Opinion Dynamics: Disciplinary Origins, Recent Developments, and a View on Future Trends

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Abstract: Opinion Dynamics is a research field which utilizes mathematical and physical models and computational tools, especially the agent-based computational modeling tools, to investigate the diffusion and evolution of opinions in a population of human beings. This research field is a confluence of various disciplines in social sciences, especially the social influence models developed in social psychology and sociology. In this paper, a short review is given to trace the historical roots of this field, to summarize its current progresses, and to envision a few key issues that worthy of further investigations.

Introduction

In the last decade, agent-based computational modeling has become a major method to study various complex social phenomena (Epstein 2007). One noticeable application field of this method is so-called "opinion dynamics" (Hoylst et al. 2001). As one key sub-field of the wider "social dynamics" (Castellano et al.), opinion dynamics is a research field in which mathematical and physical models and computational tools are utilized to explore the dynamical processes of the diffusion and evolution of opinions in a population of human beings. Essentially, the spreading of attitudes and opinions can be modeled as a global process of massive individuals, which emerges from the communications and mutual influences within local groups, e.g., between the direct neighbors in a closed community (Sznajd-Weron & Sznajd 2000); and thus agent-based modeling is an appropriate approach to cope with the complexity of such social dynamical process. In recent years, there are growing interests on opinion dynamic through agent-based modeling; however, today this research field is still at the very early stage due to the intrinsic complexity of human society. For developing this research field, it is deserved to look backward to trace its academic origins and development trails, and to envision its future trends.

Tracking the Disciplinary Upstream of Opinion Dynamics

The studies on opinion dynamics reflect a wide range of social phenomena, e.g. emergence of fashions and fads, minority opinion survival and spreading, collective decision making, consensus building, emergence of political parties, diffusion of rumor, expansion of extremism, and propagation of cult. These phenomena are the persisting topics of interest in many areas of social sciences. In this section, we try to take a brief retrospect to examine where opinion dynamics comes from.

Various studies in sociology and social psychology have great impact on the birth of opinion dynamics. One major theme of sociology is to explore the collective action of populations. Since the 19th century, the irrational and sometime maniac collective behaviors have been noticed and discussed, which were called "madness of crowds" by Mackey (1841) and "stupidity of crowds" by Le Bon (1895). This observation was then followed by a series of experimental studies in social psychology on "conformity", including the "social norm" by Sherif (1936), the "group press" by Asch (1951), and the "obedience" by Milgram (1974), and so on. These studies reveal that there is a common tendency that individuals' attitudes and behaviors conform to the majority of the belonging group. In Economics, this conformity is often termed as "herd behavior", as firstly introduced in Keynes' (1936) "General Theory" and extensively investigated thereafter in the context of, e.g., corporate investment, financial markets and intra-firm decision-making (Scharfstein & Stein 1990, Banerjee 1992). To explore the underlying mechanisms of these collective behaviors, various "threshold" or "critical mass" models have been developed in sociology and economics, e.g. Granovettor (1978), Schelling (1978), and Brock & Durlauf (2001). These models of collective behaviors have prominently influenced the growth of opinion dynamics studies since the formation of public opinions and in some cases social norms is often the premise of collective actions; and to some extent, the studies of collective behaviors and of public opinions are inseparable.

Besides the aforementioned studies on psychological conformity and collective behaviors, the topic of opinions is more directly involved in a series of sociological and social-psychological endeavors on the issues like group polarization (Moscovici & Zavalloni 1969), minority influence (Moscovici et al. 1969) and social influence models. The endeavors on group polarization and minority influence phenomenologically reveal the intrinsic complexity of opinion spreading in society; and these phenomena are also the key subjects of interest in today's

opinion dynamics researches. The endeavors on social influence models, on the other hand, provide the early disciplinary treatments on the underlying mechanisms of opinion spreading from the sociological and social-psychological perspectives. These endeavors are the direct ancestors of today's opinion dynamics researches. One breakthrough work on social influence modeling is the two-step flow model of influence that was formulated and elaborated by Lazarsfeld and his colleagues at Columbia University in the 1940-50s (Katz & Lazarsfeld 1955). This two-step flow model stresses that the massive individuals, instead of being directly persuaded by the media, usually follow a smaller number of opinion leaders who bridge the media and the mass public. Another famous early work on social influence modeling is French's (1956) social power model. In this work, French proposed a Markov chain model to describe the diffusion of social influence and the formation of public opinions in social networks, based on a discussion and classification of "social power". French's social power model was further developed in French and Raven (1959). French and Raven's original research was concerned with situations in which a supervisor influences a worker. A wide variety of other social interactions have been examined by following endeavors. This social power model has become a good source for subsequent social influence studies, which fall into five main areas, as summarized by Rashotte (2007): (1) minority influence in group settings, (2) research on persuasion, (3) dynamic social impact theory, (4) structural approach to social influence, and (5) social influence in expectation states theory.

Further progresses were made on the social influence models in the 1980 and 1990s. Latané and his collaborators and subsequent scholars made noticeable contributions in this direction. In 1981, he proposed a "social impact theory", which provides a microscopic analysis on how an individual is influenced by the surrounding social environments. For Latané, the impact of any information source on the individual relies on three factors: the number of others who make up that source, their immediacy, and their social strength (i.e. salience or power). Based on this social impact theory, Latané and his collaborators furthermore used cellular automata modeling and computer simulation to study the global effects of the individual attitude change (Nowak et al. 1990). Nowak et al.'s work was more systematically treated as "dynamic social impact theory" by Latané (1996). In the dynamic social impact theory, social structure self-organizes and evolves through the dynamic and iterative interpersonal influences. The overall process of opinion diffusion begins from a random distribution of attitudes and beliefs. Within the social network, one is more likely to be influenced by someone nearby than by the ones far away; and localized cultures of beliefs may be a result of such influences. It can be argued that in the dynamic social impact theory a solid foundation has been established for the more recent opinion dynamics researches. During the same period, Friedkin and his colleagues took a structural approach to examine interpersonal influence within a larger social network of influence. Their endeavors constitute a "social influence network theory" (Friedkin 1998). This theory utilizes mathematical models and quantification to measure the process of social influence. Within the social influence network, a "norm" opinion is formed as weighted average of the individuals' private opinions on some issue; and in turn, the individuals modify their own opinions in response to this norm by averaging their initial opinions and the social norm. The interpersonal influences are represented as the weights that determine the formation of the norm and the change of the personal opinions. Friedkin's work makes remarkable contribution to formalize social influence and opinion evolution in social networks; however, it is criticized by some authors, e.g. Hegselmann & Krause (2002), due to the proposed model is essentially a linear model which oversimplifies the real-world situations.

The prior introduction indicates that the social-influence studies carried out in sociology and social psychology have profound impact on the development of opinion dynamics. Various other disciplines, nevertheless, have also contributed to the rising of opinion dynamics, since this research field actually involves in a wide range of interests. One series of work that is closely related to opinion dynamics is the inquiries on consensus building in decision science (particularly in group decision-making studies). The studies on consensus building in the context of decision-making were initialized by De Groot (1974) and Lehrer (1975) and a plenty of subsequent inquiries followed in the 1970-80s (Chatterjee & Seneta 1977, Wagner 1978, Cohen & Newman 1986). The focus of these efforts is to examine the process of consensus-building in a group and to develop methods to facilitate consensus-building. The modeling of non-consensus situations (i.e. disagreements) is less concerned. In the contexts of economics and politics, or perhaps more precisely economic psychology and political psychology, one issue that critically influences the development opinion dynamics is the voter or voting problem. For the voting problem, Herbert Simon (1954) proposed an early model which treated the "bandwagon and underdog effects". Simon's model was later extended by Timur Kuran (1987). Since the 1970s, the voter (voting) model has been widely discussed, e.g. cc. (Holley & Liggett 1975, Dornic et al. 2001, Lambiotte et al. 2009). Some recent work has explicitly connected voter modeling to opinion dynamics (Ben-Naim 2005). What's more, a political science scholar Axelrod (1997) proposed a well-cited culture dissemination model, through which he elaborately studied the evolution of cultures through the communications between neighbors in a lattice space. In this work, the term "culture" was used to refer to a broad range of things that are subject to social influence, such as beliefs, attitudes, language, art, technical standards, and social norms. Axelrod's model

can be regarded as an opinion-dynamical model, though he himself may prefer to use the broader term "culture". Axelrod's work has been followed by many researchers, who either gave further analysis on the cultural dynamics model, or suggested model extensions (Castellano et al. 2000, Klemm et al. 2005, Centora et al. 2007).

The previously-discussed models are all concerned about the aggregate dynamics of attitudes, voter choices, and cultures upon the interpersonal social influences. Opinion dynamics is growing from those endeavors as a research field that covers the common features behind the phenomena that are conventionally studies in multiple disciplines of social sciences. In this academic confluence, the use of mathematical, physical and computational tools plays a vital role. Mathematically, opinion dynamics means studying the diffusion of opinions as a dynamic system that can essentially be modeled as differential and/or difference equations. To this end, the mathematical studies on the spreading of infectious diseases, most famously the studies on the SIS and SIR models (Hethcote 2000), may give implications to the studies on opinion dynamics. As noted, the key social force that enables the diffusion of opinions in a population is all sorts of "social influence" or "social contagion" (Dodds & Watts 2004), which can to some extent be analogous to the "infection" of some disease; thus, the mathematical models of the spreading of infectious diseases can be borrowed and revised to study the diffusion of opinions through contagions, e.g., cc. (Moreno et al. 2005, Grönlund & Holme 2004). The social processes have also come into many physicists' view since the 1970s. In this arena, the dynamics of group opinions have persistently been investigated by using statistical-mechanics models, e.g. the probability model of group polarization by Weidlich (1971) and the Ising model for attitude change and consensus formation proposed by Galam & Moscovici (1991). More recently, with the blooming of complex adaptive system researches, agent-based computational modeling has become a new weapon for taming the social processes of opinion diffusion and evolution (Stauffer 2005). Correspondingly, there have been increasing "opinion dynamics" studies reported since the mid 1990s.

In all, we can see that the rising of opinion dynamics is the confluence of multiple disciplinary sources. Especially, two basic points characterize this research field. First, there is some common phenomenon that was originally concerned in divergent disciplines, i.e. the diffusion and evolution of some ideological attributes (beliefs, attitudes, cultures, rumors, fashions, fads, etc.) that are subject to social influences. In opinion dynamics, we try to pursue a unified treatment to this phenomenon. Second, we use the mathematical, physical, and computational models and tools, which are traditionally used in just natural sciences and engineering, to study the social processes of opinions. Nowadays, opinion dynamics is flourishing as a sub-field of complex social system studies, particularly using the agent-based modeling approach. For example, in "Journal of Artificial Societies and Social Simulation", which is one major communication platform of complex social system studies, opinion dynamics is one of the key themes. In the next section, we turn to examine the more recent developments of opinion dynamics.

A View on the Recent Work and Future Trends

Following the aforementioned studies on social influence models, consensus-building models, voter models and cultural models etc., noticeable progresses have been made in opinion dynamics. The representative contributions include Sznajd-Weron & Sznajd's (2000) opinion evolution model that is established upon the Ising model (Sznajd model), Deffuant model (Deffuant et al. 2000) and Hegselmann-Krause model (Hegselmann & Krause 2002) that are based on the hypothesis of "bounded confidence". These models have been densely discussed and a plenty of variants and extensions have been reported. For the recent progresses on opinion dynamics, a thorough review was made by Castellano et al. (2009). So in this paper we do not go into the detailed descriptions of the present day's opinion dynamics models. Instead, we hope to sketch an overall picture of the current researches. It is our view that the current researches in this field rely on the basic settings of the opinion dynamics models. Thus we can summarize the research progresses and simultaneously envision the future directions by examining the model settings in the different endeavors.

Firstly, an opinion dynamics model depends on the representation of opinions. In some models, the opinions are represented as binary values (e.g. 0 or 1). This is the simplest form of opinion representation; and this representation fits the situations where people's opinions are simply binary (e.g. agree or disagree, right or left, buy or sell, etc.). Especially, in the voter models and Ising-model-based models, this binary representation is commonly adopted. The opinions can also be represented as discrete integer values, continuous real values (e.g. in the bounded confidence models of Deffuant et al.'s and Hegselmann-Krause's), or vectors/bit-strings (e.g. in Axelrod's cultural dynamics model), and their combinations. Based on the distinction in opinion representation, we can classify the opinion dynamics models into binary opinion dynamics models, discrete opinion dynamics models, continuous opinion dynamics models, and multi-dimensional (i.e. vector) opinion dynamics models, etc. In the past years, the binary and discrete models have been extensively studied. One possible future direction of

research may set the focus on the continuous and multi-dimensional models so as to cope with the intrinsic multitude of human attitudes and values.

Secondly, an opinion dynamics model should specify the local communication mechanism through which the individuals influence each other and the personal opinions are exchanged. Two issues are associated with this mechanism. First, the opinion dynamics model should specify who participates in one single activity of opinion exchange. For this issue, the current models generally fall into two categories, namely: 1) the models in which opinion exchange takes place between neighbors in a social community or network, or 2) the models that tackles opinion exchange in random groups or pairs in the entire population. Another issue is whether and how the individuals influence each other through opinion exchange. In many opinion dynamics models, the concept of "bounded confidence" plays a critical role to measure the possibility for opinion influence between individuals. Bounded confidence is a hypothesis that only people with similar opinions can influence each other so as to make their opinions more similar. In different models, the bounded confidence has different measures, indicating the differences in openness to different opinions. Furthermore, there are basically three models for describe how social influence takes place, namely "missionaries", "opportunists" and "negotiators" (Stauffer 2005). In the missionary model, one individual (i.e. the "missionary") requests the counterpart(s) to adopt the missionary's opinion. The Sznajd model is a typical missionary model, where one forces his/her own opinion onto all the neighbors whose opinions are within the bounded confidence. In the opportunist model, one individual changes his/her own opinion to conform to the communication counterparts' opinions, by, for example, updating his/her opinion to the average of the counterparts' opinions. The opportunist model was used by Hegselmann & Krause (2002) under the bounded confidence limit. Axelrod's cultural diffusion model is another example that adopts the opportunist local exchange mechanism. Finally, the negotiator model suggests that at each time step, two communication partners discuss their opinions with each other. If the opinions differ by less than the confidence bound, both of their opinions change to get closer. This negotiator model is used in Deffuant's opinion dynamics model. In all, scholars have extensively studied the local opinion exchange mechanisms to fulfill opinion dynamics modeling. However, more work is still required to deepen the research.

The third key setting for the opinion dynamics models is the social environments in which the diffusion and evolution of opinions take place, or in the other words the structure of the agent society. In some researches, the structure of the agent society is not explicitly concerned, e.g. the Deffuant model. Some other earlier efforts are mostly focused on studying opinion dynamics in human society which is modeled as a regular lattice or grid. For example, in Axelrod's model and the Ising-model-based models (e.g. the Sznajd model), each agent resides in a cell of a lattice; one may then influence and be influenced by the others within the von Neumann or Moore neighborhoods. With the growing interests on the complex networks (Albert & Barabsi 2002), one recent trend is to explore the diffusion and evolution of opinions in the context of complex social networks, especially within small-world networks or scale-free networks (Grabowski & Kosinski 2006, Pluchino et al. 2006, Watts & Dodds 2007, Suo & Chen 2008). From another aspect, the diffusion of opinions, together with the diffusions of innovations and knowledge etc., has become a recent hotspot in the complex network field.

To sum up, scholars have developed various opinion dynamics models, with the divergent model settings in the opinion representation, the local communication mechanism, and the social context of opinion evolution. Accordingly, noticeable progresses have been made in this research field. However, as also noted in Castellano et al. (2009)'s review, "the development of opinion dynamics so far has been uncoordinated and based on individual attempts, … without a general shared framework and often with no reference to real sociological studies". Opinion dynamics is still at its infancy. With respect to the prior overview of the research progresses, we can envision that three issues are noteworthy in the near future in order to develop this research field. One key issue is to improve the local communication and opinion exchange model via closer connection to the socio-psychological studies. In social psychology, a good number of researches have been conducted in the past half century to improve our understandings of social influence and personal attitude change (Zimbardo & Leippe 1991). It would be promising to incorporate the interpersonal influence and attitude change models developed in social psychology to improve the local communication mechanism of the opinion dynamics model. Second, from the perspective of complex networks, it is worthwhile to study the co-evolution of public opinions and the dynamic social network in which the opinions spread. Currently, the marriage of complex network studies and opinion dynamics studies has been a major trend. The mainstream paradigm of inquiry is to design a network of some topological structure (regular, random, hierarchical, small-world, scale-free, etc.) in *a priori*, and then to investigate the dynamics of opinions in this pre-designed network. This paradigm is suitable for the situations where the structural change of the network can be negligible. However, it is often the case that the network of social interactions co-evolves with the actors' opinions. On one hand, the diffusion and evolution of the opinions are subject to the structure of the social network; on the other hand, the diffusion and evolution of the opinions may in turn influence the adjustment of the network structure. To cope with this sort of co-evolution,

some researches have been reported (Holme & Newmann 2006, Zanette & Gil 2006, Fu & Wang 2008, Banisch et al. 2009), but more are required. The third issue that deserves attention is on opinion dynamics in the Internet-based cyber-societies. With the increasing prominence of the cyber-societies such as blogsphere, BBS forums, USENET, and Social Networking Sites, the propagation of opinions and ideas in such cyber-society has caught much attention (Grul et al. 2004, Godes & Mayzlin 2004, Nowell & Kleinberg 2008). Nevertheless, in the current studies of opinion dynamics in cyber-society, the "cyber" features that distinguish an Internet community from a traditional physical community are not well addressed. We believe that it is of both theoretical and practical value to deepen the investigation on opinion dynamics in cyber-society.

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