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A Novel Method for the Design of Takagi-Sugeno Fuzzy Controllers with Stability Analysis using Genetic Algorithm

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Abstract

This paper presents a reliable method for the evolution of Takagi-Sugeno fuzzy controller rules with stability analysis using genetic algorithm. This method does not have the burden of involving n^2 terms of common positive definite matrix in the chromosomes. This is achieved by using a fitness function involving absolute mean square error (AMSE) of state variables, the status of feasibility of LMI (FLMI) and the number of stable subsystems. Nonlinear benchmarking problems like inverted pendulum and cart pole balancing problems are used for simulation study. The performance and efficiency of the proposed design method is good as can be observed from the simulation examples.

Keywords: fuzzy-model, fuzzy-controller, genetic-algorithm, stability-analysis, linear-matrix-inequality

1 Introduction

Design and stability analysis of Takagi-Sugeno fuzzy controller has been a hot research topic since the first paper based on Takagi-Sugeno-Kang (TSK) fuzzy plant model [1]. Several approaches have been proposed for the design of consequent part (feedback gains) of Takagi-Sugeno fuzzy controllers and its stability analysis.

The most important works among which are the design of output feedback controllers [2], output stabilization of Takagi-Sugeno fuzzy systems [3], parallel distributed compensation [4, 5], an LMI approach with pole-placement constraint [6], quadratic stability conditions & H_∞ control design [7], stability issues [8], generalization of stability criterion [9], relaxed LMI based design [10], stability analysis & systematic design [11], and, a method of modeling which incorporates input variable selecting and estimating consequent part in different stages suggested by Hadjili & et.al.[12]. Genetic algorithm has also been used in the design of T-S fuzzy controllers [13, 14].

Most of the design procedures of Takagi-Sugeno fuzzy controllers described above fall into one of the following categories: (i) identification of consequent part, (ii) identifying antecedent part and consequent part in stages, and (iii) stability analysis. Hence there is a need of simultaneous design and stability analysis of Takagi-Sugeno fuzzy controllers.

Recently, genetic algorithm design of feedback gain involving stability analysis was proposed by Lam et. al. [15] in which each chromosome includes a matrix of size $n \times n$ in order to search for a common positive stability matrix in addition to the feedback gains. This paper addresses the evolution of Takagi-Sugeno fuzzy controller with stability analysis using genetic algorithm which does not have burden of including n^2 terms of common positive definite matrix and is different from the design methods specified by Lam et. al. [15].

This paper is organized as follows. Section 2 presents a general review of T-S fuzzy models and model based controller. Section 3 presents the proposed method for the design of Takagi-Sugeno fuzzy controllers with stability analysis using GA. Simulations and results based on standard benchmarking problem of inverted pendulum system and cart pole balancing problem are presented in section 4. Section 5 deals with conclusion.

2 Takagi-Sugeno Fuzzy Models and Model Based Fuzzy Controllers - A Review

Consider the nonlinear dynamical system

$$\dot{x} = A(x) + B(x)u \tag{1}$$

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Takagi-Sugeno (TS) fuzzy system of system (1) is formed by approximating (1) in each of its local regions and the ith rule is of the form.

$$R_i: \text{ IF } x_1 \text{ is } M_{1i} \cdots \text{and } x_n \text{ is } M_{ni} \quad \text{THEN } \dot{x} = A_i x + B_i u \tag{2}$$

where (A_i, B_i) is the i^{th} local linear model.

The possibility of firing the i^{th} rule is given by the product of all membership functions associated with the i^{th} rule and is $\mu_i(x) = \prod_{j=1}^{n} M_{ji}(x_j)$ where M_{ji} is the membership function of x_j in the i^{th} rule. The membership functions can be either triangular, bell-shaped or any other membership function described in the literature.

By using center of gravity method for defuzzification, TSFS can be represented as:

$$\dot{x} = \sum_{i=1}^{r} h_i(x)(A_i\, x + B_i\, u) \tag{3}$$

where $h_i(x) = \frac{\mu_i(x)}{\sum_{j=1}^{r} \mu_j(x)}$

The open loop system corresponding to (3) is

$$\dot{x} = \sum_{i=1}^{r} h_i(x) A_i x \tag{4}$$

Theorem 1 *[16] The equilibrium of a fuzzy system (4) is asymptotically stable in the large if there exists a common positive definite matrix P such that*

$$A_i^T P + P A_i < 0 \quad i = 1, 2, \cdots, r \tag{5}$$

It is obvious that a necessary condition for the existence of a common symmetric positive definite P satisfying (5) is that each A_i be asymptotically stable; that is, the eigenvalues of each A_i be in the open left-hand complex plane.

The concept of parallel distributed compensation is utilized to design fuzzy controllers to stabilize fuzzy system (3). The idea is to design a compensator for each rule of the fuzzy model using linear controller design techniques. The i^{th} rule of the Takagi-Sugeno Fuzzy Controller (TSFC) is given by:

$$R_i: \text{ IF } x_1 \text{ is } M_{1i}, \cdots \text{ and } x_n \text{ is } M_{ni} \quad \text{THEN } u_i = -K_i\, x \tag{6}$$

where, u_i is the control force required for the i^{th} local linear model.

Hence the overall TSFC is

$$u = -\sum_{i=1}^{r} h_i(x) K_i\, x \tag{7}$$

Substituting (7) into the fuzzy system (3), we obtain

$$\dot{x} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_j(x) h_i(x) \{A_i - B_i K_j\}\, x \tag{8}$$

Therefore the following sufficient condition is obtained.

Theorem 2 *[5] The equilibrium of a fuzzy control system (8) [i.e., Fuzzy controller (7)+ Fuzzy Model (3)] is asymptotically stable in the large if there exists a common positive definite matrix P such that the following conditions are satisfied:*

$$\{A_i - B_i K_j\}^T P + P\{A_i - B_i K_j\} < 0 \quad i, j = 1, 2, \cdots, r \tag{9}$$

It is obvious that a necessary condition for the existence of a common symmetric positive definite P satisfying (9) is that each $A_i - B_i K_j$ be asymptotically stable; that is, the eigenvalues of each $A_i - B_i K_j$ be in the open left-hand complex plane.

3 Genetic Algorithmic Model for the Design of T-S Fuzzy Controllers with Stability

GAs are derivative-free search algorithms, based on natural genetics, that provide robust search capabilities in complex spaces, and thereby offer a valid approach to problems requiring efficient and effective search process [17, 18]. Genetic learning processes cover different levels of complexity according to structural changes produced by the algorithm [19], from the simplest case of parameter optimization to the highest level of complexity of learning the rule set of a rule based system.

A genetic algorithm with binary chromosome, shuffle crossover, binary mutation, and selection with stochastic universal sampling is used for finding the consequent values of rules.

3.1 Structure of a chromosome

Let there be r number of rules. Each gene contains the gain values (consequent parameters), i.e., the i^{th}

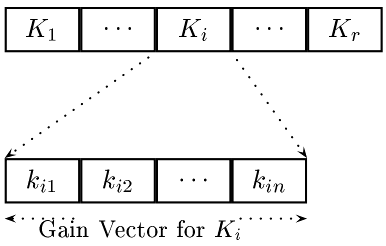

Figure 1: Hierarchical Structure of a Chromosome

gene contains the consequent gains of i^{th} rule namely $K_i = [k_{i1}\ k_{i2}\ \cdots\ k_{in}]$. The hierarchical structure of a chromosome is shown in figure 1, where K_1, K_2, $\cdots$, K_r represents the consequent gain vectors appearing in the consequent part of the r^{th} rules.

3.2 Fitness function

A fitness function involving absolute mean square error (AMSE) of state variables, status of feasibility of LMI (FLMI) and the number of stable subsystems (no_s) is used.

$$fit = \frac{func(FLMI, no_s)}{AMSE} \tag{10}$$

The $func$ must be chosen in such a way that, its value increases with the increase in the value of no_s and $FLMI$.

4 Simulation Results

4.1 Example 1 - Inverted Pendulum

A bench marking problem of inverted pendulum [4] is used for the study. The dynamic equation of the inverted pendulum is given by

$$\ddot{x} = f_1(x)x + f_2(x)u \tag{11}$$

where

$$\begin{aligned} f_1(x) &= \frac{g - amlx_2^2\cos(x_1)}{4l/3 - aml\cos^2(x_1)}\left(\frac{\sin(x_1)}{x_1}\right) \\ f_2(x) &= -\frac{a\cos(x_1)}{4l/3 - aml\cos^2(x_1)} \end{aligned}$$

where $x = [x_1\ x_2]^T = [\theta\ \dot{\theta}]^T$, θ is the angular displacement of the pendulum, $\dot{\theta}$ is the angular velocity of the pendulum, $g = 9.8m/s^2$ is the acceleration due to gravity, $m = 2Kg$ is the mass of the pendulum, $M = 8Kg$ is the mass of the cart, $2l = 1m$ is the length of the pendulum, u is the force applied to the cart and $a = 1/(m+M)$. The range of θ and $\dot{\theta}$ is given by $\theta \in [\theta_{min}\ \theta_{max}] = \left[\frac{-22\pi}{45}\ \frac{22\pi}{45}\right]$ and $\dot{\theta} \in [\dot{\theta}_{min}\ \dot{\theta}_{max}] = [-5\ 5]$.

The objective of this application example is to design a fuzzy controller to close the feedback loop of (11) such that $\theta = 0$ at steady state.

The nonlinear plant is represented by a fuzzy model with four fuzzy rules where the i^{th} rule is given by

$$\begin{aligned} &\text{Rule } i: \text{ If } f_1(x) \text{ is } M_1^i \text{ AND } f_2(x) \text{ is } M_2^i \\ &\qquad\qquad \text{THEN } \dot{x} = A_i x + B_i u \end{aligned} \tag{12}$$

for $i = 1, 2, 3, 4$ so that the system dynamics is described by

$$\dot{x} = \sum_{i=1}^{4} w_i(A_i x + B_i u) \tag{13}$$

where

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ f_{1_{min}} & 0 \end{bmatrix}$$

$$A_3 = A_4 = \begin{bmatrix} 0 & 1 \\ f_{1_{max}} & 0 \end{bmatrix}$$

$$B_1 = B_3 = \begin{bmatrix} 0 \\ f_{2_{min}} \end{bmatrix}$$

$$B_2 = B_4 = \begin{bmatrix} 0 \\ f_{2_{max}} \end{bmatrix}$$

$$f_{1_{min}} = 9, \quad f_{1_{max}} = 18$$

$$f_{2_{min}} = -0.1765, \quad f_{2_{max}} = -0.0052$$

The membership functions M_1^1, M_1^2, M_1^3, M_1^4, M_2^1, M_2^2, M_2^3, and M_2^4 are shown in figure 2.

Hence the controller rules are

$$\begin{aligned} &\text{Rule } i: \text{ If } f_1(x) \text{ is } M_1^i \text{ AND } f_2(x) \text{ is } M_2^i \\ &\qquad\qquad \text{THEN } u = -K_i\ x \end{aligned} \tag{14}$$

Genetic algorithm with population size = 20, binary chromosome with length = 80 bits (10 bits $\times$ 8 genes), crossover rate = 0.7 and mutation rate = 1/80 is used to find out the values of $K_i s$. The fitness function used is

$$f = \frac{2^{-sign(t_{min}) + no_s}}{AMSE} \tag{15}$$

where $sign(t_{min})$ is the sign of minimized t obtained by solving the LMI conditions

$$(A_i - B_i K_j)^T P + P(A_i - B_i K_j) < t \times I \tag{16}$$

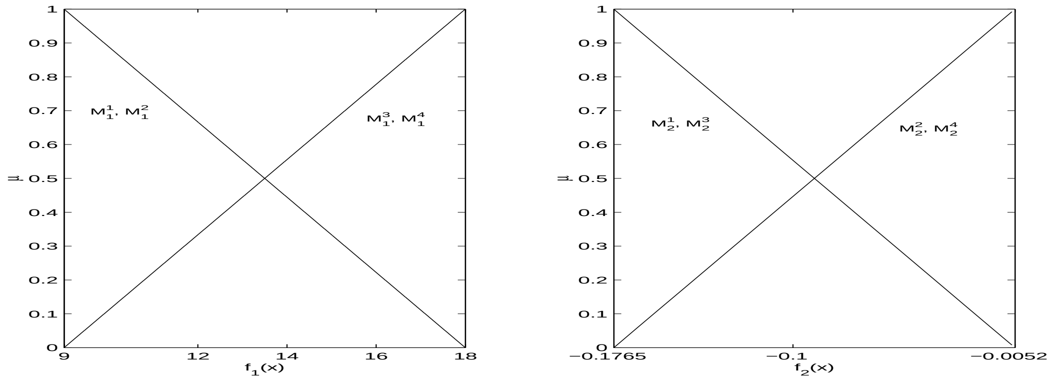

Figure 2: Plot of membership function. $M_1^1 = M_1^2 =$ MF(ls = -inf, c = 9, rs = 9), $M_1^3 = M_1^4 =$ MF(ls = 9, c = 18, rs = inf), $M_2^1 = M_2^3 =$ MF(ls = -inf, c = -0.1765, rs = 0.1713), $M_2^2 = M_2^4 =$ MF(ls = 0.1713, c = -0.0052, rs = inf), where ls, c, rs represents left spread, center right spread respectively.

using the $feasp$ function of MATLAB (remember t must be negative for feasibility), no_s is the number of stable subsystems $(A_i - B_iK_j)$ and

$$AMSE = \sum_{t=0.01:0.01:3} \frac{100x_1^2 + x_2^2 + 200(300 - 100t)}{300} \tag{17}$$

The model is simulated using genetic algorithm for 100 generations. The simulations are carried out on Celeron 2GHz machine using MATLAB 6. To simulate the inverted pendulum dynamics, a fourth-order Runge-Kutta method (ode45 [20]) is used with an integration time step size of 0.01 sec. Observation of the system states were made every iteration (integration step). The controllers are assumed to be continuous, therefore, the sampling time of the controller was set equal to the integration time step size. Figure 3 shows the plot of maximum and average fitness value obtained in each generation. Figure 4 shows the plot of the number of chromosomes satisfying stability conditions in each generation. The increasing maximum fitness, the increasing average fitness and their convergence observed from the figure clearly indicate the convergence of the proposed method. The increase in the number of stable chromosomes within 12 generations shows the efficiency of the proposed method in achieving stable chromosomes.

The values of K_i obtained after 100 generations are $K_1 = [-4231.7 \;\; -558.7]$, $K_2 = [-4500 \;\; -281.5]$, $K_3 = [-4165.7 \;\; -417.9]$, $K_4 = [-4486.8 \;\; -167.2]$.

The control system is simulated with the four rules for several initial conditions. The results are plotted for the initial conditions of $\theta = 85°$, $\theta = 60°$, and $\theta = 25°$. Figure 5 & 6 show the plot of θ and f respectively. It is clear from the figures that the angle of the pendulum (θ) balances within 1 seconds.

Sudden external disturbances are applied, (i) to the force with magnitude 500 Newton at 2 sec., (ii) to the angle (θ) with magnitude 25 deg at 5 sec., and (iii) to the angular velocity $(\dot{\theta})$ with magnitude -200 deg/sec at 8 sec. In all the three cases it can be seen from the figures of the response of θ (Figure 9) and f (Figure 10) that the angle of the pendulum balances. White Gaussian noise with signal to noise ratio of 25dB is added to the pendulum angle θ before giving to the fuzzy controller and the response of pendulum angle and force are plotted respectively in Figures 7 and 8. It can be seen from the figures that the pendulum balances with small oscillations even in the presence of noise.

The simulation tests reveal that the fuzzy controller not only successfully balances the pendulum but also does well under small perturbations on the controlled system. The fuzzy controller balances the pendulum with small oscillations even in the presence of noise in the measured state variables.

4.2 Example 2 - Cart pole balancing problem

A bench marking control problem namely the cart pole balancing (inverted pendulum with 4 variables) [21] is used for the study. The dynamic equation of

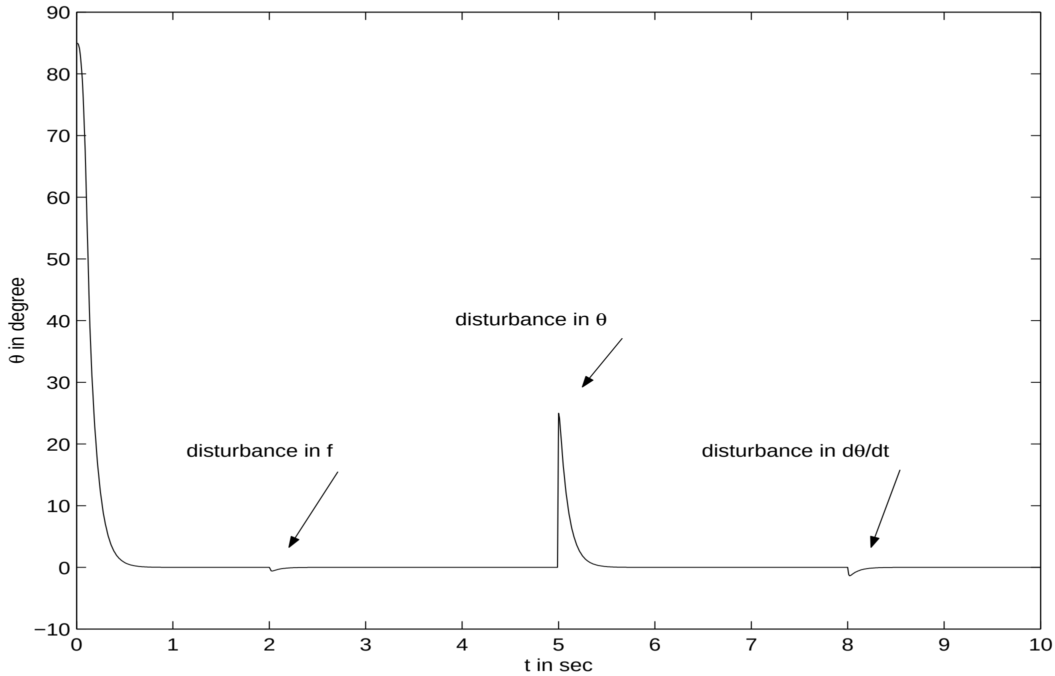

Figure 9: Response of θ for initial angle 85 deg. Sudden external disturbances are applied, to the force with magnitude 500 Newton at 2 sec., to the angle (θ) with magnitude 25 deg at 5 sec., and to the angular velocity ($\dot{\theta}$) with magnitude -200 deg/sec at 8 sec.

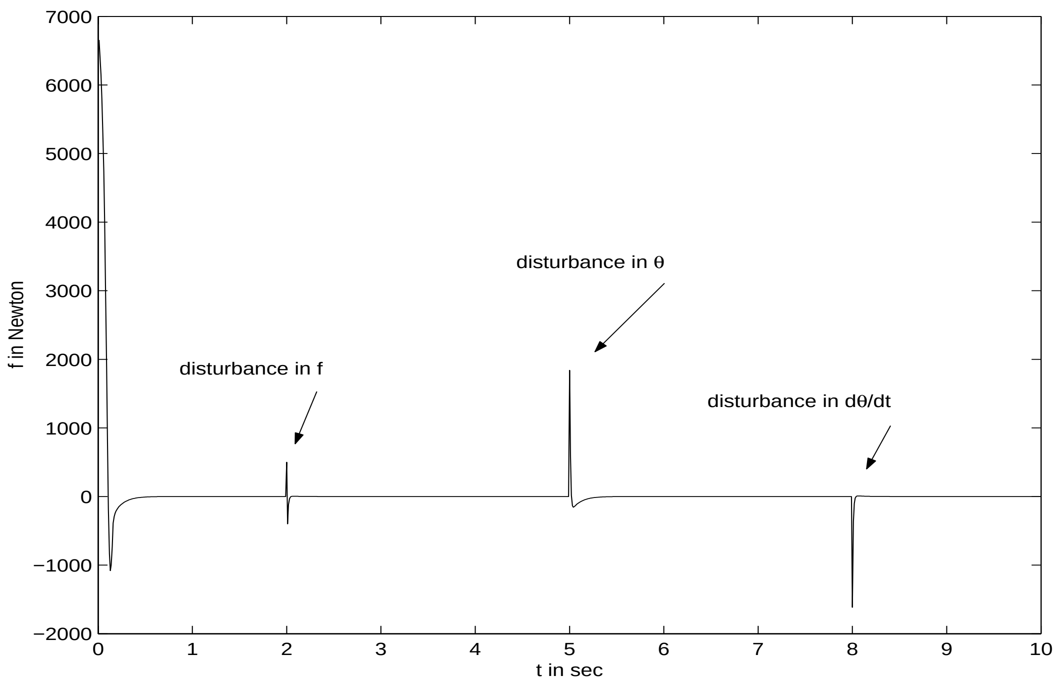

Figure 10: Control force for initial angle 85 deg. Sudden external disturbances are applied, to the force with magnitude 500 Newton at 2 sec., to the angle (θ) with magnitude 25 deg at 5 sec., and to the angular velocity ($\dot{\theta}$) with magnitude -200 deg/sec at 8 sec.

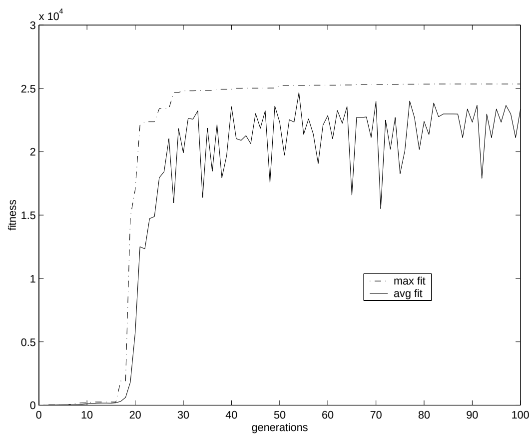

Figure 3: Plot of maximum fitness and average fitness in each generation

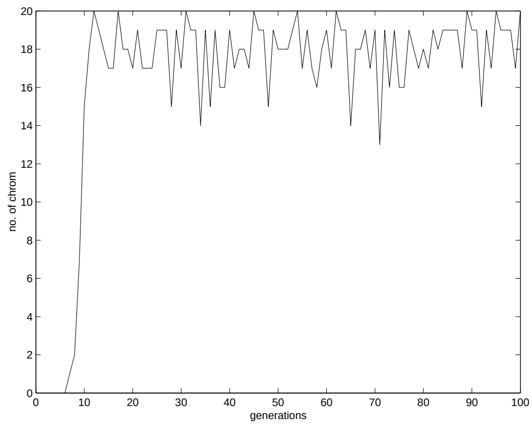

Figure 4: Plot of no. of stable chromosomes in each generation

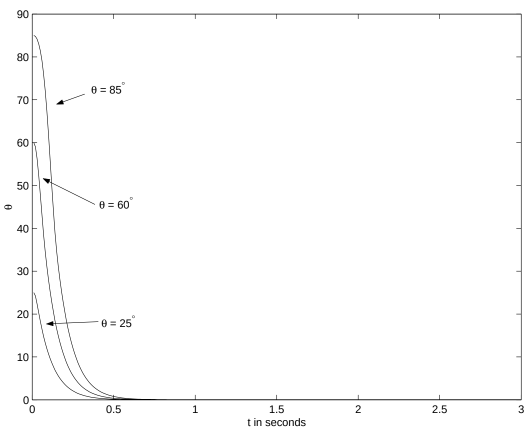

Figure 5: Plot of θ for initial angles 85°, 60° and 25°

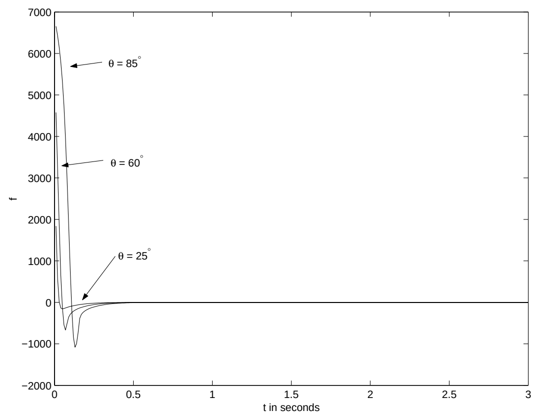

Figure 6: Plot of f for initial angles 85°, 60° and 25°

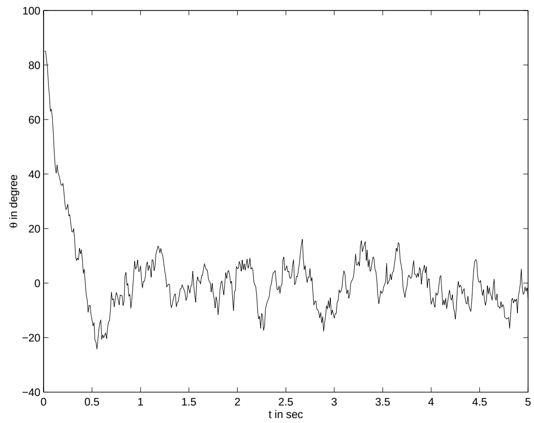

Figure 7: Response of θ for initial angle 85 deg. White Gaussian noise with signal to noise ratio of 25dB is added to the pendulum angle θ before giving to the fuzzy controller.

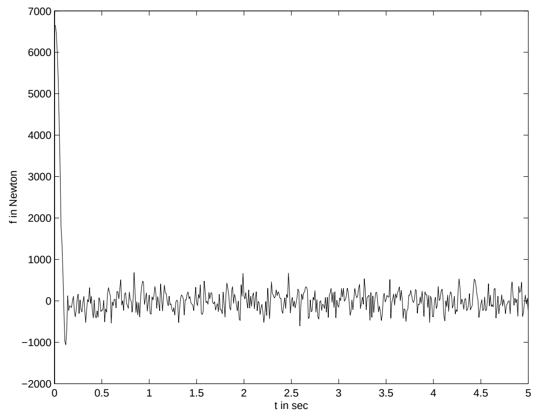

Figure 8: Control force for initial angle 85 deg. White Gaussian noise with signal to noise ratio of 25dB is added to the pendulum angle θ before giving to the fuzzy controller.

the cart pole is given by

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ f_3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ f_2 \\ 0 \\ f_4 \end{bmatrix} u \quad (18)$$

where

$$\begin{aligned} f_1 &= \frac{g - aml\dot{\theta}^2\cos(\theta)}{l(4/3 - am\cos^2(\theta))}\left(\frac{\sin(\theta)}{\theta}\right) \\ f_2 &= \frac{-a\cos(\theta)}{l(4/3 - am\cos^2(\theta))} \\ f_3 &= \frac{am(4l\dot{\theta}^2 - 3g\cos(\theta))}{4 - 3ma\cos^2(\theta)}\left(\frac{\sin(\theta)}{\theta}\right) \\ f_4 &= \frac{4a}{4 - 3ma\cos^2(\theta)} \end{aligned}$$

and θ is the angular displacement of the pendulum, $\dot{\theta}$ is the angular velocity of the pendulum, x is the position of the cart, $\dot{x}$ is the velocity of the cart, $g = 9.8m/s^2$ is the acceleration due to gravity, $m = 2Kg$ is the mass of the pendulum, $M = 8Kg$ is the mass of the cart, $2l = 1m$ is the length of the pendulum, u is the force applied to the cart and $a = 1/(m+M)$. The range of θ and $\dot{\theta}$ is given by $\theta \in [\theta_{min}\ \theta_{max}] = [\frac{-\pi}{3}\ \frac{\pi}{3}]$ and $\dot{\theta} \in [\dot{\theta}_{min}\ \dot{\theta}_{max}] = [-5\ 5]$. Let $x = [\theta\ \dot{\theta}\ x\ \dot{x}]$.

The objective of this application example is to design a fuzzy controller to close the feedback loop of the cart pole system such that $\theta = 0$ at steady state.

The nonlinear plant is represented by a fuzzy model with 16 fuzzy rules where the i^{th} rule is given by

$$\begin{aligned} \text{Rule } i: \ \text{If } f_1(x) \text{ is } M_1^i \text{ and } f_2(x) \text{ is } M_2^i \\ \text{and } f_3(x) \text{ is } M_3^i \text{ and } f_4(x) \text{ is } M_4^i \\ \text{THEN } \dot{x} = A_i x + B_i u \end{aligned}$$

for $i = 1, 2, \cdots, 16$ so that the system dynamics is described by

$$\dot{x} = \sum_{i=1}^{16} w_i(A_i x + B_i u) \quad (19)$$

where

$$A_1 = A_2 = A_5 = A_6 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_{1_{min}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ f_{3_{min}} & 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = A_4 = A_7 = A_8 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_{1_{min}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ f_{3_{max}} & 0 & 0 & 0 \end{bmatrix}$$

$$A_9 = A_{10} = A_{13} = A_{14} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_{1_{max}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ f_{3_{min}} & 0 & 0 & 0 \end{bmatrix}$$

$$A_{11} = A_{12} = A_{15} = A_{16} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ f_{1_{max}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ f_{3_{max}} & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} B_1 = B_3 = B_9 = B_{11} &= [0\ f_{2_{min}}\ 0\ f_{4_{min}}]^T \\ B_2 = B_4 = B_{10} = B_{12} &= [0\ f_{2_{min}}\ 0\ f_{4_{max}}]^T \\ B_5 = B_7 = B_{13} = B_{15} &= [0\ f_{2_{max}}\ 0\ f_{4_{min}}]^T \\ B_6 = B_8 = B_{14} = B_{16} &= [0\ f_{2_{max}}\ 0\ f_{4_{max}}]^T \end{aligned}$$

$$\begin{aligned} f_{1_{min}} &= 12.5660 & f_{1_{max}} &= 17.2941 \\ f_{2_{min}} &= -0.1765 & f_{2_{max}} &= -0.0779 \\ f_{3_{min}} &= -1.7294 & f_{3_{max}} &= -0.5456 \\ f_{4_{min}} &= 0.1039 & f_{4_{max}} &= 0.1176 \end{aligned}$$

The membership functions are shown in figure 11 where

$$\begin{aligned} M_1^i &= M_1^a & &\text{for } i = 1, \cdots, 8 \\ M_1^i &= M_1^b & &\text{for } i = 9, \cdots, 16 \\ M_2^i &= M_2^a & &\text{for } i = 1, \cdots, 4, 9, \cdots, 12 \\ M_2^i &= M_2^b & &\text{for } i = 5, \cdots, 8, 13, \cdots, 16 \\ M_3^i &= M_3^a & &\text{for } i = 1, 2, 5, 6, 9, 10, 13, 14 \\ M_3^i &= M_3^b & &\text{for } i = 3, 4, 7, 8, 11, 12, 15, 16 \\ M_4^i &= M_4^a & &\text{for } i = 1, 3, 5, 7, 9, 11, 13, 15 \\ M_3^i &= M_3^b & &\text{for } i = 2, 4, 6, 8, 10, 12, 14, 16 \end{aligned}$$

Hence the controller rules are

$$\begin{aligned} \text{Rule } i: \ \text{If } f_1(x) \text{ is } M_1^i \text{ and } f_2(x) \text{ is } M_2^i \\ \text{and } f_3(x) \text{ is } M_3^i \text{ and } f_4(x) \text{ is } M_4^i \\ \text{THEN } u = -K_i\ x \end{aligned}$$

Genetic algorithm with population size = 25, real value chromosome with length = 64 (16 rules × 4 coefficients), crossover rate = 0.7 and mutation rate = 1/64 is used to find out the values of $K_i s$. The fitness function used is

$$f = \frac{100 \times (-sign(t_{min}) + no_s)}{AMSE} \quad (20)$$

where $sign(t_{min})$ is the sign of minimized t obtained by solving the LMI conditions

$$(A_i - B_i K_j)^T P + P(A_i - B_i K_j) < t \times I \quad (21)$$

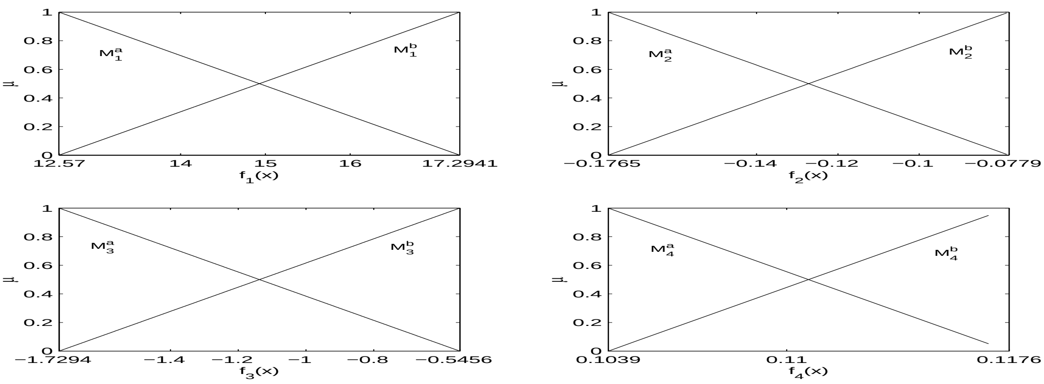

Figure 11: Plot of membership function. M_1^a = MF(ls = -inf, c = 12.5660, rs = 4.7281), M_1^b = MF(ls = 4.7281, c = 17.2941, rs = inf), M_2^a = MF(ls = -inf, c = -0.1765, rs = 0.0986), M_2^b = MF(ls = 0.0986, c = -0.0779, rs = inf), M_3^a = MF(ls = -inf, c = -1.7294, rs = 1.1838), M_3^b = MF(ls = 1.1838, c = -0.5456, rs = inf), M_4^a = MF(ls = -inf, c = 0.1039, rs = 0.0137), M_4^b = MF(ls = 0.0137, c = 0.1176, rs = inf), where ls, c, rs represents left spread, center and right spread respectively.

using the $feasp$ function of MATLAB (remember t must be negative for feasibility), no_s is the number of stable subsystems $(A_i - B_i K_j)$ and

$$AMSE = \sum_{t=0.01:0.01:25} \frac{100x_1^2 + x_2^2 + 100000(25-t)}{2500} \tag{22}$$

The model is simulated using genetic algorithm for 100 generations. The simulations are carried out on Celeron 2GHz machine using MATLAB 6. To simulate the dynamics of the system, a fourth-order Runge-Kutta method (ode45 [20]) is used with an integration time step size of 0.01 sec. Observation of the system states were made every iteration (integration step). The controllers are assumed to be continuous, therefore, the sampling time of the controller was set equal to the integration time step size. Figure 12 shows the plot of maximum and average fitness value obtained in each generation. Upto 700 generations, the fitness values increments very slowly (not clearly visible in the picture) due to the unstable chromosomes. The fitness value shows a sudden increase when some of the chromosomes satisfy stability conditions. Figure 13 shows the plot of the number of chromosomes satisfying stability conditions in each generation. The increasing maximum fitness, the increasing average fitness and their convergence observed from the figure clearly indicates the convergence achieved in the proposed method. The increase in the number of stable chromosomes with generation shows the efficiency of the proposed method in achieving stable chromosomes.

K_1 =	[-973.5600, -546.8342, -52.5238, -112.4410]
K_2 =	[-987.6997, -843.1480, -10.0466, -42.8366]
K_3 =	[-968.5482, -363.9544, -27.3352, -182.7170]
K_4 =	[-969.1127, -677.6675, -59.6569, -323.5316]
K_5 =	[-994.2449, -407.0647, -2.2464, -118.9776]
K_6 =	[-925.0248, -828.7409, -33.9427, -86.3122]
K_7 =	[-997.4092, -532.1673, -101.5708, -253.3797]
K_8 =	[-989.1540, -843.4428, -23.8058, -425.6207]
K_9 =	[-971.0495, -544.2801, -1.6111, -30.4194]
K_{10} =	[-953.6995, -207.0134, -0.7932, -9.1531]
K_{11} =	[-950.6810, -168.0409, -27.0666, -66.1644]
K_{12} =	[-968.8456, -865.8883, -0.5597, -412.7820]
K_{13} =	[-968.3291, -767.5462, -14.0625, -104.7118]
K_{14} =	[-968.0909, -471.0025, -8.9932, -19.4069]
K_{15} =	[-995.9967, -793.1754, -79.5778, -338.6939]
K_{16} =	[-898.0100, -762.1072, -13.9839, -375.7827]

Table 1: The values of K_i obtained after 1000 generations

The values of K_i obtained after 1000 generations are given in Table 1.

The control system is simulated with the sixteen rules for several initial conditions. The results are plotted for the initial conditions of $\theta = 60°$, and $\theta = 25°$. Figure 14, 15, & 16 show the plot of θ, x, and f respectively. It is clear from the figures that the angle of the pendulum (θ) balances within 4 seconds. The cart position (x) also attains zero after 150 seconds.

Sudden external disturbances are applied, (i) to the force with magnitude 500 Newton at 4 sec., (ii) to

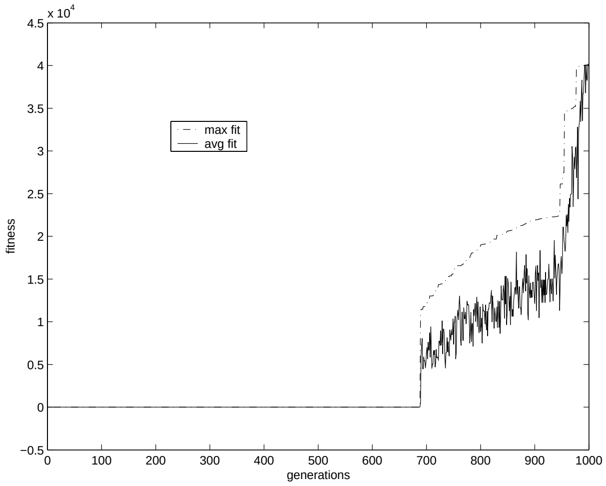

Figure 12: Plot of maximum fitness and average fitness in each generation

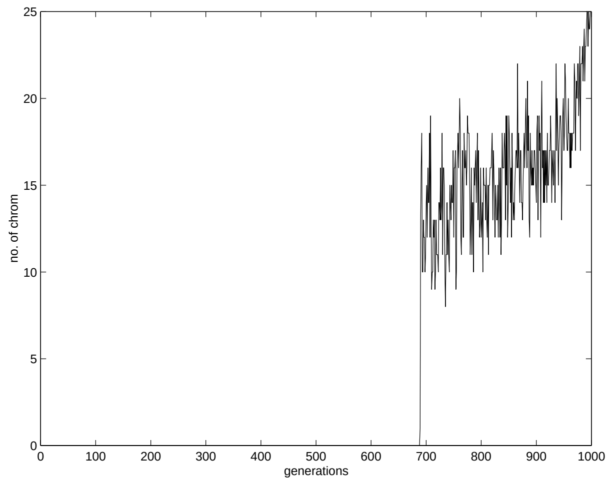

Figure 13: Plot of no. of stable chromosomes in each generation

the angle (θ) with magnitude 30 deg at 7 sec., and (iii) to the angular velocity ($\dot{\theta}$) with magnitude -100 deg/sec at 10 sec. In all the three cases it can be seen from the figures of the response of θ (Figure 20), x (Figure 21), and f (Figure 22) that the angle of the pendulum balances and attains steady state. White Gaussian noise with signal to noise ratio of 35dB is added to the pendulum angle θ before giving to the fuzzy controller and the response of pendulum angle, cart position, and force are plotted respectively in figures 17, 18, and 19. It can be seen from the figures that the pole balances with small oscillations in the presence of noise.

The simulation tests reveal that the fuzzy controller not only successfully balances the pendulum but also does well under some perturbations on the controlled system. The fuzzy controller balances the pole with small oscillations even in the presence of noise in the measured state variables.

5 Conclusion

A new algorithm has been proposed for the design of consequent parts of Takagi-Sugeno fuzzy controllers with guaranteed stability. The simulation results of inverted pendulum and cart-pole balancing examples show that the proposed method is effective and efficient.

The advantages of the proposed methods are summarized below.

1. It finds optimal consequent parts of Takagi-Sugeno fuzzy controllers.

2. No need to include a matrix of size $n \times n$ in the chromosome for searching a common positive definite matrix as in Lam et. al. [15].

3. Ensures stability of Takagi-Sugeno fuzzy controller by including a status term for stability in the fitness function.

References

[1] T. Takagi, M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Tans. System, Man and Cybernetics*, Vol. SMC-15, No. 1, pp. 116-132, 1985.

[2] J. Yoneyama, M. Nishikawa, H. Katayama, A. Ichikawa, "Design of output feedback controllers for Takagi-Sugeno fuzzy controllers," *Fuzzy sets and systems*, Vol. 121, pp. 127-148, 2001.

[3] J. Yoneyama, M. Nishikawa, H. Katayama, A. Ichikawa, "Output stabilization of Takagi-Sugeno fuzzy systems," *Fuzzy sets and systems*, Vol. 111, pp. 253-266, 2000.

[4] H.O. Wang, K. Tanaka, M. Griffin, "Parallel distributed compensation of nonlinear systems by Takagi-Sugeno fuzzy model," *IEEE ...*, pp. 531-538, 1995.

[5] H.O. Wang, K. Tanaka, M. Griffin, "An approach to fuzzy control of nonlinear systems: stability

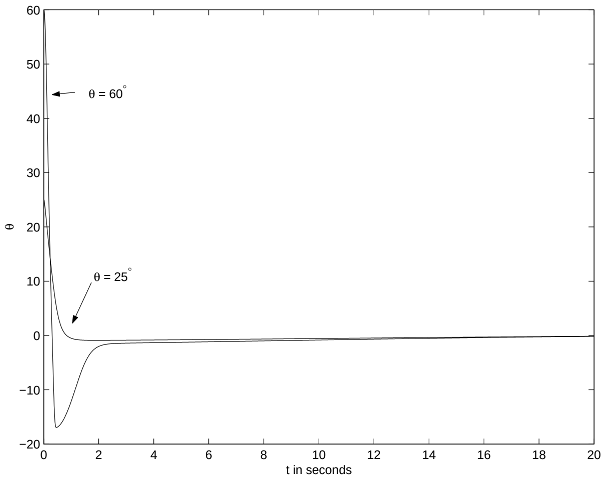

Figure 14: Plot of θ corresponding to the initial angles 60° and 25°

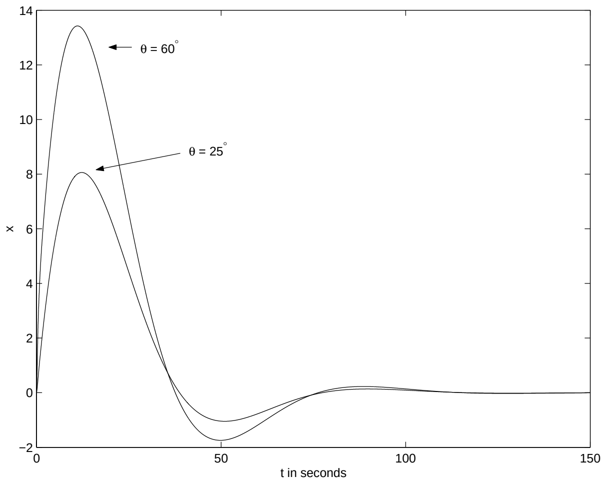

Figure 15: Plot of x corresponding to the initial angles 60° and 25°

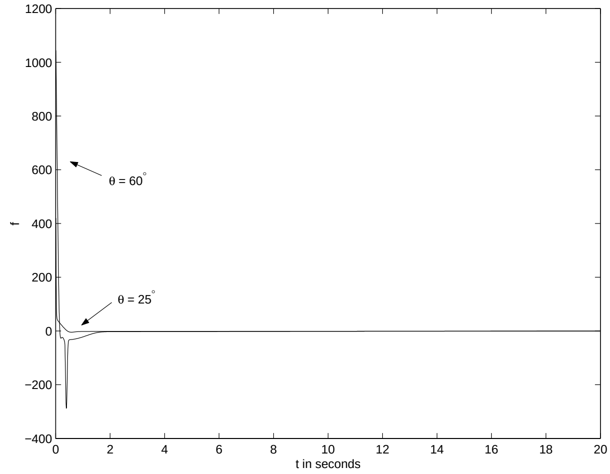

Figure 16: Plot of force corresponding to the initial angles 60° and 25°

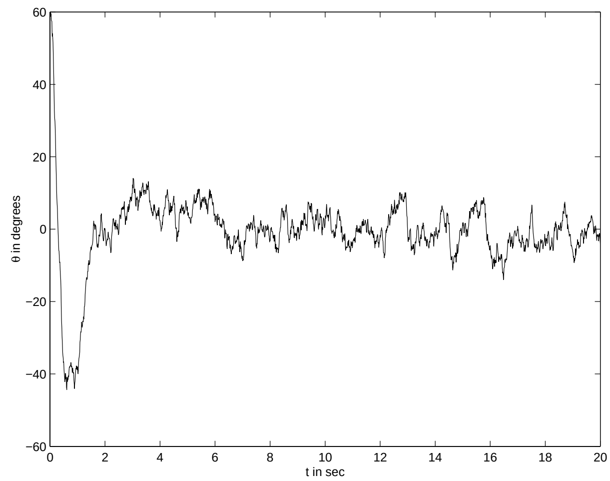

Figure 17: Response of θ for initial angle 60 deg. White Gaussian noise with signal to noise ratio of 35dB is added to the pendulum angle (θ) before giving to the fuzzy controller.

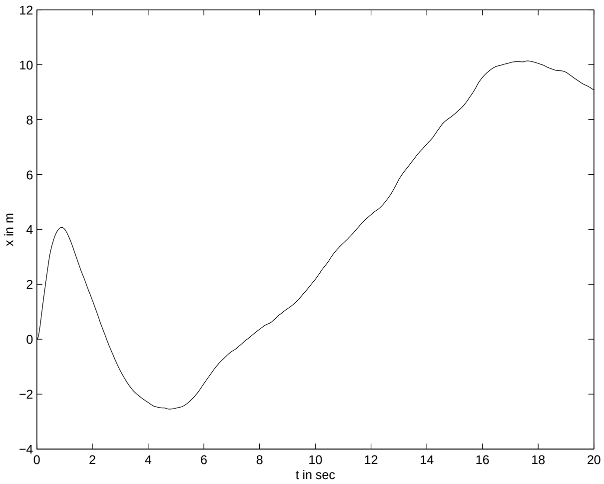

Figure 18: Response of x for initial angle 60 deg. White Gaussian noise with signal to noise ratio of 35dB is added to the pendulum angle (θ) before giving to the fuzzy controller.

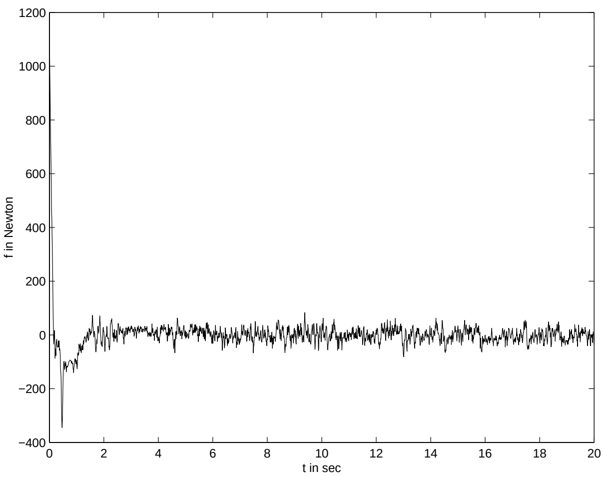

Figure 19: Control force for initial angle 60 deg. White Gaussian noise with signal to noise ratio of 35dB is added to the pendulum angle (θ) before giving to the fuzzy controller.

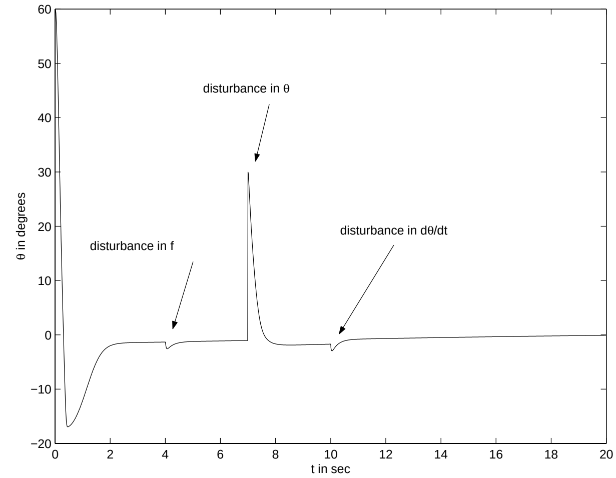

Figure 20: Response of θ for initial angle 60 deg. Sudden external disturbances are applied, to the force with magnitude 500 Newton at 4 sec., to the angle (θ) with magnitude 30 deg at 7 sec., and to the angular velocity ($\dot{\theta}$) with magnitude -100 deg/sec at 10 sec.

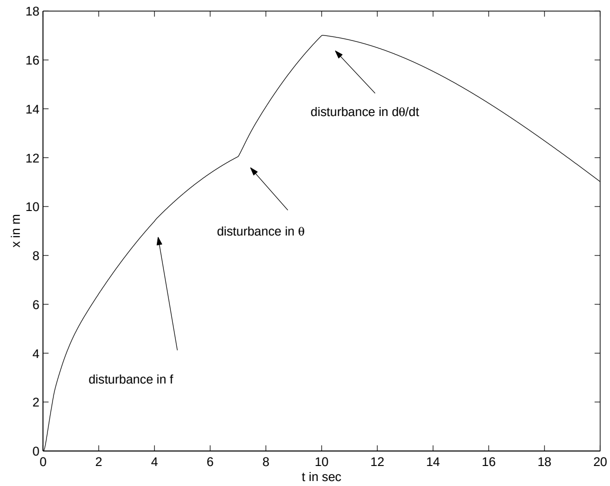

Figure 21: Response of x for initial angle 60 deg. Sudden external disturbances are applied, to the force with magnitude 500 Newton at 4 sec., to the angle (θ) with magnitude 30 deg at 7 sec., and to the angular velocity ($\dot{\theta}$) with magnitude -100 deg/sec at 10 sec.

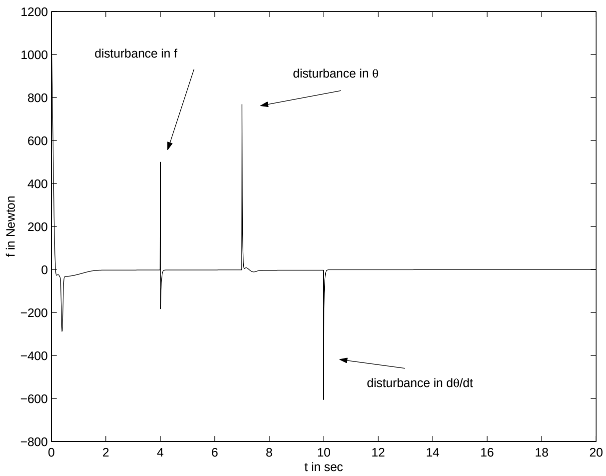

Figure 22: Control force for initial angle 60 deg. Sudden external disturbances are applied, to the force with magnitude 500 Newton at 4 sec., to the angle (θ) with magnitude 30 deg at 7 sec., and to the angular velocity ($\dot{\theta}$) with magnitude -100 deg/sec at 10 sec.

and design issues," *IEEE Trans. on fuzzy systems*, pp. 14-23, 1996.

[6] S.K. Hong, Y. Nam, "Stable fuzzy control system design with pole-placement constraint: an LMI approach," *Computers in Industry*, Vol. 51, pp. 1-11, 2003.

[7] X. Liu, Q. Zhang, "Approaches to quadratic stability conditions and H_∞ control designs for T-S fuzzy systems," *IEEE Trans. on fuzzy systems*, Vol. 11, No. 6, pp. 830-838, 2003.

[8] J. Joh, Y.-H. Chen, R. Langari, "On the stability issues of linear Takagi-Sugeno fuzzy models," *IEEE Trans. on fuzzy systems*, Vol. 6, No. 3, pp. 402-410, 1998.

[9] F. Matía, B.M. Al-Hadithi, A. Jiménez, "Generalization of stability criterion for Takagi-Sugeno continuous fuzzy model," *Fuzzy sets and systems*, Vol. 129, pp. 295-309, 2002.

[10] M.C.M. Teixeira, E. Assuncão, R.G. Avellar, "On Relaxed LMI-Based design for fuzzy regulators and fuzzy observers," *IEEE Trans. Fuzzy Systems*, Vol. 11, No. 5, pp. 613-623, 2003.

[11] J.-M. Zhang, R.-H. Li, P.-A, Zhang, "Stability analysis and systematic design of fuzzy control systems," *Fuzzy sets and systems*, Vol. 120, pp. 65-72, 2001.

[12] M.L. Hadjili, V. Wertz, "Takagi-Sugneo fuzzy modeling incorporating input variable selection," *IEEE Trans. on fuzzy systems*, Vol. 10, No. 6, pp. 728-742, 2002.

[13] O. Cordón, F. Gomide, F. Herrera, F. Hoffmann, L. Magdalena, "Ten years of genetic fuzzy systems: current framework and new trends," *Fuzzy Sets and Systems*, Vol. 141, pp. 5-31, 2004.

[14] W. Chang, J.B. Park, Y.H. Joo, "GA-based intelligent digital redesign of fuzzy-model-based controllers," *IEEE Trans. on fuzzy systems*, Vol. 11, No. 1, pp. 35-44, 2003.

[15] H.K. Lam, F.H. Leung, P.K.S. Tam, "Design and stability analysis of fuzzy model-based nonlinear controller for nonlinear systems using genetic algorithm," *IEEE Trans. on System, Man and Cybernetics*, Vol. 33, No. 2, pp. 250-257, 2003.

[16] K. Tanaka, M. Sugeno, "Stability analysis and design of fuzzy control systems," *Fuzzy sets and systems*, vol. 45, no. 2, pp. 135-156, 1992.

[17] M. Srinivas, L.M. Patnaik, "Genetic Algorithms: A Survey," *IEEE Computer*, pp. 17-26, Jun. 1994.

[18] D.E. Golberg, *Genetic Algorithms in search, optimization and machine learning*, Addison Wesley Longman, Inc., 1999.

[19] K. DeJong, "Learning with genetic algorithms: an overview," *Mach. Learning*, vol.3, no. 3, pp. 121-138, 1988.

[20] L.F. Shampine, M.W. Reichelt, "The MATLAB ODE suite," SIAM Journal on scientific computing, vol. 18, no.1, 1997.

[21] S. Zeng, Y. He, "Learning and tuning fuzzy logic controllers through genetic algorithm," *IEEE ...*, pp. 1632-1637, 1994.