Interval Type-2 Fuzzy Logic Toolbox

Juan R. Castro, Oscar Castillo, Luis G. Martínez

***Abstract*—This paper presents the development and design of a graphical user interface and a command line programming toolbox for construction, edition and observation of Interval Type-2 Fuzzy Inference Systems. The Interval Type-2 Fuzzy Logic System Toolbox (IT2FLS), is an environment for interval type-2 fuzzy logic inference system development. Tools that cover the different phases of the fuzzy system design process, from the initial description phase, to the final implementation phase, build the Toolbox. The Toolbox's best qualities are the capacity to develop complex systems and the flexibility that permits the user to extend the availability of functions for working with the use of type-2 fuzzy operators, linguistic variables, interval type-2 membership functions, defuzzification methods and the evaluation of Interval Type-2 Fuzzy Inference Systems.**

***Index Terms*— Interval Type-2 Fuzzy Inference Systems, Interval Type-2 Fuzzy Logic Toolbox, Interval Type-2 Membership Functions, Footprint of Uncertainty.**

I. INTRODUCTION

On the past decade, fuzzy systems have displaced conventional technology in different scientific and system engineering applications, especially in pattern recognition and control systems. The same fuzzy technology, in approximation reasoning form, is resurging also in the information technology, where it is now giving support to decision making and expert systems with powerful reasoning capacity and a limited quantity of rules.

The fuzzy sets were presented by L.A. Zadeh in 1965 [1,2] to process / manipulate data and information affected by unprobabilistic uncertainty / imprecision. These were designed to mathematically represent the vagueness and uncertainty of linguistic problems; thereby obtaining formal tools to work with intrinsic imprecision in different type of problems; it is considered a generalization of the classic set theory.

Intelligent Systems based on fuzzy logic are fundamental tools for nonlinear complex system modeling. The fuzzy sets and fuzzy logic are the base for fuzzy systems, where their objective has been to model how the brain manipulates inexact information.

Type-2 fuzzy sets are used for modeling uncertainty and imprecision in a better way. These type-2 fuzzy sets were originally presented by Zadeh in 1975 and are essentially "fuzzy fuzzy" sets where the fuzzy degree of membership is a type-1 fuzzy set [4,6]. The new concepts were introduced by Mendel and Liang [8,10] allowing the characterization of a type-2 fuzzy set with a superior membership function and an inferior membership function; these two functions can be represented each one by a type-1 fuzzy set membership function. The interval between these two functions represent the footprint of uncertainty (FOU), which is used to characterize a type-2 fuzzy set.

The uncertainty is the imperfection of knowledge about the natural process or natural state. The statistical uncertainty is the randomness or error that comes from different sources as we use it in a statistical methodology.

There are different sources of uncertainty in the evaluation and calculus process. The five types of uncertainty that emerge from the imprecise knowledge natural state are:

- Measurement uncertainty. It is the error on observed quantities.
- Process uncertainty. It is the dynamic randomness.
- Model uncertainty. It is the wrong specification of the model structure.
- Estimate uncertainty. It is the one that can appear from any of the previous uncertainties or a combination of them, and it is called inexactness and imprecision.
- Implementation uncertainty. It is the consequence of the variability that results from sorting politics, i.e. incapacity to reach the exact strategic objective.

II. INTERVAL TYPE-2 FUZZY SET THEORY

A. Type-2 Fuzzy Sets Concept

A type-2 fuzzy set [6,7] expresses the non-deterministic truth degree with imprecision and uncertainty for an element that belongs to a set. A type-2 fuzzy set denoted by $\tilde{\tilde{A}}$, is characterized by a type-2 membership function $\mu_{\tilde{\tilde{A}}}(x,u)$, where x∈X, u $\in J_x^u \subseteq [0,1]$ and $0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1$ defined in equation (1).

$$\tilde{\tilde{A}} = \{(x, \mu_{\tilde{\tilde{A}}}(x)) \mid x \in X\}$$

$$\tilde{\tilde{A}} = \{(x,u,\mu_{\tilde{\tilde{A}}}(x,u)) \mid \forall x \in X, \forall u \in J_x^u \subseteq [0,1]\} \quad (1)$$

An example of a type-2 membership function constructed in the IT2FLS toolbox was composed by a Pi primary and a Gbell secondary type-1 membership functions, these are depicted in Figure 1.

Oscar Castillo is with the Division of Graduate Studies and Research in Tijuana Institute of Technology, Mexico (corresponding author phone: 52664-623-6318; fax: 52664-623-6318; e-mail: ocastillo@tectijuana.mx).

Juan R. Castro is a graduate student in Science and Engineering with the Division of Graduate Studies and Research in Baja California Autonomous University, Tijuana Campus, Mexico (e-mail: jrcastror@uabc.mx).

Luis G. Martinez is a graduate student in Science and Engineering with the Division of Graduate Studies and Research in Baja California Autonomous University, Tijuana Campus, Mexico (e-mail: luisgmo@uabc.mx).

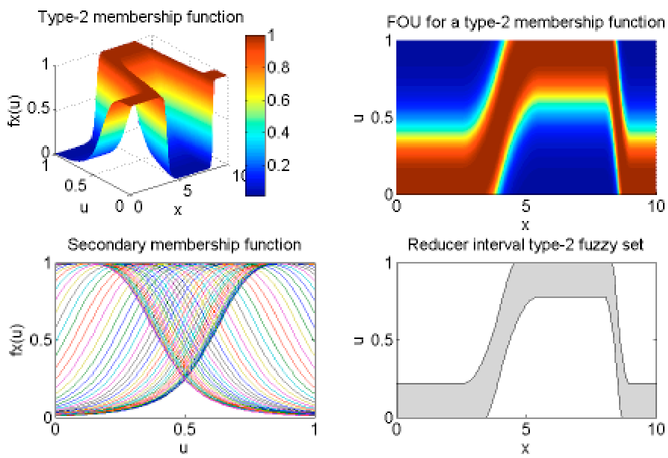

Fig. 1. FOU for Type-2 Membership Functions.

If $\tilde{\tilde{A}}$ is continuous it is denoted in equation (2).

$$\tilde{\tilde{A}} = \left\{ \int_{x\in X} \left[\int_{u\in J_x^u \subseteq [0,1]} f_x(u)/u \right] / x \right\} \tag{2}$$

where $\iint$ denotes the union of x and u. If $\tilde{\tilde{A}}$ is discrete then it is denoted by equation (3).

$$\tilde{\tilde{A}} = \left\{ \sum_{x\in X} \mu_{\tilde{A}}(x)/x \right\} = \left\{ \sum_{i=1}^{N} \left[\sum_{k=1}^{M_i} f_{x_i}(u_{ik})/u_{ik} \right] / x_i \right\} \tag{3}$$

where $\sum\sum$ denotes the union of x and u.

If $f_x(u) = 1, \forall u \in [\underline{J}_x^u, \overline{J}_x^u] \subseteq [0,1]$, the type-2 membership function $\mu_{\tilde{A}}(x,u)$ is expressed by one type-1 inferior membership function, $\underline{J}_x^u \equiv \underline{\mu}_A(x)$ and one type-1 superior, $\overline{J}_x^u \equiv \overline{\mu}_A(x)$ (Fig. 2), then it is called an interval type-2 fuzzy set [8] denoted by equations (4) and (5).

$$\tilde{\tilde{A}} = \begin{cases} (x,u,1) \mid \forall x \in X, \\ \qquad \forall u \in [\underline{\mu}_A(x), \overline{\mu}_A(x)] \subseteq [0,1] \end{cases} \tag{4}$$

or

$$\tilde{\tilde{A}} = \left\{ \int_{x\in X} \left[\int_{u\in[\underline{J}_x^u, \overline{J}_x^u]\subseteq[0,1]} 1/u \right] / x \right\}$$
$$= \left\{ \int_{x\in X} \left[\int_{u\in[\underline{\mu}_A(x), \overline{\mu}_A(x)]\subseteq[0,1]} 1/u \right] / x \right\} \tag{5}$$

If $\tilde{\tilde{A}}$ is a type-2 fuzzy Singleton, the membership function is defined by equation (6).

$$\mu_{\tilde{A}}(x) = \begin{cases} 1/1 & si\ x = x' \\ 1/0 & si\ x \neq x' \end{cases} \tag{6}$$

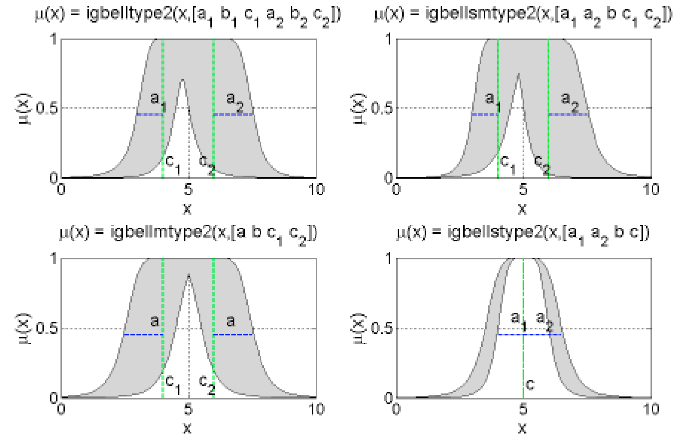

Fig. 2. FOU for Gbell Primary Interval Type-2 Membership Functions.

B. Fuzzy Set Operations

We can apply some operators to the fuzzy sets, or we can make some operations between them [4,10,11]. When we apply an operator to one fuzzy set we obtain another fuzzy set; by the same manner when we combine an operation with two or more sets we obtain another fuzzy set. If we have two type-2 fuzzy subsets identified by the letters $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, associated to a linguistic variable, we can define three basic operations: complement, union and intersection (TABLE I).

TABLE I
Interval Type-2 Fuzzy Set Operations.

Name	Operator	Operation
Union	$\sqcup$ = join	$\tilde{\tilde{A}} \sqcup \tilde{\tilde{B}} = \{ \int_{x\in X} \mu_{\tilde{A}}(x) \sqcup \mu_{\tilde{B}}(x) \}$ $= \left\{ \int_{x\in X} \left[\int_{\alpha\in[\underline{\mu}_{\tilde{A}}(x)\vee\underline{\mu}_{\tilde{B}}(x), \overline{\mu}_{\tilde{A}}(x)\vee\overline{\mu}_{\tilde{B}}(x)]} 1/\alpha \right] / x \right\}$
Intersectio n	$\sqcap$ = meet	$\tilde{\tilde{A}} \sqcap \tilde{\tilde{B}} = \{ \int_{x\in X} \mu_{\tilde{A}}(x) \sqcap \mu_{\tilde{B}}(x) \}$ $= \left\{ \int_{x\in X} \left[\int_{\alpha\in[\underline{\mu}_{\tilde{A}}(x)\wedge\underline{\mu}_{\tilde{B}}(x), \overline{\mu}_{\tilde{A}}(x)\wedge\overline{\mu}_{\tilde{B}}(x)]} 1/\alpha \right] / x \right\}$
Negation	$\neg$	$\neg\tilde{\tilde{A}} = \left\{ \int_{x\in X} \mu_{\tilde{A}}(x)/x \right\}$ $= \left\{ \int_{x\in X} \left[\int_{\alpha\in[1-\overline{\mu}_{\tilde{A}}(x), 1-\underline{\mu}_{\tilde{A}}(x)]} 1/\alpha \right] / x \right\}$

C. *Fuzzy Inference System*

The human knowledge is expressed in fuzzy rule terms with the next syntaxes:

IF *<fuzzy proposition>* **THEN** *<fuzzy proposition>*

The fuzzy propositions are divided in two types, the first one is named **atomic: x is A**, where x is a linguistic variable and A is a linguistic value; the second one is called **compounded: x is A AND y is B OR z is NOT C**, this is a compounded atomic fuzzy proposition with the "AND", "OR" and "NOT" connectors, representing fuzzy intersection, union and complement respectively. The compounded fuzzy propositions are fuzzy relationships. The membership function of the rule IF-THEN is a fuzzy relation determined by a fuzzy implication operator. The fuzzy rules combine one or more fuzzy sets of entry, called antecedent, and are associated with one output fuzzy set, called consequents. The Fuzzy Sets of the antecedent are associated by fuzzy operators AND, OR, NOT and linguistic modifiers. The fuzzy rules permit expressing the available knowledge about the relationship between antecedent and consequents. To express this knowledge completely we normally have several rules, grouped to form what it is known a rule base, that is, a set of rules that express the known relationships between antecedent and consequents. The fuzzy rules are basically IF <Antecedent> THEN <Consequent> and expresses a fuzzy relationship or proposition.

In fuzzy logic the reasoning is imprecise, it is approximated, that means that we can infer from one rule a conclusion even if the antecedent doesn't comply completely. We can count on two basic inference methods between rules and inference laws, Generalized Modus Ponens (GMP) [5,6,8,13] and Generalized Modus Tollens (GMT), that represent the extensions or generalizations of classic reasoning. The GMP inference method is known as direct reasoning and is resumed as:

Rule	*IF x is A THEN y is B*
Fact	*x is A'*
Conclusion	*y es B'*

Where A, A', B and B' are fuzzy sets of any kind. This relationship is expressed as $B' = A' \circ (A \rightarrow B)$. Figure 3 shows an example of Interval Type-2 direct reasoning with Interval Type-2 Fuzzy Inputs.

An Inference Fuzzy System is a rule base system that uses fuzzy logic, instead of Boolean logic utilized in data analysis [4,10,20]. Its basic structure includes four components (Fig. 4):

- ***Fuzzificator.*** Translates inputs (real values) to fuzzy values.
- ***Inference System.*** Applies a fuzzy reasoning mechanism to obtain a fuzzy output.
- ***Type Defuzzificator/Reductor.*** The defuzzificator traduces one output to precise values; the type reductor transforms a Type-2 Fuzzy Set into a Type-1 Fuzzy Set.
- ***Knowledge Base.*** Contains a set of fuzzy rules, and a membership functions set known as the database.

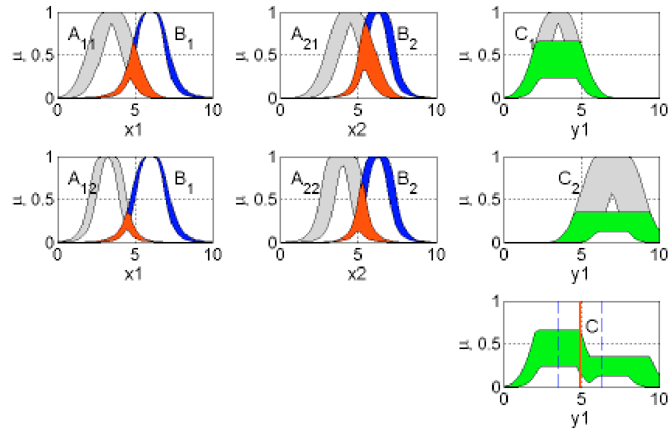

Fig. 3. Interval Type-2 Fuzzy Reasoning.

The decision process is a task that identifies parameters by the inference system using the rules of the rule base data. These fuzzy rules define the connection between the input and output fuzzy variables. A fuzzy rule has the form: IF <Antecedent> THEN <Consequent>, where antecedent is a compound fuzzy logic expression of one or more simple fuzzy expressions connected with fuzzy operators; and the consequent is an expression that assigns fuzzy values to output variables. The inference system evaluates all the rules of the rule base and combines the weights of the consequents of all relevant rules in one fuzzy set using the aggregate operation. This operation is analog in fuzzy logic to the S-norm operator.

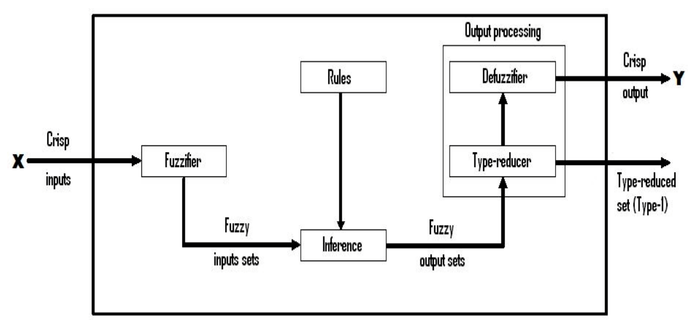

Fig. 4. Type-2 Inference Fuzzy System Structure.

Fuzzy modeling is a task for parameter identification in a fuzzy inference system to obtain an adequate behavior. A fuzzy model with the direct view is constructed with the knowledge of an expert. This task becomes more difficult when the available knowledge is incomplete or when space is a problem, then the use of automatic views are recommended for the fuzzy model. It can be considered different point of views for fuzzy modeling, based on neural networks, genetic algorithms and hybrid methods. The selection of relevant variables and adequate rules is critical for generating a good system. One of the biggest problems occurring in fuzzy modeling is dimensionality, that is, when the computational requirements grow exponentially in relation of the quantity of variables.

III. INTERVAL TYPE-2 FUZZY LOGIC SYSTEM DESIGN

The Mamdani and Takagi-Sugeno-Kang (TSK) Interval Type-2 Fuzzy Inference Models [10] and the design of Interval Type-2 membership functions and operators are implemented in the IT2FLS Toolbox (Interval Type-2 Fuzzy Logic Systems) reused from the Matlab® commercial Fuzzy Logic Toolbox.

The IT2FLS Toolbox includes a series of folders called dit2mf, it2fis, it2mf and it2op (Fig. 5). This folders contain the functions to create Mamdani and TSK Interval Type-2 Fuzzy Inference Systems (newfistype2.m), functions to add input-output variables and their ranges (addvartype2.m), it has functions to add 22 types of Interval Type-2 Membership functions for input-outputs (addmftype2.m), functions to add the rule matrix (addruletype2.m), it can evaluate the Interval Type-2 Fuzzy Inference Systems (evalifistype2.m), evaluate Interval Type-2 Membership functions (evalimftype2.m), it can generate the initial parameters of the Interval Type-2 Membership functions (igenparamtype2.m), it can plot the Interval Type-2 Membership functions with the input-output variables (plotimftype2.m), it can generate the solution surface of the Fuzzy Inference System (gensurftype2.m), it plots the Interval type-2 membership functions (plot2dtype2.m, plot2dctype2.m), a folder to evaluate the derivatives of the Interval type-2 Membership Functions (dit2mf) and a folder with different and generalized Type-2 Fuzzy operators (it2op, t2op).

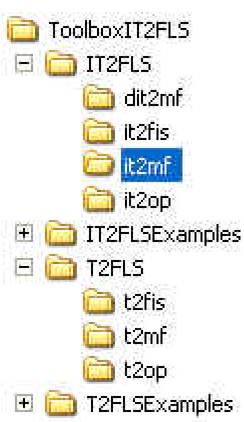

Fig. 5. Toolbox Folder.

The Interval Type-2 Fuzzy Inference Systems (IT2FIS) structure is the MATLAB object that contains all the interval type-2 fuzzy inference system information. This structure is stored inside each GUI tool. Access functions such as getifistype2 and setifistype2 make it easy to examine this structure.

All the information for a given fuzzy inference system is contained in the IT2FIS structure, including variable names, membership function definitions, and so on. This structure can itself be thought of as a hierarchy of structures, as shown in the following diagram (Fig. 6).

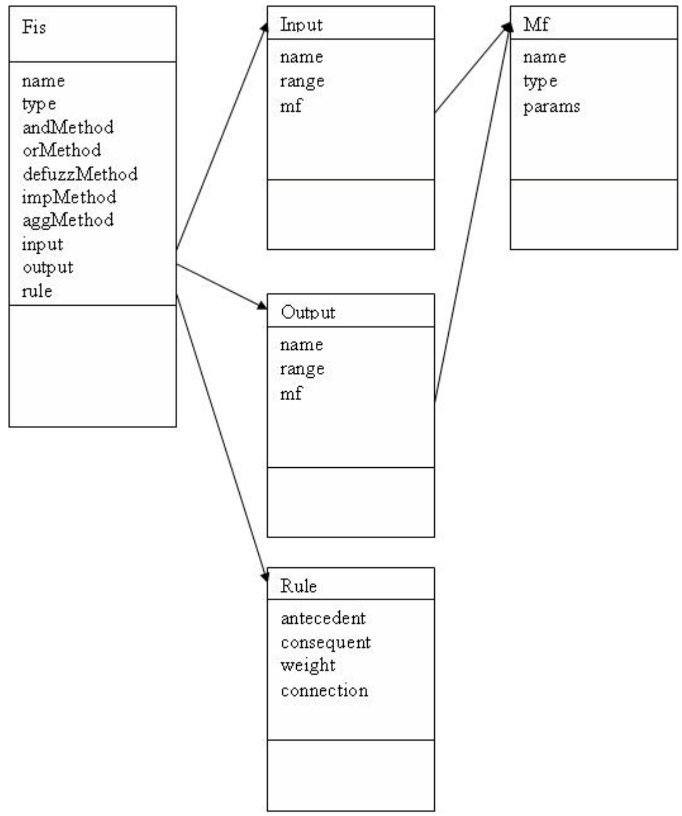

Fig. 6. Hierarchy of IT2FIS structures diagram.

The implementation of the IT2FLS GUI is analogous to the GUI used for Type-1 FLS in the Matlab® Fuzzy Logic Toolbox, thus permitting the experienced user to adapt easily to the use of IT2FLS GUI. Figures 7 and 8 show the main viewport of the Interval Type-2 Fuzzy Inference Systems Structure Editor called IT2FIS (Interval Type-2 Fuzzy Inference Systems).

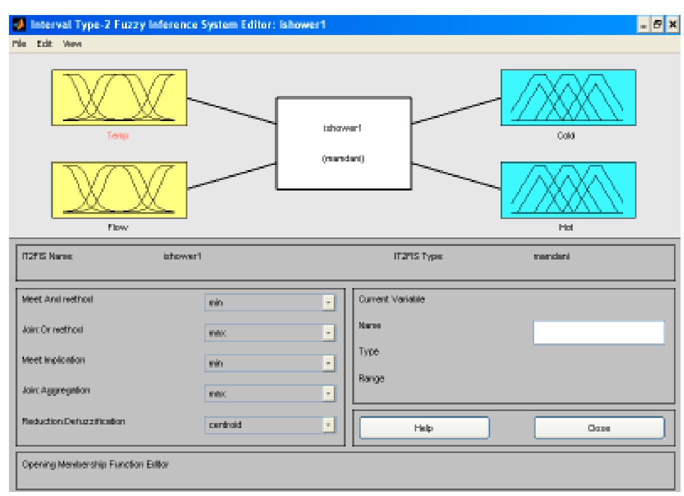

Fig. 7. IT2FIS Editor.

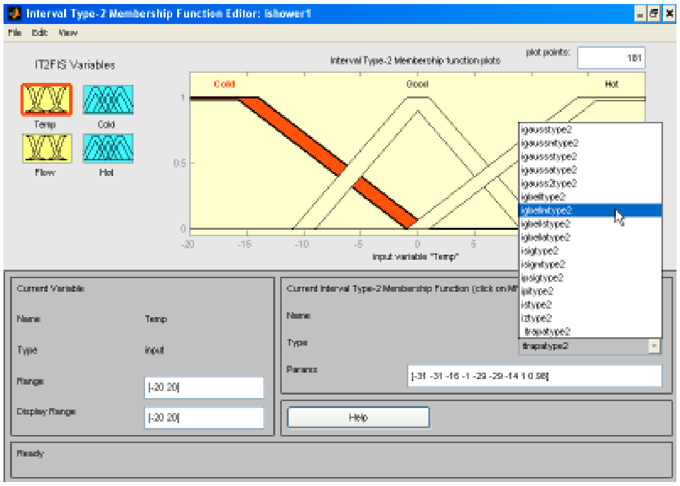

Fig. 8. Interval Type-2 MF's Editor.

A. Mamdani Interval Type-2 Fuzzy Inference System

The Mamdani IT2FIS, is designed with **n** inputs, **m** outputs and **r** rules. The *k*th rule with interval type-2 fuzzy antecedents $\tilde{\tilde{A}}_{k,j} \in \{\mu_{i,l_{k,i}}\}$, interval type-2 fuzzy consequent $\tilde{\tilde{C}}_{k,j} \in \{\sigma_{j,l_{k,j}}\}$ and interval type-2 fuzzy facts $\tilde{\tilde{A}}_i'$ are inferred as a direct reasoning [10].

$$R^k : IF\ x_1\ is\ \tilde{\tilde{A}}_{k,1}\ and \ldots and\ xn\ is\ \tilde{\tilde{A}}_{k,n}\ THEN\ y_1\ is\ \tilde{\tilde{C}}_{k,j}\ and \ldots and\ y_m\ is\ \tilde{\tilde{C}}_{k,m}$$

$$H\ : IF\ x_1\ is\ \tilde{\tilde{A}}_1'\ and \ldots and\ x_n\ is\ \tilde{\tilde{A}}_n'$$

$$C: \qquad y_1\ is\ \tilde{\tilde{C}}_1'\ and \ldots and\ y_m\ is\ \tilde{\tilde{C}}_m'$$

The evaluation of this reasoning is:

$$\tilde{\tilde{R}}_{k,j} = \tilde{\tilde{A}}_{k,1} \times \ldots \times \tilde{\tilde{A}}_{k,n} = (\tilde{\tilde{A}}_{k,1} \to \tilde{\tilde{C}}_{k,j}) \times \ldots \times (\tilde{\tilde{A}}_{k,n} \to \tilde{\tilde{C}}_{k,j})\ , k\text{th rule.}$$

$$\tilde{\tilde{H}} = \tilde{\tilde{A}}_1' \times \ldots \times \tilde{\tilde{A}}_n'\ , \text{facts.}$$

$$\tilde{\tilde{C}}_{k,j}'' = \tilde{\tilde{H}} \circ \tilde{\tilde{R}}_{k,j} = \sqcap_{i=1}^{n}[\tilde{\tilde{A}}_i' \circ (\tilde{\tilde{A}}_{k,i} \to \tilde{\tilde{C}}_{k,j})]$$

$$= \left\{ \int_Y \left[\int_{\alpha \in [\underline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j), \overline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j)] \subseteq [0,1]} 1/\alpha \right] / y_j \right\}$$

$$\underline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j) = \left[\tilde{*}_{i=1}^{n} \left(\underline{\mu}_{\tilde{\tilde{A}}_i'}(x_i)\ \tilde{*}\ \underline{\mu}_{\tilde{\tilde{A}}_{k,i}}(x_i) \right) \right] \tilde{*}\ \underline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j)$$

$$\overline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j) = \left[\tilde{*}_{i=1}^{n} \left(\overline{\mu}_{\tilde{\tilde{A}}_i'}(x_i)\ \tilde{*}\ \overline{\mu}_{\tilde{\tilde{A}}_{k,i}}(x_i) \right) \right] \tilde{*}\ \overline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j)$$

$$\tilde{\tilde{C}}_j' = \sqcup_{k=1}^{r} \tilde{\tilde{C}}_{k,j}'' = \sqcup_{k=1}^{r}[\sqcap_{i=1}^{n}[\tilde{\tilde{A}}_i' \circ (\tilde{\tilde{A}}_{k,i} \to \tilde{\tilde{C}}_{k,j})]]$$

$$= \left\{ \int_Y \left[\int_{\alpha \in [\underline{\mu}_{\tilde{\tilde{C}}_j'}(y_j), \overline{\mu}_{\tilde{\tilde{C}}_j'}(y_j)] \subseteq [0,1]} 1/\alpha \right] / y_j \right\}$$

$$\underline{\mu}_{\tilde{\tilde{C}}_j'}(y_j) = \bigvee_{k=1}^{r} \left(\underline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j) \right)$$

$$= \bigvee_{k=1}^{r} \left(\left[\tilde{*}_{i=1}^{n} \left(\underline{\mu}_{\tilde{\tilde{A}}_i'}(x_i)\ \tilde{*}\ \underline{\mu}_{\tilde{\tilde{A}}_{k,i}}(x_i) \right) \right] \tilde{*}\ \underline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j) \right)$$

$$\overline{\mu}_{\tilde{\tilde{C}}_j'}(y_j) = \bigvee_{k=1}^{r} \left(\overline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j) \right)$$

$$= \bigvee_{k=1}^{r} \left(\left[\tilde{*}_{i=1}^{n} \left(\overline{\mu}_{\tilde{\tilde{A}}_i'}(x_i)\ \tilde{*}\ \overline{\mu}_{\tilde{\tilde{A}}_{k,i}}(x_i) \right) \right] \tilde{*}\ \overline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j) \right)$$

The defuzzification of the interval type-2 fuzzy aggregated output set $\tilde{\tilde{C}}_j'$ is: $\hat{y}_j = idefuzztype2(\mu_{\tilde{\tilde{C}}_j}(y_j),'type')$

where *type* is the name of the defuzzification technique. If $\tilde{\tilde{A}}_i'$ are interval type-2 fuzzy singletons then:

$$\mu_{\tilde{\tilde{C}}_{k,j}''}(y_j) = \left[\sqcap_{i=1}^{n}[\mu_{\tilde{\tilde{A}}_{k,i}}(x_i)] \right] \sqcap \mu_{\tilde{\tilde{C}}_{k,j}}(y_j)$$

$$= \left\{ \int_Y \left[\int_{\alpha \in [\underline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j), \overline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j)] \subseteq [0,1]} 1/\alpha \right] / y_j \right\}$$

$$\underline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j) = \left[\tilde{*}_{i=1}^{n} \left(\underline{\mu}_{\tilde{\tilde{A}}_{k,i}}(\hat{x}_i) \right) \right] \tilde{*}\ \underline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j)$$

$$\overline{\mu}_{\tilde{\tilde{C}}_{k,j}''}(y_j) = \left[\tilde{*}_{i=1}^{n} \left(\overline{\mu}_{\tilde{\tilde{A}}_{k,i}}(\hat{x}_i) \right) \right] \tilde{*}\ \overline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j)$$

$$\mu_{\tilde{\tilde{C}}_j'}(y_j) = \sqcup_{k=1}^{r} \mu_{\tilde{\tilde{C}}_{k,j}''}(y_j)$$

$$= \sqcup_{k=1}^{r}[\sqcap_{i=1}^{n}[\mu_{\tilde{\tilde{A}}_{k,i}}(x_i)] \sqcap \mu_{\tilde{\tilde{C}}_{k,j}}(y_j)]$$

$$= \left\{ \int_Y \left[\int_{\alpha \in [\underline{\mu}_{\tilde{\tilde{C}}_j'}(y_j), \overline{\mu}_{\tilde{\tilde{C}}_j'}(y_j)] \subseteq [0,1]} 1/\alpha \right] / y_j \right\}$$

$$\underline{\mu}_{\tilde{\tilde{C}}'_j}(y_j) = \bigvee_{k=1}^{r}\left(\underline{\mu}_{\tilde{\tilde{C}}''_{k,j}}(y_j)\right)$$

$$= \bigvee_{k=1}^{r}\left([\widetilde{\circledast}_{i=1}^{n} \underline{\mu}_{\tilde{\tilde{A}}_{k,i}}(\hat{x}_i)] \,\widetilde{\circledast}\, \underline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j)\right)$$

$$\overline{\mu}_{\tilde{\tilde{C}}'_j}(y_j) = \bigvee_{k=1}^{r}\left(\overline{\mu}_{\tilde{\tilde{C}}''_{k,j}}(y_j)\right)$$

$$= \bigvee_{k=1}^{r}\left([\widetilde{\circledast}_{i=1}^{n} \overline{\mu}_{\tilde{\tilde{A}}_{k,i}}(\hat{x}_i)] \,\widetilde{\circledast}\, \overline{\mu}_{\tilde{\tilde{C}}_{k,j}}(y_j)\right)$$

The next case study shows the command line editing procedure of the Mamdani interval type-2 fuzzy logic inference system structure implemented in the IT2FLS Toolbox.

```
% ishowery1.m
% Interval type-2 fuzzy inference system for shower control
%
clear all
x1 = -20:2:20;
x2 = -1:0.1:1;
y1 = -1:0.1:1;
y2 = -1:0.1:1;
data = [x1' x2' y1' y2'];
% Define Interval type-2 fuzzy inference system
fis = newfistype2('ctrlshower','mamdani');
% Define input variables and ranges
fis = addvartype2(fis,'input','Temp',[-20 20]);
fis = addvartype2(fis,'input','Flow',[-1 1]);
% Define output variables and ranges
fis = addvartype2(fis,'output','Cold',[-1 1]);
fis = addvartype2(fis,'output','Hot',[-1 1]);
% Define mf's of first input variable
fis = addmftype2(fis,'input',1,'Cold','itrapatype2',[-31 -31 -16 -
1 -29 -29 -14 1 0.98]);
fis = addmftype2(fis,'input',1,'Good','itritype2',[-11 -1 9 -9 1
11]);
fis = addmftype2(fis,'input',1,'Hot','itrapatype2',[-1 14 29 29 1
16 31 31 0.98]);
% Define mf's of second input variable
fis = addmftype2(fis,'input',2,'Soft','itrapatype2',[-3.1 -3.1 -0.9
-0.1 -2.9 -2.9 -0.7 0.1 0.98]);
fis = addmftype2(fis,'input',2,'Good','itritype2',[-0.45 -0.05
0.35 -0.35 0.05 0.45]);
fis = addmftype2(fis,'input',2,'Hard','itrapatype2',[-0.1 0.7 2.9
2.9 0.1 0.9 3.1 3.1 0.98]);
% Define mf's of first output variable
fis = addmftype2(fis,'output',1,'closeFast','itritype2',[-1.05 -
0.65 -0.35 -0.95 -0.55 -0.25]);
fis = addmftype2(fis,'output',1,'closeShow','itritype2',[-0.65 -
0.35 -0.05 -0.55 -0.25 0.05]);
fis = addmftype2(fis,'output',1,'Steady','itritype2',[-0.35 -0.05
0.25 -0.25 0.05 0.35]);
fis = addmftype2(fis,'output',1,'openShow','itritype2',[-0.05
0.25 0.55 0.05 0.35 0.65]);
fis = addmftype2(fis,'output',1,'openFast','itritype2',[0.25 0.55
0.95 0.35 0.65 1.05]);
% Define mf's of second output variable
fis = addmftype2(fis,'output',2,'closeFast','itritype2',[-1.05 -
0.65 -0.35 -0.95 -0.55 -0.25]);
fis = addmftype2(fis,'output',2,'closeShow','itritype2',[-0.65 -
0.35 -0.05 -0.55 -0.25 0.05]);
fis = addmftype2(fis,'output',2,'Steady','itritype2',[-0.35 -0.05
0.25 -0.25 0.05 0.35]);
fis = addmftype2(fis,'output',2,'openShow','itritype2',[-0.05
0.25 0.55 0.05 0.35 0.65]);
fis = addmftype2(fis,'output',2,'openFast','itritype2',[0.25 0.55
0.95 0.35 0.65 1.05]);
% Define rule table
ruleList=[1 1 4 5 1 1;
        1 2 2 4 1 1;
        1 3 1 2 1 1;
        2 1 4 4 1 1;
        2 2 3 3 1 1;
        2 3 2 2 1 1;
        3 1 5 4 1 1;
        3 2 4 2 1 1;
        3 3 2 1 1 1];
% Add rule table
fis=addruletype2(fis,ruleList);
% Prints the rules
showrule(fis)
% Visualize input/output linguistic variables
figure;
subplot(4,1,1);plotimftype2(fis,'input',1); grid on
xlabel('Temp');
subplot(4,1,2);plotimftype2(fis,'input',2); grid on
xlabel('Flow');
subplot(4,1,3);plotimftype2(fis,'output',1); grid on
xlabel('Cold');
subplot(4,1,4);plotimftype2(fis,'output',2); grid on
xlabel('Hot');
% Evaluate the Interval type-2 fuzzy inference system
[xx1,xx2] = meshgrid(x1,x2);
input = [xx1(:) xx2(:)];
tic
out   = evalifistype2(input,fis);
toc
%
% Visualize surface solutions
figure;
gensurftype2(fis,[1 2],[1])
figure;
gensurftype2(fis,[1 2],[2])
```

B. Interval Type-2 Takagi-Sugeno-Kang Fuzzy Inference System.

The IT2FIS de Takagi-Sugeno-Kang system is designed with **n** inputs, **m** outputs and **r** rules. The *k*th rule with interval type-2 fuzzy antecedents $\tilde{\tilde{A}}_{k,j} \in \{\mu_{i,l_{k,i}}\}$, interval type-1 fuzzy set are used for the consequents sets, $f_{j,k} = \theta_{0,j}^k + \sum_{i=1}^{n} \theta_{i,j}^k \cdot x_i$ and real facts are inferred as a direct reasoning [10]:

$$
\begin{array}{ll}
R^k: IF\ x_1\ is\ \tilde{A}_{k,1}\ and \ldots and\ xn\ is\ \tilde{A}_{k,n}\ THEN\ y_1\ is\ f_{1,k}\ and \ldots and\ y_m\ is\ f_{m,k} \\
H\ : IF\ x_1\ is\ \hat{x}_1\ and \ldots and\ x_n\ is\ \hat{x}_n \\
\hline
C: \qquad y_1\ is\ \hat{y}_1\ and \ldots and\ y_m\ is\ \hat{y}_m
\end{array}
$$

The evaluation of this reasoning is:

$$\alpha_k = [\underline{\alpha}_k, \overline{\alpha}_k] = \sqcap_{i=1}^{n} [\mu_{\tilde{A}_{k,i}}(\hat{x}_i)]$$

$$= \left[\mathop{\tilde{*}}_{i=1}^{n} \left(\underline{\mu}_{\tilde{A}_{k,i}}(\hat{x}_i) \right), \mathop{\tilde{*}}_{i=1}^{n} \left(\overline{\mu}_{\tilde{A}_{k,i}}(\hat{x}_i) \right) \right]$$

where $\alpha_k = [\underline{\alpha}_k, \overline{\alpha}_k]$ is the firing set of the interval type-1 fuzzy antecedent of the *k*th rule.

$$f_{j,k} = \theta_{0,j}^k + \sum_{i=1}^{n} \theta_{i,j}^k \cdot x_i$$

where $f_{j,k} = [{}^l f_{j,k}, {}^r f_{j,k}]$ is a real function of the interval consequents of the *k*th rule. If $\theta_{i,j}^k = [c_{i,j}^k - s_{i,j}^k, c_{i,j}^k + s_{i,j}^k]$ $\forall\ i = 0, \ldots, n$, where $c_{i,j}^k$ is the center and $s_{i,j}^k$ denotes the spread, then ${}^l f_{j,k}, {}^r f_{j,k}$ is expressed as:

$${}^l f_{j,k} = \sum_{i=1}^{n} c_{i,j}^k \cdot x_i + c_{0,j}^k - \sum_{i=1}^{n} s_{i,j}^k \cdot |x_i| - s_{0,j}^k$$

$${}^r f_{j,k} = \sum_{i=1}^{n} c_{i,j}^k \cdot x_i + c_{0,j}^k + \sum_{i=1}^{n} s_{i,j}^k \cdot |x_i| + s_{0,j}^k$$

With the Karnik and Mendel algorithm [10] the ${}^l\alpha_k$ and ${}^r\alpha_k$ are evaluated to obtain the FIS output variables, these are expressed as

$$\hat{y}_j^l = \frac{\sum_{k=1}^{r} {}^l\alpha_k \cdot {}^l f_{j,k}}{\sum_{k=1}^{r} {}^l\alpha_k} = \frac{\sum_{k=1}^{L} \overline{\alpha}_k \cdot {}^l f_{j,k} + \sum_{k=L+1}^{r} \underline{\alpha}_k \cdot {}^l f_{j,k}}{\sum_{k=1}^{L} \overline{\alpha}_k + \sum_{k=L+1}^{r} \underline{\alpha}_k}$$

$$\hat{y}_j^{rl} = \frac{\sum_{k=1}^{r} {}^r\alpha_k \cdot {}^r f_{j,k}}{\sum_{k=1}^{r} {}^r\alpha_k} = \frac{\sum_{k=1}^{R} \underline{\alpha}_k \cdot {}^r f_{j,k} + \sum_{k=R+1}^{r} \overline{\alpha}_k \cdot {}^r f_{j,k}}{\sum_{k=1}^{R} \underline{\alpha}_k + \sum_{k=R+1}^{r} \overline{\alpha}_k}$$

$$\hat{y}_j = \frac{\hat{y}_j^l + \hat{y}_j^r}{2}$$

The next case study shows the command line editing procedure of the Takagi-Sugeno-Kang interval type-2 fuzzy inference system structure implemented in the IT2FLS Toolbox.

```
% ejfistsk22.m
% TKS Interval Type-2 Fuzzy Inference System
% Basic Case Study
clear all
% Define Interval type-2 fuzzy inference system
fis = newfistype2('tks22','sugeno');
% Define first input variables, ranges and mf's
fis = addvartype2(fis,'input','x1',[0 10]);
fis = addmftype2(fis,'input',1,'F11','igaussmtype2',[1.30 4.85
6.15]);
fis = addmftype2(fis,'input',1,'F12','igaussmtype2',[1.10 3.95
5.05]);
fis = addmftype2(fis,'input',1,'F13','igaussmtype2',[0.80 5.80
6.60]);
% Define second input variables, ranges and mf's
fis = addvartype2(fis,'input','x2',[0 10]);
fis = addmftype2(fis,'input',2,'F21','igaussmtype2',[1.20 2.40
3.60]);
fis = addmftype2(fis,'input',2,'F22','igaussmtype2',[1.00 5.50
6.50]);
fis = addmftype2(fis,'input',2,'F23','igaussmtype2',[1.50 4.35
5.85]);
% Define first output  variables, ranges and mf's
fis = addvartype2(fis,'output','y1',[0 10]);
fis = addmftype2(fis,'output',1,'G11','linear',[0.0 0.0 6.1396
0.0 0.0 2.6666]);
fis = addmftype2(fis,'output',1,'G12','linear',[0.0 0.0 3.1627
0.0 0.0 2.1417]);
fis = addmftype2(fis,'output',1,'G13','linear',[0.0 0.0 4.7708
0.0 0.0 0.6308]);
% Define second  output variables, ranges and mf's
fis = addvartype2(fis,'output','y2',[0 10]);
fis = addmftype2(fis,'output',2,'G21','linear',[0.0 0.0 7.16645
0.0 0.0 1.83085]);
fis = addmftype2(fis,'output',2,'G22','linear',[0.0 0.0 4.31100
0.0 0.0 4.18390]);
fis = addmftype2(fis,'output',2,'G23','linear',[0.0 0.0 0.90615
0.0 0.0 0.57895]);
% Define rule table
ruleList=[1 1 1 1 1 1;
          2 2 2 2 1 1;
         3 3 3 3 1 1];
% Add rules to the IT2FIS
fis=addruletype2(fis,ruleList);
% Input Data (X)
X=[4.7 6.0;
   6.1 3.9;
   2.9 4.2;
   7.0 5.5];
% Desired Output Data (D)
D=[3.52 4.02;
   5.43 6.23;
   4.95 5.76;
   4.70 4.28];
% Evaluate interval type-2 fuzzy inference system
out = evalifistype2(X,fis)
% Evaluate error differences
e = D - out;
% Evaluate the root media square root
rmse = sqrt(mse(e))
```

```
% Generate solution surfaces
figure;
gensurftype2(fis,[1 2],[1]);
axis([fis.input(1).range(1) fis.input(1).range(2) ...
    fis.input(2).range(1),fis.input(2).range(2) ...
    fis.output(1).range(1),fis.output(1).range(2)])
xlabel('x1'); ylabel('x2'); zlabel('y1');
figure;
gensurftype2(fis,[1 2],[2]);
axis([fis.input(1).range(1) fis.input(1).range(2) ...
    fis.input(2).range(1),fis.input(2).range(2) ...
    fis.output(2).range(1),fis.output(2).range(2)])
xlabel('x1'); ylabel('x2'); zlabel('y2');
% Prints the rules
showrule(fis)
```

TABLE II
Forecasting of Time Series.

Methods	Mackey-Glass			
	RMSE	***trn/chk***	***epoch***	***cpu(s)****
NNFF†**	0.0595	500/500	200	13.36
CANFIS	0.0016	500/500	50	7.34
NNFF-GA†	0.0236	500/500	150	98.23
FLS(TKS)-GA†	0.0647	500/500	200	112.01
FLS(MAM)-GA†	0.0693	500/500	200	123.21
IT2FLS	0.0335	500/500	6	20.23

* POWER BOOK G4 1.5 Ghz / 512 MB RAM
** Architecture: x-13-1 † 30 samples average

IV. RESULTS

The results are a comparative analysis of a Mackey-Glass chaotic time-series forecasting study using intelligent cooperative architecture hybrid methods, with neural networks, (Mamdani, Takagi-Sugeno-Kang) type-1 fuzzy inference systems and genetic algorithms (neuro-genetic, fuzzy-genetic and neuro-fuzzy) and an interval type-2 fuzzy logic model, for the implicit knowledge acquisition in a time series behavioral data history [23]. Also we present a shower simulation and a truck backer-upper simulation with interval type-2 fuzzy logic systems using the IT2FLS Toolbox.

A. Mackey-Glass chaotic time-series

To identify the model we make an exploratory series analysis with 5 delays, $L^5x(t)$, 6 periods and 500 training data values to forecast 500 output values. The IT2FLS system works with 4 inputs, 2 interval type-2 membership functions (igbellmtype2) for each input, 16 rules (Fig. 9) and one output with 16 interval lineal functions, it is evaluated with unnormalized values. The root mean square error (RMSE) forecasted is 0.0335 (TABLE II).

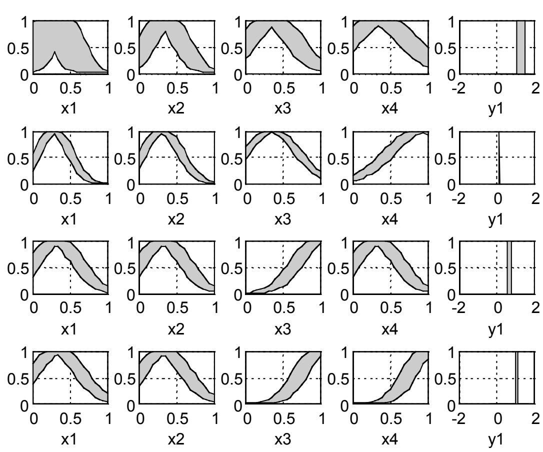

Fig. 9. IT2FLS (TSK) First Four Rules.

B. Shower Control Simulation.

In figure 10 it shows an interval type-2 fuzzy control scheme and in figures 11 and 12 the control answers.

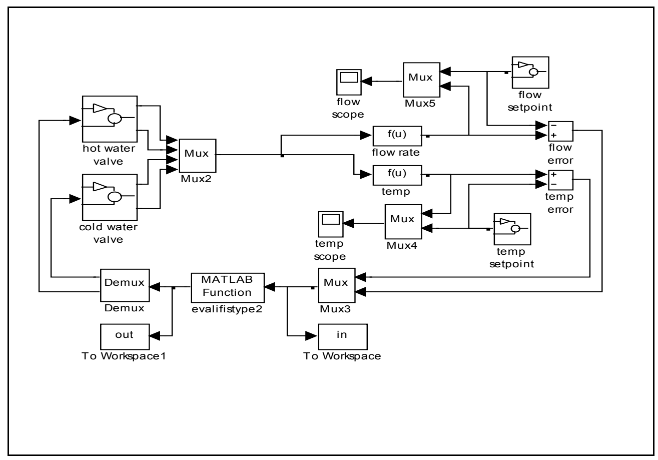

Fig. 10. Simulink interval type-2 fuzzy control scheme.

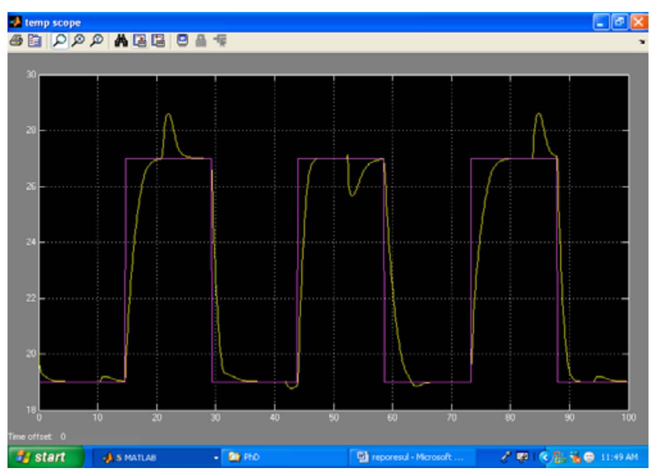

Fig. 11. Temperature interval type-2 fuzzy control.

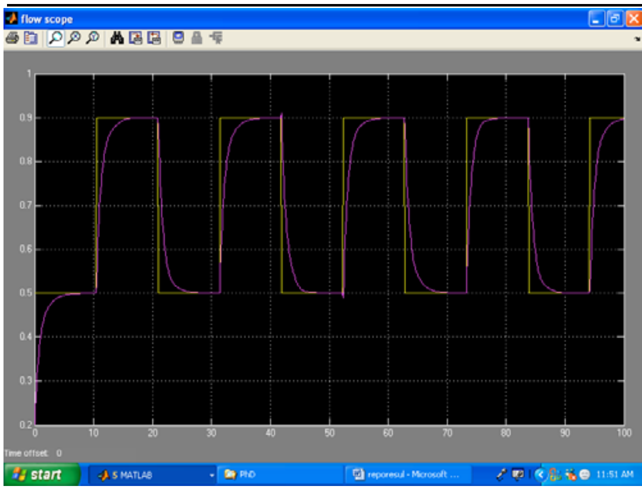

Fig. 12. Interval type-2 fuzzy control flow.

C. Truck backer-upper control simulation.

In figure 13 have the interval type-2 fuzzy control scheme and in figure 14-17 the control answers of the car made trajectories.

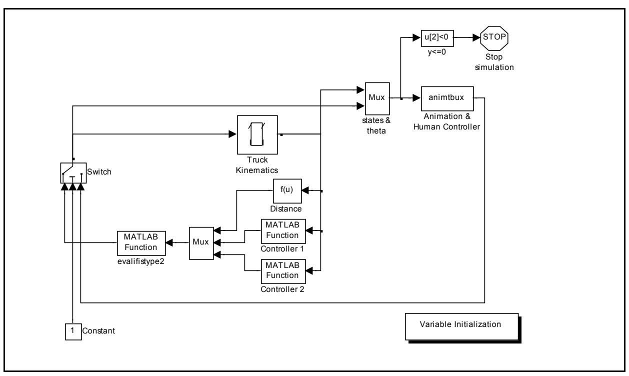

Fig. 13. Simulink interval type-2 fuzzy control scheme.

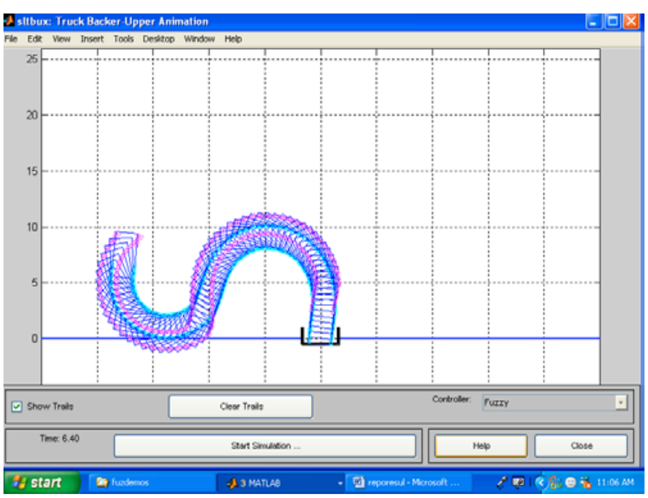

Fig. 14. Trajectory 1 Interval type-2 fuzzy control.

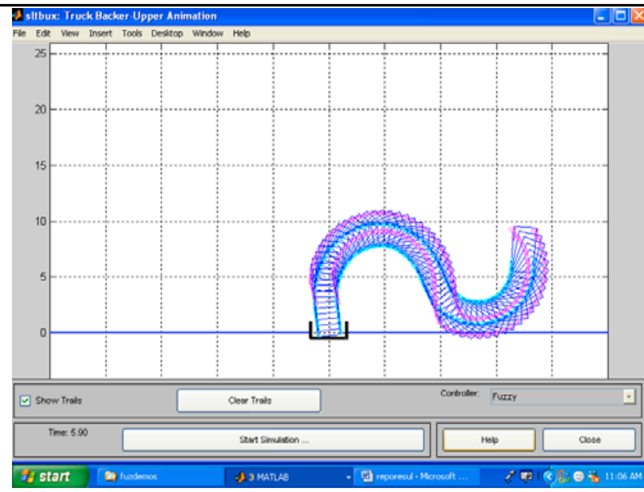

Fig. 15. Trajectory 2 interval type-2 fuzzy control.

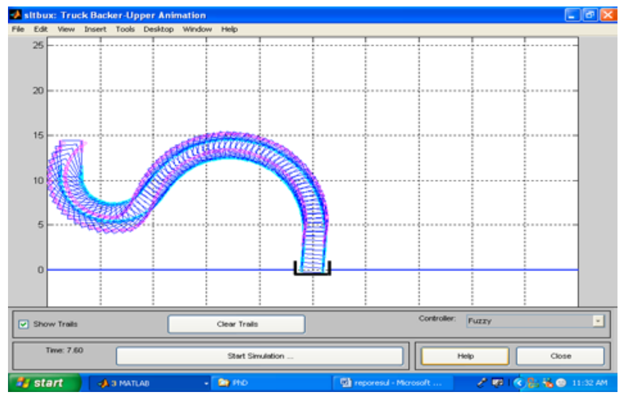

Fig. 16. Interval type-2 fuzzy control trajectory 3.

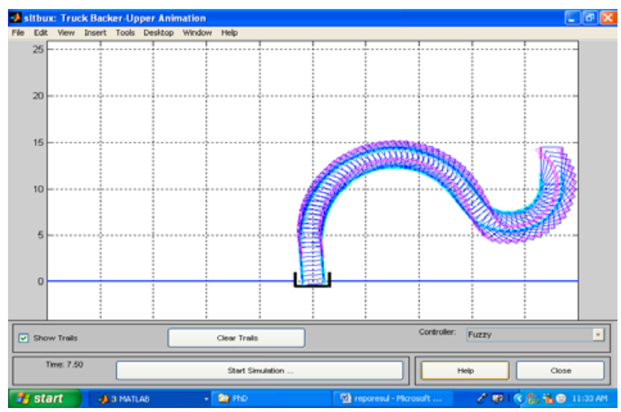

Fig. 17. Interval type-2 fuzzy control trajectory 4.

V. Conclusions

The time series results show that intelligent hybrid methods and interval type-2 fuzzy models can be derived as a generalization of the autoregressive non-lineal models in the context of time series. This derivation permits a practical specification for a general class of prognosis and identification time series model, where a set of input-output variables are part of the dynamics of the time series knowledge base.

This helps the application of the methodology to a series of diverse dynamics, with a very low quantity of causal variables to explain behavior. In consequence we can obtain models with a lower number of rules, therefore a lower number of parameters than those obtained on the derivation of traditional methods like Box-Jenkins. The interpretation as a temporal series model, also enables the use of statistical tools in the variable selection process, in the diagnostic and meta-diagnostic models, thus enriches the individual artificial intelligence methodologies when applied to this type of problems. The proposed approximation methodology not only gives information in numeric precision terms about de prognosis capability, but also permits a detailed analysis of the model quality.

The results in the interval type-2 fuzzy control case studies of the shower and truck backer-upper have similar results to the type-1 fuzzy control with moderate uncertain footprints. To better characterize the interval type-2 fuzzy models we need to generate more case studies with better knowledge bases for the proposed problems, therefore classify the interval type-2 fuzzy model application strengths.

The design and implementation done in the IT2FLS Toolbox is potentially important for research in the interval type-2 fuzzy logic area, thus solving complex problems on the different applied areas.

Our future work is to improve the IT2FLS Toolbox with a better graphics user interface (GUI), to have compiled code, and integrate a learning technique Toolbox to optimize the knowledge base parameters of the interval type-2 fuzzy inference system, and design interval type-2 fuzzy neural network hybrid models.

References

[1] Zadeh, L.A., "Fuzzy sets," *Information and Control*, Vol. 8, pp. 338-353, 1965.

[2] Zadeh, L.A., "Outline of a new approach to the analysis of complex systems and decision processes," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 3, No. 1, pp. 28-44, Jan. 1973.

[3] Zadeh, L.A., "The concept of a linguistic variable and its application to approximate reasoning, Parts 1, 2, and 3," *Information Sciences*, 1975, 8:199-249, 8:301-357, 9:43-80.

[4] Zadeh, L.A., "Fuzzy Logic," *Computer*, Vol. 1, No. 4, pp. 83-93, 1988.

[5] Zadeh, L.A., "Knowledge representation in fuzzy logic," *IEEE Transactions on Knowledge and Data Engineering*, Vol. 1, pp. 89-100, 1989.

[6] Karnik, N.N. and J.M. Mendel. An Introduction to Type-2 Fuzzy Logic Systems, Univ. of Southern Calif., Los Angeles, CA., June 1998b.

[7] L. Zadeh, "Fuzzy logic = computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 2, pp. 103–111, 1996.

[8] Q. Liang and J. Mendel, "Interval type-2 fuzzy logic systems: Theory and design," *IEEE Transactions Fuzzy Systems*, vol. 8, pp. 535–550, 2000.

[9] Dubois, D. and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, 1980.

[10] J. Mendel, *Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions*. NJ: Prentice-Hall, 2001.

[11] Mizumoto, M. and K. Tanaka, "Some Properties of Fuzzy Sets of Type-2," Information and Control, vol. 31, pp. 312-340, 1976.

[12] Wang, L.-X., *Adaptive fuzzy systems and control: design and stability analysis*, Prentice Hall, 1994.

[13] Yager, R. R., "On the Implication Operator in Fuzzy logic," Information Sciences, vol. 31, pp. 141-164, 1983.

[14] Yager, R.R., "A Caracterization of the Fuzzy Extension Principle," J. Fuzzy Sets and Systems, vol. 18, pp. 205-217, 1986.

[15] Mamdani, E.H. and S. Assilian, "An experiment in linguistic synthesis with a fuzzy logic controller," *International Journal of Man-Machine Studies*, Vol. 7, No. 1, pp. 1-13, 1975.

[16] Mamdani, E.H., "Applications of fuzzy logic to approximate reasoning using linguistic synthesis," *IEEE Transactions on Computers*, Vol. 26, No. 12, pp. 1182-1191, 1977.

[17] M. Sugeno and G. T. Kang. Structure identification of fuzzy model. Fuzzy Sets and Systems, 28:15-33, 1988.

[18] Sugeno, M., *Industrial applications of fuzzy control*, Elsevier Science Pub. Co., 1985.

[19] Y. Tsukamoto. An approach to fuzzy reasoning method. In Madan M. Gupta, Rammohan K. Ragade, and Ronald R. Yager, editors, Advances in fuzzy set theory and applications, pages 137-149. North-Holland, Amsterdam, 1979.

[20] Yager, R., "On a general class of fuzzy connectives," *Fuzzy Sets and Systems*, 4:235-242, 1980.

[21] Mamdani, E.H., "Advances in the linguistic synthesis of fuzzy controllers," *International Journal of Man-Machine Studies*, Vol. 8, pp. 669-678, 1976.

[22] T. Takagi and M. Sugeno. Fuzzy identification of systems and its applications to modeling and control. IEEE Transactions on Systems, Man, and Cybernetics, 15:116-132, 1985.

[23] Juan R. Castro. Hybrid Intelligent Architecture Development for Time Series Forecasting. Masters Degree Thesis. Tijuana Institute of Technology. December 2005.

Oscar Castillo is a Professor of Computer Science in the Graduate Division, Tijuana Institute of Technology, Tijuana, Mexico. In addition, he is serving as Research Director of Computer Science and head of the research group on fuzzy logic and genetic algorithms. Currently, he is President of HAFSA (Hispanic American Fuzzy Systems Association) and Vice-President of IFSA (International Fuzzy Systems Association) in charge of publicity. Prof. Castillo is also Vice-Chair of the Mexican Chapter of the Computational Intelligence Society (IEEE). Prof. Castillo is also General Chair of the IFSA 2007 World Congress to be held in Cancun, Mexico. He also belongs to the Technical Committee on Fuzzy Systems of IEEE and to the Task Force on "Extensions to Type-1 Fuzzy Systems". His research interests are in Type-2 Fuzzy Logic, Intuitionistic Fuzzy Logic, Fuzzy Control, Neuro-Fuzzy and Genetic-Fuzzy hybrid approaches. He has published over 50 journal papers, 5 authored books, 10 edited books, and 160 papers in conference proceedings.

Juan R. Castro is a graduate in Computer Science (2005) from the Division of Graduate Studies and Research in Tijuana Institute of Technology, Mexico. He is a Professor in Baja California Autonomous University since 1987, teaches for the Computer Engineering Program since 1991 and currently a student of Science and Engineering with the Division of Graduate Studies and Research in Baja California Autonomous University, Mexico. His present research interests include: hybrid intelligent systems, encompassing neural networks, type-1 fuzzy logic systems, type-2 fuzzy logic systems, genetic algorithms and their applications to a wide range of problems.

Luis G. Martínez received his Masters Degree in Computer Engineering (2006) with the Division of Graduate Studies and Research in Baja California Autonomous University, Mexico. He is a Professor in the same University since 1988, teaches for the Computer Engineering Program since 1991. His present research interests include: software engineering, software process methodology, personal software process and fuzzy logic systems.