Robust Suppression Sliding Mode Control for Uncertain Duffing-Holmes Chaotic Systems

T. C. Kuo, *Member, IAENG,* and Y. J. Huang, *Member, IAENG,* C. H. Chang, and C. Y. Chen

***Abstract*—This paper proposes a robust suppression sliding mode controller design for uncertain Duffing-Holmes chaotic systems. A form of time-varying second-order differential equation is organized to describe the dynamical system. Control system performance is proved robust against parametric uncertainties and external disturbances. The control input involves a discontinuous switching control input which is used to deal with the uncertainties and disturbances. Illustrative example is given. Input chattering is remarkably eliminated and trajectory tracking is effectively achieved.**

***Index Terms*—Chaos, Duffing-Holmes equation, robustness, sliding mode control.**

I. Introduction

Robust stabilization of uncertain systems is an important topic in the field of control. Many approaches account for the uncertainties under various hypotheses. Sliding mode control (SMC) is one of the popular strategies to deal with uncertain control systems [1-6]. The main feature of SMC is that it consists of a discontinuous control that drives the control system onto a specified sliding surface and maintains the system on this surface. When the system trajectory reaches the sliding surface, the robustness against parameter variations and external disturbances can be obtained. Various applications of SMC have been found, such as robotic manipulators, aircrafts, DC motors, and so on.

Chaos exists in many engineering systems such as electronic circuits, power converters, chemical systems, and so on [7]. A fundamental characteristic of a chaotic system is its extreme sensitivity to initial conditions; that is, small differences in the initial state can lead to extraordinary differences in the system state. Chaos control has been of broad interest since the early 1990s. A pioneering work of Ott, *et al*. proposed the well-known OGY control method [8]. Soon the OGY method was modified by Shinbort *et al*. to reduce the time required for stabilizing the target orbit [9]. Later, the control of chaos in a Bonhoeffer-van de Pol oscillator using a feed-forward back-propagating neural network trained on two different control schemes, the OGY control algorithm and the Pyragas method of delayed continuous feedback control, was proposed [10]. Recently, various methods have been proposed to control chaotic systems, such as neural network, fuzzy control, adaptive control, sliding mode control, etc [10-13].

In this paper, a systematic robust sliding mode control for Duffing-Holmes chaotic system is proposed. The goal is to achieve system robustness against parameter variations and external disturbances. The control input consists of a continuous nominal control part and a discontinuous switching control part. The former is the equivalent control for the nominal system and latter deals with the parametric variation and disturbance. To reduce the high frequency chattering in the controller, the boundary layer technique was used [14]. Theoretical analysis and numerical simulation verify the effectiveness of the proposed method. Further, it is worth noting that using the proposed method the input chattering does not appear.

This paper is organized as follows. Section 2 describes the robust controller design for Duffing-Holmes chaotic system. Section 3 shows simulation results of proposed method. Finally, conclusion is given.

II. Robust Controller Design

Consider the Duffing-Holmes chaotic system. In 1918, Duffing introduced a nonlinear oscillator [15], with a cubic stiffness term, to describe the hardening spring effect observed in many mechanical problems. Later, Duffing's equation has been modified in different manners by many researchers, for example, Moon and Holmes. In this paper, to be more general we consider a modified Duffing equation of the form named Duffing-Holmes [16]. For a Duffing-Holmes chaotic system, its dynamic equation can be described as

$$\ddot{x} + p_1\dot{x} + p_2 x + x^3 - q\cos(w_1 t) = 0 \,, \tag{1}$$

where $p_1 = 0.25$, $p_2 = -1$, $q = 0.3$, and $w_1 = 1$. This Duffing-Holmes chaotic system displays obvious chaotic behavior as shown in Fig. 1 when no control input is applied. In Fig. 1, the initial conditions are $x(0) = 2$ and $\dot{x}(0) = 2$. The sampling time is 0.001 sec.

In order to solve this problem, first we rewrite the chaotic system (1) with a form of time-varying second-order

Manuscript received June 30, 2008. This work was supported in part by National Science Council, Taiwan, R.O.C., under Contract NSC95-2221-E-231-013 and Contract NSC95-2221-E-155- 066-MY2.

T. C. Kuo is with the Department of Electrical Engineering, Ching Yun University, Chungli, Taiwan, R.O.C. (e-mail: tck@mail.cyu.edu.tw).

Y. J. Huang, C. H. Chang, and C. Y. Chen are with the Department of Electrical Engineering, Yuan-Ze University, Chungli, Taiwan, R.O.C. (e-mail: eeyjh@saturn.yzu.edu.tw, s939111@mail.yzu.edu.tw, s919101@mail.yzu.edu.tw).

(Advance online publication: 20 August 2008)

differential equation with uncertainties and disturbances:

$$\ddot{x} + a_1(t)\dot{x} + a_2(t)x = b(t)(u + d(t,x)), \tag{2}$$

where $x \in R$ denotes the system state, $u \in R$ is the system input, and $d(t,x)$ is the lumped disturbance. Let the upper and lower bounds of the uncertain system parameters $a_1(t)$, $a_2(t)$ and $b(t)$, and the disturbance $d(t)$ be specified as

$$\begin{cases} \beta_{\min} \le b^{-1}(t) \le \beta_{\max}, \\ \alpha_{1\min} \le b^{-1}(t)a_1(t) \le \alpha_{1\max}, \\ \alpha_{2\min} \le b^{-1}(t)a_2(t) \le \alpha_{2\max}, \\ \max|d(t,t)| < D|x|. \end{cases} \tag{3}$$

In the following, a robust suppression method is developed. The design procedure is divided into two steps. The first step is to define a sliding surface function such that in the sliding mode the system behaves equivalently as a linear system. The second step is to determine a control law such that the system will reach and stay on the sliding surface $s = 0$.

First, define the sliding surface function as

$$s = \dot{e} + ce, \tag{4}$$

where

$$e = x - r. \tag{5}$$

The symbol e is the tracking error, r is the desired path, and c is a positive constant.

The control input consists of two parts. Let the control input u be

$$u = u_o + u_s, \tag{6}$$

where u_o is the continuous nominal control, and u_s is the discontinuous switching control. The former is the equivalent control for the nominal system and the latter deals with the parametric variation and disturbances. In order to deal with the uncertainties and disturbances in (3), let $b^{-1}(t)$, $b^{-1}(t)a_1(t)$ and $b^{-1}(t)a_2(t)$ be divided into two parts: nominal part $(\hat{\beta}, \hat{\alpha}_1, \hat{\alpha}_2)$ and uncertain part $(\Delta\beta, \Delta\alpha_1, \Delta\alpha_2)$, i.e.,

$$\begin{cases} \hat{\beta} = \dfrac{\beta_{\max} + \beta_{\min}}{2}, \Delta\beta = \dfrac{\beta_{\max} - \beta_{\min}}{2}, \\ \hat{\alpha}_1 = \dfrac{\alpha_{1\max} + \alpha_{1\min}}{2}, \Delta\alpha_1 = \dfrac{\alpha_{1\max} - \alpha_{1\min}}{2}, \\ \hat{\alpha}_2 = \dfrac{\alpha_{2\max} + \alpha_{2\min}}{2}, \Delta\alpha_1 = \dfrac{\alpha_{2\max} - \alpha_{2\min}}{2}. \end{cases} \tag{7}$$

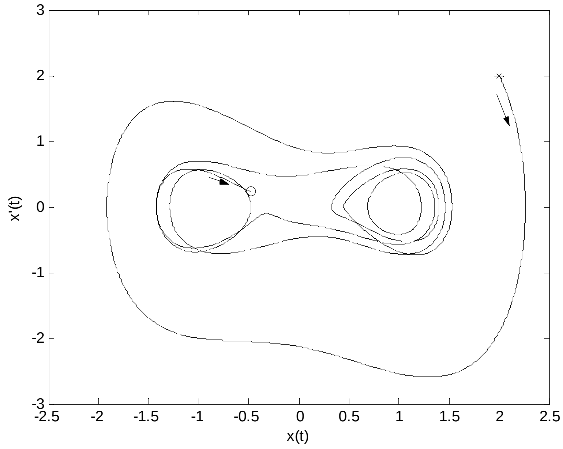

Fig. 1. The phase-plane plot of unforced Duffing-Holmes chaotic system with $x(0) = 2$ and $\dot{x}(0) = 2$.

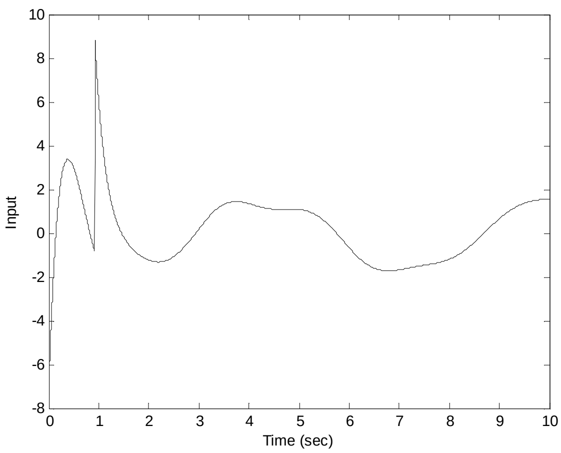

Fig. 2. The time response of control input.

Once the sliding surface function is designed, the next step is to design the control law accordingly. The control law u_o and u_s are formulated as

$$u_o = (\hat{\alpha}_1 - \hat{\beta}c)\dot{x} + \hat{\alpha}_2 x + \hat{\beta}\ddot{r} + \hat{\beta}c\dot{r}. \tag{8}$$

$$u_s = -(\Delta\alpha_1|\dot{x}| + \Delta\alpha_2|x| + \Delta\beta|\ddot{r} + c\dot{r} - c\dot{x}| + D|x|)\mathrm{sgn}(s) \tag{9}$$

where

$$\mathrm{sgn}(s) = \begin{cases} +1, & s > 0, \\ -1, & s < 0. \end{cases} \tag{10}$$

System (2) is said to be in "sliding mode" when the sliding surface $s \equiv 0$ is reached. It can be reached in finite time if

$$s\dot{s} < 0 \tag{11}$$

holds for all time $t > 0$.

Taking derivative of (4) yields

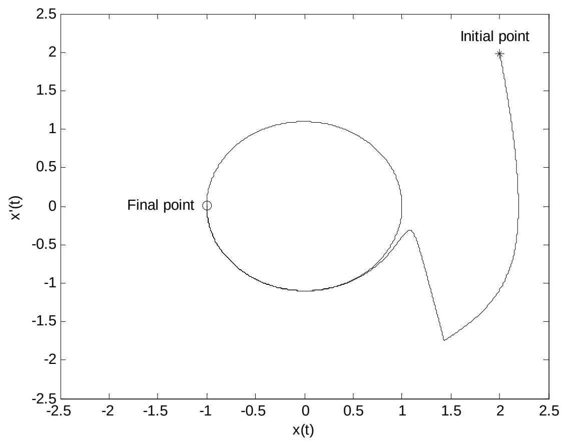

Fig. 3. The phase-plane plot of controlled Duffing-Holmes chaotic system.

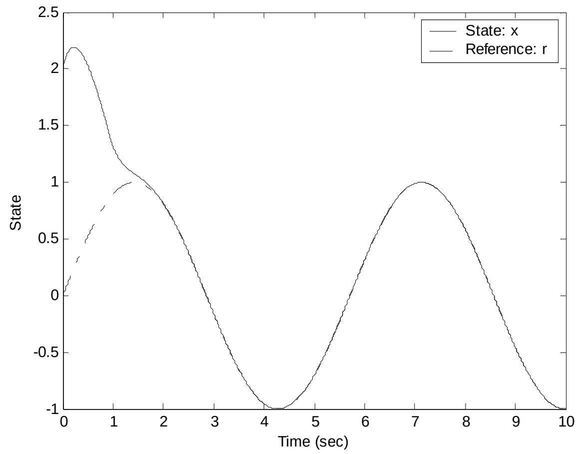

Fig. 4. The time response of the state.

$$\dot{s} = b(t)u + (c - a_1(t))\dot{x} - a_2(t)x - \ddot{r} - c\dot{r} + b(t)d(t,x) \,. \quad (12)$$

Substituting (8)-(10) into the equation of (12) and multiplying with s yields

$$s\dot{s} = b(t)[s(\hat{\alpha}_1 - \hat{\beta}c)\dot{x} + s\hat{\alpha}_2 x + s\hat{\beta}\ddot{r} + s\hat{\beta}c\dot{r}$$

$$-(\Delta\alpha_1|\dot{x}| + \Delta\alpha_2|x| + \Delta\beta|\ddot{r} + c\dot{r} - c\dot{x}| + D|x|)|s|]$$

$$+s[(c - a_1(t))\dot{x} - a_2(t)x - \ddot{r} - c\dot{r} + b(t)d(t,x)] \,,$$

$$\leq |b(t)||s|\Big[|\hat{\alpha}_1 - \Delta\alpha_1||\dot{x}| + |\hat{\alpha}_2 - \Delta\alpha_2||x|$$

$$+|\hat{\beta} - \Delta\beta||\ddot{r} + c\dot{r} - c\dot{x}| - D|x|\Big] \; + |s|\Big[-|a_1(t)||\dot{x}|$$

$$-|a_2(t)||x| - |\ddot{r} + c\dot{r} - c\dot{x}| + |b(t)||d(t,x)|\Big]$$

$$= |b(t)||s|(|d(t,x)| - D|x|) \,,$$

$$< 0 \,. \quad (13)$$

Thus, the control law given by (8)-(10) guarantees the reaching and sustaining of the sliding mode.

In general, the inherent high-frequency chattering of the control input may limit the practical application of the developed method. We further replace $\operatorname{sgn}(s)$ in (9) by the function $\operatorname{sat}(\frac{s}{\delta})$,

$$\operatorname{sat}(\frac{s}{\delta}) = \begin{cases} 1, & \frac{s}{\delta} \geq 1, \\ \frac{s}{\delta}, & -1 < \frac{s}{\delta} < 1, \\ -1, & \frac{s}{\delta} \leq -1, \end{cases} \quad (14)$$

where δ is the width of the boundary layer. With this replacement, the sliding surface function s with an arbitrary initial value will reach and stay within the boundary layer $|s| \leq \delta$.

III. Simulation Results

To verify the proposed method, the following uncertain Duffing-Holmes chaotic system [16] is considered,

$$\ddot{x} = -p_1\dot{x} - p_2 x + x^3 - q\cos(w_1 t) + d + f + u \,. \quad (15)$$

Assume that the parameter uncertainty f and disturbance d satisfy $|f| \leq 0.1|x|$ and $|d| \leq 0.2$, respectively. The sampling time is 0.001 sec. The initial condition is $x(0) = \dot{x}(0) = 2$. The aim here is to control the uncertain Duffing-Holmes chaotic system such that system trajectory follows a prescribed trajectory $r = \sin(1.1t)$. According to (8), (9) and (14), the control law is chosen to be

$$u = -2.75\dot{x} - x + \ddot{r} + 3\dot{r} \; -15sat(s/0.01) \,, \quad (15)$$

where the sliding surface function is $s = \dot{e} + 3e$.

Figures 2 to 6 show the simulation results. It is obvious that the proposed robust sliding mode control can effectively reduce input chattering as shown in Fig. 2. The trajectory of the system in the phase-plane is shown in Fig. 3. The state tracking response is shown in Fig. 4. The tracking performance is smooth and satisfactory comparing with Fig. 1. The time response of the error signal converges to zero as shown in Fig. 5. Further, as shown in Fig. 6, the sliding surface function using the proposed robust SMC does not chatter in the sliding mode.

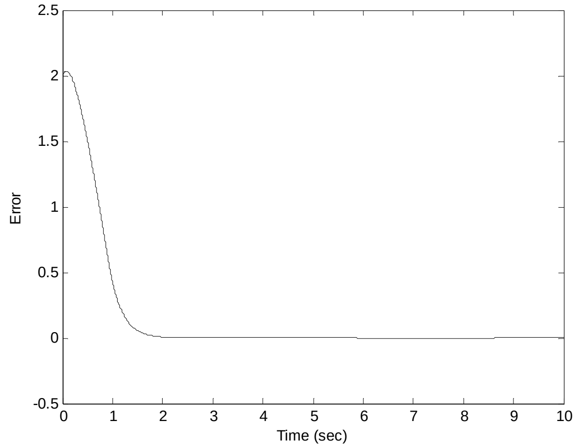

Fig. 5. The time response of the trajectory error.

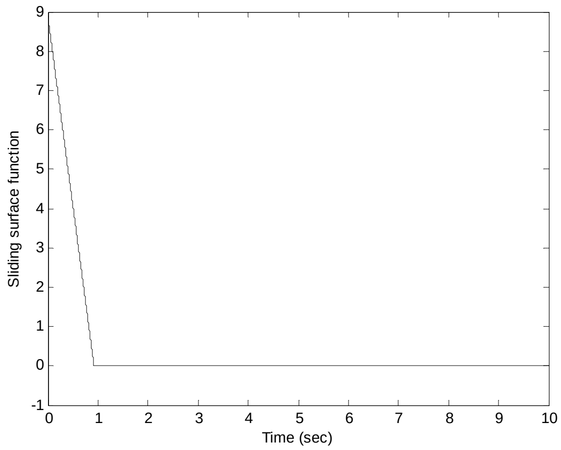

Fig. 6. The time response of the sliding surface function.

IV. CONCLUSIONS

In this paper, a schematic robust suppression sliding mode control design for chaotic systems is proposed. The control law consists of a continuous nominal control part and a discontinuous switching control input. The high frequency chattering in the control input is eliminated. System stability is assured. The uncertain Duffing-Holmes chaotic system is investigated. The advantages of the proposed method are the good tracking performance, insensitive to uncertainties, simple design procedure, and effectiveness in eliminating the input chattering. Therefore, this method can be easily applied to many mechanical systems.

REFERENCES

[1] G. Bartolini, A. Pisano, E. Punta, and E. Usai, "A survey of applications of second order sliding mode control to mechanical systems," *Int. J. Control*, vol. 76, 2003, pp. 875-892.

[2] Y. J. Huang, "Discrete fuzzy variable structure control for pantograph position control," *Electr. Eng.*, vol. 86, 2004, pp. 171-177.

[3] Y. J. Huang and T. C. Kuo, "Robust position control of DC servomechanism with output measurement noise," *Electr. Eng.*, vol. 88, 2006, pp. 223-238.

[4] J. Y. Hung, W. Gao, and J. C. Hung, "Variable structure control: a survey," *IEEE Trans. Ind. Electr.*, vol. 40, 1993, pp. 2-22.

[5] K. D. Young, V. I. Utkin, and Ü. Özgüner, "A control engineer's guide to sliding mode control," *IEEE Trans. Control Sys. Tech.*, vol. 7, 1999, pp. 328-342.

[6] A. S. I. Zinober, *Variable Structure and Lyapnuov Control*. Berlin: Springer-Verlag, 1994.

[7] G. Chen, *Controlling Chaos and Bifurcations in Engineering Systems.* Boca Raton: CRC Press, 1999.

[8] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," *Physical Review Letters*, vol. 64, 1990, pp. 1196-1199.

[9] T. Shinbort, E. Ott, N. Grebogi, and J. A. Yorke, "Using chaos to direct trajectories to target," *Physical Review Letters*, vol. 65, 1990, pp. 3215-3218.

[10] M. Ramesh and S. Narayanan, "Chaos control of Bonhoeffer-van der Pol oscillator using neural networks," *Chaos Solitions & Fractals*, vol. 12, 2001, pp. 2395-2405.

[11] H. Guo, S. Lin, and J. Liu, "A radial basis function sliding mode controller for chaotic Lorenz system," *Physics Letters A*, vol. 351, 2006, pp. 257-261.

[12] L. Udawatta, K. Watanabe, K. Kiguchi, and K. Izumi, "Fuzzy-chaos hybrid controller for controlling of nonlinear systems," *IEEE Trans. Fuzzy Sys.*, vol. 10, 2002, pp. 401-411.

[13] J. X. Xu, Y. J. Pan, and T. H. Lee, "Sliding mode control with closed-loop filtering architecture for a class of nonlinear systems," *IEEE Trans. Circ. Sys. II: Exp. Briefs*, vol. 51, 2004, pp. 168-173.

[14] J. J. Slotine and S. S. Sastry, "Tracking control of non-linear system using sliding surfaces with application to robot manipulators," *Int. J. Control*, vol. 38, 1983, pp. 465-492.

[15] G. Duffing, *Enwungene Schwingungen bei Verinderlickr Eigenfiequenz*. Germany: Braunschweig, 1918.

[16] J. J. Yan, "Design of robust controllers for uncertain chaotic systems with nonlinear inputs," *Chaos Solitions & Fractals*, vol. 19, 2004, pp. 541-547.

T. C. Kuo received the B.Sc. degree, M.Sc. degree, and Ph. D. in Electrical Engineering from Yuan Ze University, Taiwan, R.O.C., in 1998, 2000 and 2004, respectively. She has won the Yu-Ziang Scholarship, the top performance scholarship, five times. From February 2004 to August 2004, she served as Assistant Professor with the Department of Electrical Engineering, Yuan Ze University, Taiwan, R.O.C. She is currently an Assistant Professor with the Department of Electrical Engineering, Ching Yun University, Taiwan, R.O.C. Her research interests include intelligent control, automatic control, robotic control, and robust control.

Y. J. Huang received the B.Sc. degree and M.Sc. degree in Control Engineering from National Chiao Tung University, Taiwan, R.O.C., in 1981 and 1983 respectively, and Ph. D. in Electrical Engineering from University of Texas at Arlington, U.S.A., in 1990. Currently, he serves as Full Professor with the Department of Electrical Engineering, Yuan Ze University, Taiwan, R.O.C. He is also the Director of Robotics Research Lab and Control Engineering Lab. Professor Huang has published over one hundred research papers and seven text books. He owns seven patents. He also co-founded two companies in industry and conducted many research projects. He has obtained Outstanding Research Award and Outstanding Service Award many times from Yuan Ze University, and many medals from nationwide robotics competition. His research interests include robotics, intelligent control, security and monitoring technology, and unmanned autonomous vehicle.

C. H. Chang received the B.Sc. degree in Electrical Engineering from Chung-Yuan University, Taiwan, R.O.C., in 1985, and M.Sc. degree in Electrical Engineering from Yuan Ze University, Taiwan, R.O.C., Currently, he is a doctoral candidate in Yuan Ze Univeristy and a manger in the R&D Department of Tung-Thih Electric Company.

C. Y. Chen received the M.Sc. degree in Electrical Engineering from Yuan Ze University, Taiwan, R.O.C., in 2002. Currently, he is pursuing Ph.D. in Yuan Ze Univeristy. His research interests include control theory and application, motor drives, and power electronics.