

# Application of TLM and Cassie-Mayr Arc model on Transformer Aging and Incipient Faults Simulation

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*Abstract*—The development of the transformer insulation failure undergoes three stages: insulation aging, incipient faults and a short circuit. This paper presents a complete scheme to simulate single-phase transformers with insulation deterioration and arcing phenomena i.e. in the first two stages. The approach incorporates Transmission Line Methods (TLM), Jiles-Atherton Hysteretic Model, Composite Cassie-Mayr Arcing Model and Dielectric Model. A small 25kVA 11kV/220V power transformer with aging and incipient faults is taken as example for simulation.

*Keywords:* Deterioration, Incipient Fault, Insulation Aging, Transformer, Transmission Line Methods (TLM)

## 1 Introduction

One of the most important electrical units in power system is transformer, the stability of which is significant for the reliability of the whole supply. Therefore various protection and monitoring schemes were developed in the last few decades. By now the differential relay that depends on the current difference to trigger the execution units is widely applied in the transformer protection against internal faults[1]. With the aspect of transformer monitoring, lots of practical experience is accumulated during the periodical preventative experiments, while the state-based maintenance and hence online monitoring are developing rapidly. Although can not directly reflect the remaining life of the transformer, dissolved gas and partial discharge are still the most common monitored items under the major circumstance. In fact each scheme can be regarded as a recognition procedure, the key issue of which is how to effectively distinguish the major faults arising within transformers.

A survey of the modern transformer breakdowns, which took place over a period of years, showed that 70%-80% of faults could be attributed to the failure of the internal

insulation between winding turns[2]. As a result, such internal insulation faults inevitably become the main investigated subjects. Actually the development from a perfect condition to a complete breakdown undergoes several stages, which are insulation aging, incipient fault and a short circuit. After a transformer is installed on a site and because of some electrical, thermal or chemical effects, the internal insulation always weakens although it may develop very slowly. This deterioration is called aging, the main feature of which is the higher leakage current flowing through the insulation dielectric than the perfect condition. When the insulation degrades further, incipient faults appear in the form of some intermittent arcs within the insulation dielectric material. In such a case, if the transformer does not stop operating, the incipient faults will eventually turn into a permanent inter-turn short circuit that can cause serious damage and an outage. Since most faults mentioned above are destructive, it's desirable to undertake an accurate simulation to facilitate analysis and distinguish the characteristics of different stages in the deterioration of the insulation before the laboratory experiment.

D.J.Wilcox introduced a time-domain modal analysis which described how a transformer model could be converted from the frequency domain into the time domain for ATP/EMTP implementation[3], but it did not consider the existence of internal faults. For the transformer with internal faults, Patrick presented the matrix model that can be easily obtained by the calculation on the inductance of the healthy transformer, so that it can be simulated by EMTP[4]. But some idealized assumptions on which Patrick's model is based may affect its results. Hang Wang and Karen L. Butler proposed that the simulation results be acquired from the standpoint of electromagnetic fields with the assistance of finite element softwares e.g. ANSOFT's Maxwell[5]. The effects of insulation aging and incipient faults were exhibited by the parallel connection of a constant voltage source with an increasing resistance. The validity of this method was confirmed by experiments. However on the other hand, the application of more sophisticated Composite Cassie-Mayr theory in modelling arc has proved to be more successful[6][7].

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So following the authors' previous work on simulation of internal short circuit faults[8], a method, which incorporates Transmission Line Methods (TLM), Jiles-Atherton model for magnetic hysteresis and Composite Cassie-Mayr description of arc, is presented in this paper.

## 2 Transformer Model with Aging and Incipient Fault

A single-phase two-winding transformer impedance is represented by two matrices  $[R]$  and  $[L]$  as follows, where suffixes  $p$  and  $s$  are for the primary and the secondary respectively;  $R_i$  and  $L_{ii}$  are the resistance and the self inductance of winding  $i$ ;  $M_{ij}$  is the mutual inductance between winding  $i$  and  $j$ .

$$[R] = \begin{bmatrix} R_p & 0 \\ 0 & R_s \end{bmatrix} \quad [L] = \begin{bmatrix} L_{pp} & M_{ps} \\ M_{sp} & L_{ss} \end{bmatrix} \quad (1)$$

The terminal voltages and currents are then related by

$$\begin{bmatrix} u_p \\ u_s \end{bmatrix} = [R] \begin{bmatrix} i_p \\ i_s \end{bmatrix} + [L] \frac{d}{dt} \begin{bmatrix} i_p \\ i_s \end{bmatrix} \quad (2)$$

where  $u_i$  is the voltage and  $i_i$  is the current of winding  $i$ .

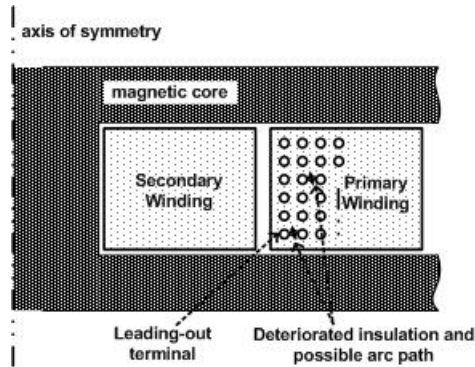


Figure 1: Diagram of deteriorated insulation and possible arcing path

After the transformer operates for a period, the insulation within the protection zone inevitably deteriorates. For a shell type transformer, the weakest insulation is usually at the locations between two adjacent layers of the winding. So the effects of aging can be severe at these points. For example, the severest deterioration may take place in the primary as shown in Figure.1 (marked points). And from the fault location, the primary winding may be divided to two (Figure.2(a)) or three sub-windings (Figure.2(b)).

Accordingly the matrices  $[R]$  and  $[L]$  in the expression (1) are converted to the matrices in the expression (3) for the configuration in Figure.2(a) and those in the expression (5) for the configuration in Figure.2(b)

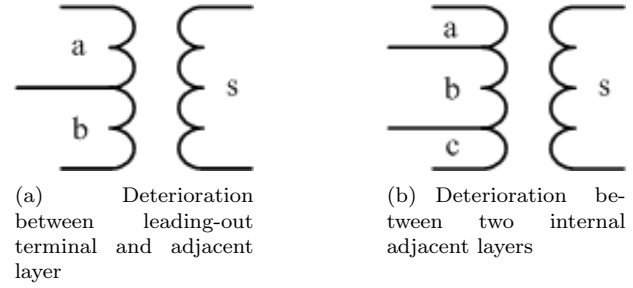


Figure 2: Diagram of sub-windings in transformer with insulation deterioration in the primary

$$[R] = \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_s \end{bmatrix} \quad [L] = \begin{bmatrix} L_{aa} & M_{ab} & M_{as} \\ M_{ba} & L_{bb} & M_{bs} \\ M_{sa} & M_{sb} & L_{ss} \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} u_a \\ u_b \\ u_s \end{bmatrix} = [R] \begin{bmatrix} i_a \\ i_b \\ i_s \end{bmatrix} + [L] \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_s \end{bmatrix} \quad (4)$$

$$[R] = \begin{bmatrix} R_a & 0 & 0 & 0 \\ 0 & R_b & 0 & 0 \\ 0 & 0 & R_c & 0 \\ 0 & 0 & 0 & R_s \end{bmatrix}$$

$$[L] = \begin{bmatrix} L_{aa} & M_{ab} & M_{ac} & M_{as} \\ M_{ba} & L_{bb} & M_{bc} & M_{bs} \\ M_{ca} & M_{cb} & L_{cc} & M_{cs} \\ M_{sa} & M_{sb} & M_{sc} & L_{ss} \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_s \end{bmatrix} = [R] \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_s \end{bmatrix} + [L] \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_s \end{bmatrix} \quad (6)$$

### 2.1 Aging Model

Traditionally the low frequency behavior of dielectric material can be represented in terms of an equivalent parallel circuit as shown in Figure.3, where  $u_d$ ,  $i_d$ ,  $R_d$  and  $C_d$  are applied voltage, leakage current through dielectric, insulation resistance and parasite capacitance respectively. According to the literature[5] and experimental results, the equivalent capacitance  $C_d$  changes little during the deterioration of insulation. Resistance  $R_d$  is large in the case of perfect insulation, while it decreases significantly with the effect of aging.

For considering the aging condition of the dielectric, the equivalent circuit in Figure.2 becomes as given in Figure.4. And the additional expression is generated ( $u_d = u_b$ ).

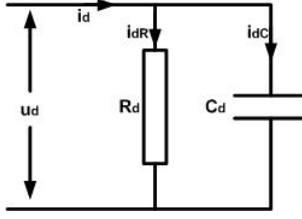
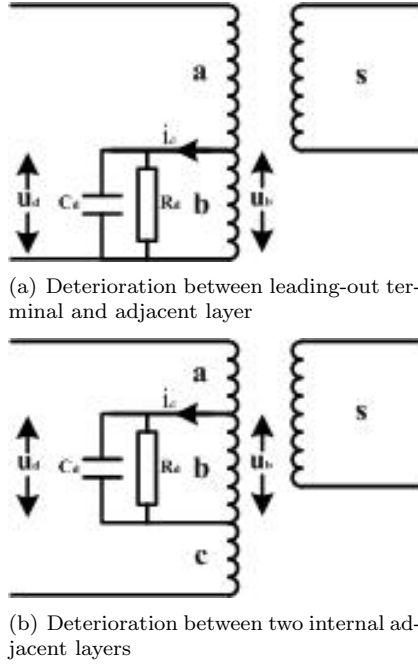


Figure 3: Equivalent circuit of insulation dielectric



(a) Deterioration between leading-out terminal and adjacent layer

(b) Deterioration between two internal adjacent layers

Figure 4: Diagram of transformer with aging model in the primary

$$i_d = i_a - i_b = \frac{1}{R_d} u_b + C_d \frac{du_b}{dt} \quad (7)$$

## 2.2 Arc Model

When the insulation degrades further, some intermittent arcs i.e. incipient faults begin to persist within the insulation. The process of arcing can be divided into extinction stage and recovery stage. The former one lasts a few microseconds from the beginning of the arcing to the moment when the arc current reaches zero, while the latter one follows in time.

As two special cases of black box arc models, Cassie and Mayr arc model have the general forms[9] given in the equations (8):

$$\begin{aligned} \frac{du}{dt} &= \Phi(u, i) \frac{di}{dt} - uF(u, i) \\ \Phi(u, i) &= \frac{u}{i} \\ F_M(u, i) &= \frac{1}{\theta} \left( \frac{ui}{N_0} - 1 \right) \text{ for Mayr Model} \end{aligned}$$

$$F_C(u, i) = \frac{1}{\theta} \left( \frac{u^2}{E_0} - 1 \right) \text{ for Cassie Model} \quad (8)$$

where  $\theta$  represents the arc time constant,  $u$  and  $i$  are the arc voltage and current respectively.

Then the composite Cassie-Mayr method models arcing phenomena by modifying the value of the arc resistance i.e. insulation resistance  $R_d$  in Figure.4. When the voltage over the resistance  $R_d$  is larger than the constant arc voltage  $E_0$  in case of steady state, the extinction stage begins. During this stage, the resistance is dominated by the Cassie's equation with the initial value of  $R_d$  being the one in the aging model:

$$\frac{1}{R_d} \frac{dR_d}{dt} = \frac{1}{\theta} \left( 1 - \frac{u_d^2}{E_0^2} \right) \quad (9)$$

After the current  $i_{dR}$  decreases to zero, the arc is completely extinguished. Then the recovery stage begins which can be represented by the Mayr's equation.

$$\frac{1}{R_d} \frac{dR_d}{dt} = \frac{1}{\theta} \left( 1 - \frac{u_d i_{dR}}{N_0} \right) \quad (10)$$

Where  $N_0 = E_0 I \omega \theta$ ,  $I$  is the rms value of the interrupted current in ampere. If the arc resistance  $R_d$  computed by Mayr's equation starts to decrease, indicating reignition, Cassie's differential equation (9) is used with a new value  $E_0 = \sqrt{N_0 R_m}$  from the point the arc resistance has reached the maximum value  $R_m$ . If the arc resistance  $R_d$  keeps increasing until it reaches the value before arc starts, then the entire arcing process ends. And after this point, the aging model returns for use.

## 2.3 Hysteresis Model

For modelling nonlinear hysteretic cores the approach applied here is to use the Jiles-Atherton model (J-A model) [10] to introduce hysteretic behavior. Based on the constitutive relationship for the flux density  $B = \mu_0(H + M)$ , where  $\mu_0$  is the permeability of free space,  $H$  is the magnetic flux density and  $M$  is the magnetising intensity, each inductance in the matrices (3) and (5) are decomposed.

Equations (4) and (6) including non-linear inductance are given in equations (11) and (12) respectively.

$$\begin{aligned} \begin{bmatrix} u_a \\ u_b \\ u_s \end{bmatrix} &= \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_s \end{bmatrix} \\ &+ \begin{bmatrix} L'_{aa} & M'_{ab} & M'_{as} & N_a L_m \\ M'_{ba} & L'_{bb} & M'_{bs} & N_b L_m \\ M'_{sa} & M'_{sb} & L'_{ss} & N_s L_m \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_s \\ i_m \end{bmatrix} \end{aligned} \quad (11)$$

$$\begin{bmatrix} u_a \\ u_b \\ u_c \\ u_s \end{bmatrix} = \begin{bmatrix} R_a & 0 & 0 & 0 \\ 0 & R_b & 0 & 0 \\ 0 & 0 & R_c & 0 \\ 0 & 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_s \end{bmatrix} + \begin{bmatrix} L'_{aa} & M'_{ab} & M'_{ac} & M'_{as} & N_a L_m \\ M'_{ba} & L'_{bb} & M'_{bc} & M'_{bs} & N_b L_m \\ M'_{ca} & M'_{cb} & L'_{cc} & M'_{cs} & N_c L_m \\ M'_{sa} & M'_{sb} & M'_{sc} & L'_{ss} & N_s L_m \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_m \end{bmatrix} \quad (12)$$

where  $i_m$  is the normalized magnetization given by  $i_m = Ml$  and  $L'_{ii}/M'_{ij}$  are self/mutual inductance in the situation where the transformer core is removed and replaced by air.  $L'_{ii}/M'_{ij}$  and  $L_m$  can either be deduced from the expressions for impedances given by Wilcox et. al [11][12] or approximated by

$$M'_{ij} = \frac{\mu_0 N_i N_j A}{l} \quad (13)$$

$$L_m = \frac{\mu_0 A}{l} \quad (14)$$

The classical J-A model [10][13] is described in the following subsections:

### 2.3.1 Weighting coefficient

The magnetization is split into two parts, the anhysteretic magnetization and the irreversible magnetization. In normalized form, this is expressed by

$$i_m = \beta_c i_{an} + (1 - \beta_c) i_{irr} \quad (15)$$

where  $\beta_c$  is the weighting coefficient with  $0 \leq \beta_c \leq 1$ ,  $i_{an}$  is the normalized anhysteretic magnetization and  $i_{irr}$  is the normalized irreversible magnetization.

### 2.3.2 Modified langevin function

The anhysteretic magnetization dependence is given by a modified Langevin function, i.e.

$$i_{an} = i_{sat} \left[ \coth \left( \frac{i_L + \alpha i_m}{i_{aht}} \right) - \frac{i_{aht}}{i_L + \alpha i_m} \right] = i_{sat} L(\gamma) \quad (16)$$

where  $i_L$  is the ampere-turn sum of all exciting currents,  $i_{sat}$  is the normalized saturation magnetization,  $\alpha$  is the interdomain coupling coefficient and  $i_{aht}$  is the normalized anhysteretic magnetization form factor. The coefficients  $i_{sat}$ ,  $\alpha$ ,  $i_{aht}$  are positive constants. Also  $L(\gamma)$

denotes the modified Langevin function with argument  $\gamma = \frac{i_L + \alpha i_m}{i_{aht}}$ . To avoid difficulties with the modified Langevin function for small arguments, a linear approximation is used where for  $|\gamma| < 0.001$  we put  $L(\gamma) \approx \frac{\gamma}{3}$ .

### 2.3.3 Differential equation for the irreversible magnetization

In the Jiles-Atherton model, the derivative of the normalized irreversible magnetization with respect to the inductor current is

$$\frac{di_{irr}}{di_L} = \left[ \frac{\delta_m (i_{an} - i_{irr})}{\delta i_{coe} - \alpha (i_{an} - i_{irr})} \right] \quad (17)$$

where the migration flag  $\delta_m$  is given by:

$$\delta_m = \begin{cases} 1 & : \text{if } \frac{di_L}{dt} > 0 \text{ and } i_{an} > i_{irr} \\ 1 & : \text{if } \frac{di_L}{dt} < 0 \text{ and } i_{an} < i_{irr} \\ 0 & : \text{otherwise} \end{cases} \quad (18)$$

## 3 TLM Modelling

The transformer equations are solved using the time domain TLM method as described in [14]. In TLM an inductor element is represented by a short circuited transmission line stub as given in Figure. 5, which is reduced to a serial voltage source and surge impedance as shown. At each time step  $n$  the following equations are solved.

$$u_n = Z_L i_n + 2u_n^i \quad (19)$$

$$u_n = u_n^i + u_n^r \quad (20)$$

$$u_{n+1}^i = -u_n^r \quad (21)$$

where  $Z_L = \frac{2L}{\Delta t}$ ;  $\Delta t$  is the time length of each step; the suffix  $n$  stands for the  $n$ th time step;  $u_n^i$  is the incident voltage in  $n$ th time step;  $u_n^r$  is the reflected voltage in  $n$ th time step.

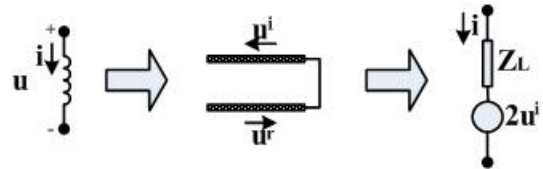


Figure 5: Representation of Inductor in TLM

Similarly a capacitor element can be described by an open circuited transmission line stub as given in Figure.6 and the iterative equations are

$$u_n = Z_C i_n + 2u_n^i \quad (22)$$

$$u_n = u_n^i + u_n^r \quad (23)$$

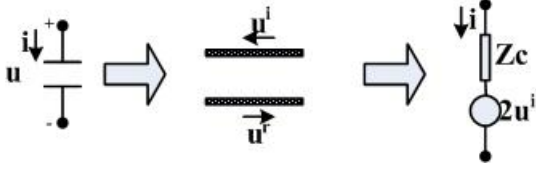


Figure 6: Representation of Capacitor in TLM

$$u_{n+1}^i = u_n^r \quad (24)$$

where  $Z_C = \frac{\Delta t}{2C}$

Therefore for the simulation of an aging or arcing transformer with a nonlinear hysteretic core and with the terminal relationships as turn to earth fault as given by equation (11) and Figure.4(a): the self impedances (inductances) can be modelled as given by equations (25), the controlled sources representing mutual terms of the type  $M'_{ij} \frac{di_j}{dt}$  are given by equation (26) and the capacitor  $C_d$  is represented by equation (27) as follows:

$$u_{ii} = Z_{ii}i_i + 2u_{ii}^i \quad (25)$$

$$u_{ij} = Z_{ij}i_j + 2u_{ij}^i \quad (26)$$

$$u_d = Z_d i_{dc} + 2u_d^i \quad (27)$$

where  $Z_{ii} = \frac{2L'_{ii}}{\Delta t}$ ;  $Z_{ij} = \frac{2M'_{ij}}{\Delta t}$ ;  $Z_d = \frac{\Delta t}{2C_d}$  and there is a term representing the magnetization  $L_m \frac{di_m}{dt}$  given by

$$u_m = Z_m i_m + 2u_m^i \quad (28)$$

where  $Z_m = \frac{2L_m}{\Delta t}$

The magnetization  $i_m$  is non-linear so that an iterative solution for the following simultaneous equations has to be found.

$$\begin{aligned} f_1 &= (Z_{aa} + R_a)i_a + Z_{ab}i_b + Z_{as}i_s \\ &+ (Z_d \parallel R_d)(i_a - i_b) + N_a Z_m i_m - u_{src} \\ &+ 2(u_{aa}^i + u_{ab}^i + u_{as}^i + \frac{u_d^i R_d}{R_d + Z_d} + N_a u_m^i) = 0 \end{aligned} \quad (29)$$

$$\begin{aligned} f_2 &= Z_{ba}i_a + Z_{bb}i_b + Z_{bs}i_s \\ &- (Z_d \parallel R_d)(i_a - i_b) + N_b Z_m i_m \\ &+ 2(u_{ba}^i + u_{bb}^i + u_{bs}^i - \frac{u_d^i R_d}{R_d + Z_d} + N_b u_m^i) = 0 \end{aligned} \quad (30)$$

$$\begin{aligned} f_3 &= Z_{sa}i_a + Z_{sb}i_b + (Z_{ss} + R_s + Z_{load} + R_{load})i_s \\ &+ N_s Z_m i_m + 2(u_{sa}^i + u_{sb}^i + u_{ss}^i + u_{load}^i + N_s u_m^i) = 0 \end{aligned} \quad (31)$$

We have chosen the Newton-Raphson technique for its efficiency and stability so the solution is found through

the following iterative procedure

$$\begin{bmatrix} i_a \\ i_b \\ i_s \end{bmatrix}_{p+1} = \begin{bmatrix} i_a \\ i_b \\ i_s \end{bmatrix}_p - \begin{bmatrix} \frac{\partial f_1}{\partial I_a} & \frac{\partial f_1}{\partial I_b} & \frac{\partial f_1}{\partial I_s} \\ \frac{\partial f_2}{\partial I_a} & \frac{\partial f_2}{\partial I_b} & \frac{\partial f_2}{\partial I_s} \\ \frac{\partial f_3}{\partial I_a} & \frac{\partial f_3}{\partial I_b} & \frac{\partial f_3}{\partial I_s} \end{bmatrix}_p^{-1} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}_p \quad (32)$$

where  $p$  is the iteration number and

The iteration is started with initial values taken from the TLM previous time step and continued until suitable convergence criteria are met. In this work this is set as

$$\begin{aligned} |(i_a)_{p+1} - (i_a)_p| &< \tau \text{ and } |(i_b)_{p+1} - (i_b)_p| < \tau \\ \text{and } |(i_s)_{p+1} - (i_s)_p| &< \tau \end{aligned}$$

Similarly for the simulation of transformer with turn to turn fault governed by equation (12) and Figure.4(b)

$$\begin{aligned} f_1 &= (Z_{aa} + Z_{ac} + R_a + Z_{ca} + Z_{cc} + R_c)i_a \\ &+ (Z_{ab} + Z_{cb})i_b + (Z_{as} + Z_{cs})i_s \\ &+ (Z_d \parallel R_d)(i_a - i_b) + (N_a + N_c)Z_m i_m \\ &- u_{src} + 2(u_{aa}^i + u_{ab}^i + u_{ac}^i + u_{as}^i + u_{ca}^i + u_{cb}^i \\ &+ u_{cc}^i + u_{cs}^i + \frac{u_d^i R_d}{R_d + Z_d} + (N_a + N_c)u_m^i) = 0 \end{aligned} \quad (33)$$

$$\begin{aligned} f_2 &= (Z_{ba} + Z_{bc})i_a + Z_{bb}i_b + Z_{bs}i_s \\ &- (Z_d \parallel R_d)(i_a - i_b) + N_b Z_m i_m \\ &+ 2(u_{ba}^i + u_{bb}^i + u_{bc}^i + u_{bs}^i - \frac{u_d^i R_d}{R_d + Z_d} + N_b u_m^i) = 0 \end{aligned} \quad (34)$$

$$\begin{aligned} f_3 &= (Z_{sa} + Z_{sc})i_a + Z_{sb}i_b \\ &+ (Z_{ss} + R_s + Z_{load} + R_{load})i_s + N_s Z_m i_m \\ &+ 2(u_{sa}^i + u_{sb}^i + u_{sc}^i + u_{ss}^i + u_{load}^i + N_s u_m^i) = 0 \end{aligned} \quad (35)$$

Note that  $i_a = i_c$  and hence only  $i_a$  is required to be solved

## 4 Simulation Results

A small 25kVA, 11kV/220V power transformer with the geometry given in Fig. 7 is modelled so as to demonstrate the modelling procedure. The Jiles-Atherton parameters are typical of a core made of FeSi sheets [15].

Figure.8 and Figure.9 show some simulation results of the transformer with deterioration insulation in the primary winding. Parameters of the aging model and Cassie-Mayr arc model are as follows:  $R_d = 1400\Omega$ ;  $C_d = 6.06\mu F$ ;  $E_0 = 397.22V$ ;  $\theta = 0.1ms$ . During the time section from 0.5s to 0.55s, three arcs take place and some characteristics of arcs can be observed e.g. the arc voltage is almost a flat-top waveform as shown in Figure.8(d) and Figure.9(c).

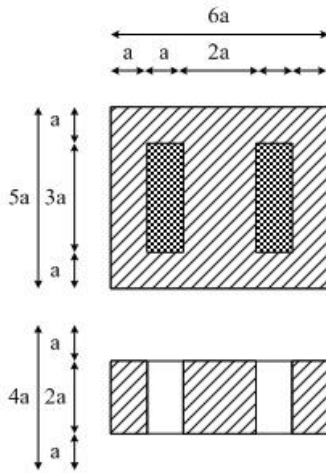


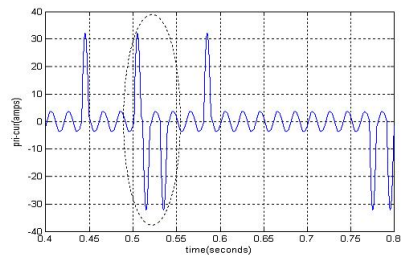
Figure 7: 25kVA 11kV/220V power transformer geometry. Unit  $a = 0.0587375m$  and magnetic path length  $l = 12a$ . Number of turns in primary  $N_p = 2509$ ; Number of turns in secondary  $N_s = 51$ . Jiles-Atherton parameters: Saturation magnetization  $M_s = 1.47 \times 10^6 A/m$ ; Anhysteretic form factor  $H_a = 40.0 A/m$ ; Interdomain coupling coefficient  $\alpha = 0.00008$ ; Coercive field magnitude  $H_c = 60.0 A/m$ ; Magnetization weighting factor  $\beta_c = 0.55$ ; Supply voltage source: resistance  $R_{src} = 1.565 \Omega$ , inductance  $L_{src} = 2.4132 \mu H$

## 5 Conclusion and Future Work

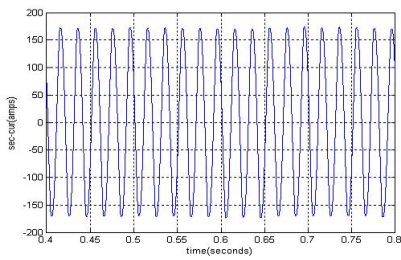
By incorporating Transmission Line Methods (TLM), Jiles-Atherton Hysteretic Model, Composite Cassie-Mayr Arcing Model and Dielectric Model, the simulation results of transformer with aging insulation and incipient faults can be acquired. Although the results seem reasonable, they need to be validated by the experiments.

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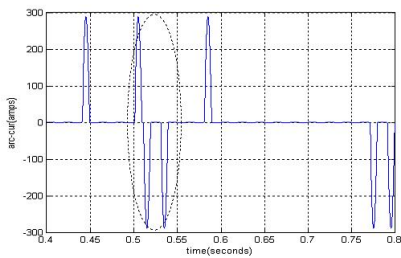
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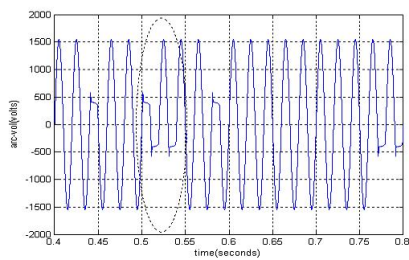
(a)



(b)

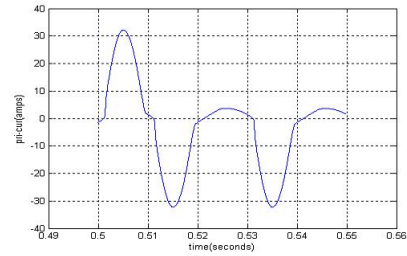


(c)

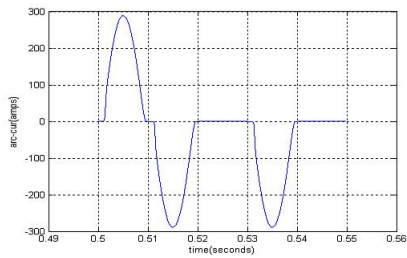


(d)

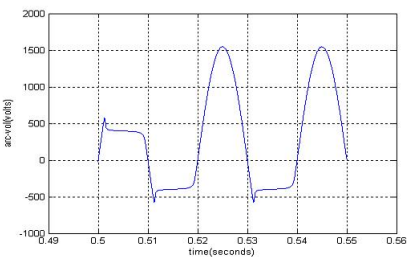
Figure 8: Terminal currents and arc voltage/current for the transformer with deterioration/aging insulation and some incipient faults between 125th and 375th turns on the primary winding. (a) Primary Current. (b) Secondary Current. (c) Arc Current. (d) Arc Voltage.



(a)



(b)



(c)

Figure 9: Zoomed in details of Figure.8 in dot circle (a) Primary Current. (b) Arc Current. (c) Arc Voltage.