $\alpha \in (\underline{\alpha}^C, \overline{\alpha}^C)$. Thus, more revenue sharing (i.e., a lower value of α) decreases c^C. This completes the proof of the proposition.

A.9 Proof of Proposition 9

In order to prove that the introduction of revenue sharing decreases aggregate club profits, we evaluate the derivative of the aggregate profit function π^c with respect to α at $\alpha = 1$ as:[41]

$$\frac{\partial \pi^c}{\partial \alpha}|_{\alpha=1} = -\frac{(1+m)\left[floor(1-m) - 2\sqrt{2mfloor} + m(2+\sqrt{2mfloor})\right]}{4m^2}.$$

One can show that $\frac{\partial \pi^c}{\partial \alpha}|_{\alpha=1} < 0$ for all $floor \in \left(\underline{floor}, \overline{floor}\right)$. For the large club, we evaluate the derivative of its profit function π_1^c with respect to α at $\alpha = 1$ as:

$$\frac{\partial \pi_1^c}{\partial \alpha}|_{\alpha=1} = \frac{1}{4}\left[2(m-1) - \sqrt{2mfloor}\right].$$

We derive $\frac{\partial \pi_1^C}{\partial \alpha}|_{\alpha=1} > 0 \Leftrightarrow floor < 2(m + 1/m - 2)$. The last inequality is fulfilled for all $floor \in \left(\underline{floor}, \overline{floor}\right)$ if m is not too small. For the small club, we evaluate the derivative of its profit function π_2^C with respect to α at $\alpha = 1$ as:

$$\frac{\partial \pi_2^C}{\partial \alpha}|_{\alpha=1} = \frac{\sqrt{2mfloor}(m+2) + floor(m^2-1) - 2(m^3+m)}{4m^2}.$$

One can show that $\frac{\partial \pi_2^C}{\partial \alpha}|_{\alpha=1} < 0$ for all $floor \in \left(\underline{floor}, \overline{floor}\right)$. This completes the proof of the proposition.

[41]Note that $\alpha = 1$ is always in the interval of feasible α in Regime C because $\overline{\alpha}^C \geq 1$ for $\alpha = 1$ and $floor \in \left(\underline{floor}, \overline{floor}\right)$.

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