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A Comparative Study of Probabilistic and Worst-case Tolerance Synthesis

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***Abstract*— The tolerance design directly influences the functionality of products and production costs. Tolerance synthesis is a procedure that distributes assembly tolerances between components. To ensure that the selected components and its tolerances give the assembly tolerance less than a desired value, worst-case analysis is commonly used. Even though the worst-case analysis is used, there is still a probability that a product performance is out of its tolerance. Moreover, using the worst-case analysis, some assumptions need to be realized. The first is, nominal value of components and assembly are at the desirable values. The second is, left-sided and right-sided tolerances are symmetric in both components and assembly. Use of probabilistic tolerance synthesis may relax those assumptions and quantify the loss due to falling out of assembly limit. This paper then gives a comparative study of worst-case and probabilistic tolerance syntheses. A numerical example is used to illustrate the use of worst-case and probabilistic tolerance syntheses. Conclusion and discussion are given.**

***Index Terms*—tolerance synthesis, quality engineering, randomness.**

I. INTRODUCTION

Due to fierce competition in the international marketplace, companies seek ever-increasing product quality as well as reducing costs. The selection of tolerance has a profound impact on the manufacturing processes, product costs, and functional quality. Hence, manufacturers have treated tolerance as a very important topic and realized that a proper selection of design tolerance is a key element in their effort to increase productivity, control product quality, and yield significant cost savings. A careful analysis and assignment of tolerance is given by manufacturers.

Tolerance is defined as the range between a specification limit and the nominal dimension. Based on the literature survey, two basic processes are considered in tolerance design. The first one is tolerance analysis -- the component tolerances are specified, and the resulting assembly variation and yield are calculated. The latter one called tolerance synthesis, which we consider through this paper, involves the allocation of the specified assembly tolerances among the component dimensions of an assembly to ensure a specified yield. The literature on tolerance synthesis has been reviewed by [1]-[3]. Two types of objectives have generally been used in the tolerance design. The first one is minimizing direct manufacturing cost and the second is minimizing the sensitivity of tolerances to variations in manufacturing processes and the service environment. This paper emphasizes in the first objective while limiting the tolerance variations. Kusiak and Feng [4]also consider the first objective and give a comparative study of deterministic tolerance synthesis. The study may be said as a worst case study. It is used to ensure that almost all assemblies meet the specified tolerance stackup or assembly limit, which may result in a high manufacturing cost. The consideration of tolerance randomness may reduce manufacturing cost while still satisfying specified assembly tolerance in an acceptable range, and it is called probabilistic tolerance synthesis. This paper gives a comparative study of those two methods (worst-case and probabilistic tolerance syntheses) through numerical examples. Finally, conclusion and discussion are given based on the examples.

II. WORST-CASE TOLERANCE SYNTHESIS

The issue of worst-case tolerance synthesis has been widely discussed. Most research assumes that nominal values of component alternatives are equal and concentrates on the tolerance of component alternatives only. The objective of worst-case tolerance synthesis usually is minimizing total manufacturing cost especially component costs. Hence, worst-case tolerance synthesis is about attempting to minimize component costs while satisfying the assembly tolerance. Several methods are used to solve the problem. The integer programming approach to discrete tolerance synthesis may have been used for the first time by [5]. Then Monte and Datseris [6] extend the IP approach given by [5] to consider a variation of the integer programming approach. Further, Lee and Woo [7] proposed a branch and bound algorithm to solve a large-scale worst-case tolerancing problem. Since the value of tolerance depends on the corresponding manufacturing costs, the deterministic tolerance synthesis problem can be formulated as the following 0-1 integer programming model [5].

Minimize

$$y = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij} x_{ij} \le T \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \tag{3}$$

$$x_{ij} = 0, 1 \quad \forall i, j \tag{4}$$

where

c_{ij} = manufacturing cost of process *j* used to produce dimension *i*

t_{ij} = three sigma normal variation of process *j* used to produce dimension *i*

T = single side tolerance assembly limit for dimensional chain

x_{ij} = 1 if process *j* is selected for dimension *i*, and 0 otherwise

m = number of dimensional tolerances

n = number of processes available for producing dimension *i*

The objective shown in (1) is to minimize the total manufacturing costs of components. The constraint in (2) ensures that the total tolerance does not exceed the assembly tolerance limit. The constraint in (3) ensures that exactly one process is selected to generate each tolerance. The constraint in (4) ensures the integrality of x_{ij}. The utilization of the model is shown in [4] who applying the IP method in three different examples. In this paper, those examples would be studied comparatively with probabilistic tolerance synthesis. The examples would be expressed later.

III. PROBABILISTIC TOLERANCE SYNTHESIS

Even though worst-case synthesis is quite simple, there are some assumptions behind. The assumptions include nominal values of components and assembly be the desirable values; left-sided and right-sided tolerances be symmetric; and the variations of tolerances be normally distributed with mean equal to 0. Moreover, in the case of assembly tolerance is out of the assembly limit, there is no consideration of losses. Use of probabilistic tolerance synthesis may relax those assumptions and quantify the loss due to assembly performance falling out of its tolerance limit. Therefore, the cost consideration is changed from manufacturing component costs to the sum of manufacturing component costs and expected loss due to out of the limit. The expected loss can be calculated by multiplying unit loss (*O*) and the probability of an assembly falling out of its limit. Assume that an assembly falling out of its limit is rejected. In the case of high component tolerance, cost of component is normally low but it incurs high probability of rejected assembly. On the contrary, when the component tolerance is low, the component cost seems to be high but it incurs low probability of rejected assembly. Therefore, the optimization model attempts to trade-off between those two costs.

Minimize

$$\begin{aligned} y = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} + O \,\mathrm{Pr}\left[\left[\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij} x_{ij} < \left(y_T - T_L\right)\right]\right. \\ \left. + \mathrm{Pr}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij} x_{ij} > \left(y_T + T_R\right)\right]\right] \end{aligned} \tag{5}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \tag{6}$$

$$x_{ij} = 0, 1 \quad \forall i, j \tag{7}$$

where

y_{ij} = component length obtaining from process *j* used to produce dimension *i*

y_T = assembly length of dimensional chain

T_L = left-side assembly tolerance limit of dimensional chain

T_R = right-side assembly tolerance limit of dimensional chain

The objective of the model is to minimize the total costs which include manufacturing component costs and the costs of rejected assembly. This is based on the assumption that if the assembly is out of the limits, it is rejected. If not, it can be used. The unit cost of rejected assembly is assigned. Hence, the expected rejected costs are the summation of unit cost of rejected assembly and its probability. And even though an assembly is rejected, the manufacturing component costs are still paid. The objective then shows the summation of those two types of costs. An associated constraint is to ensure that exactly one process is selected to generate each tolerance.

In the case that component lengths follow normal distributions with mean μ_{ij} and variance σ_{ij}^2 for all *i* and *j*. The expected mean of assembly length is $\sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{ij} x_{ij}$, while the expected variance is $\sum_{i=1}^{m}\sum_{j=1}^{n} \sigma_{ij}^{2} x_{ij}$. Knowing the expected mean and variance of assembly length, the right term of the objective function (5) can be calculated. The comparative study will give an illustration and comparison of the two models.

IV. COMPARATIVE STUDY

In the section, an example is given to illustrate the uses of the two tolerance syntheses. Figure 1 shows an example problem given by [4]. Using the worst-case tolerance synthesis, the optimization model is shown as follows [4].

Minimize

$$y = 92x_{11} + 65x_{12} + 57x_{13} + 92x_{21} + 75x_{22}$$
$$+57x_{23} + 65x_{31} + 42x_{32} + 20x_{33} \quad (8)$$

subject to

$$2x_{11} + 4x_{12} + 5x_{13} + 2x_{21} + 3x_{22} + 5x_{23}$$
$$+4x_{31} + 6x_{32} + 10x_{33} \le 16 \quad (9)$$
$$x_{11} + x_{12} + x_{13} = 1 \quad (10)$$
$$x_{21} + x_{22} + x_{23} = 1 \quad (11)$$
$$x_{31} + x_{32} + x_{33} = 1 \quad (12)$$
$$x_{ij} = 0,1 \quad i = 1,2,3;\ j = 1,2,3$$

Solving the problem, the following solution is obtained: $x_{13} = x_{23} = x_{32} = 1$ and $y = 156$.

Since t_{ij} is assumed as three sigma normal variation of process *j* used to produce dimension *i*, standard deviations and variances of components producing by process *j* and used on dimension *i* are shown in Table I. Assume that *O* equal to 500. The optimization model turns to be:

Minimize

$$y = 92x_{11} + 65x_{12} + 57x_{13} + 92x_{21} + 75x_{22}$$
$$+57x_{23} + 65x_{31} + 42x_{32} + 20x_{33}$$
$$+500\big[\Pr\big[2x_{11} + 4x_{12} + 5x_{13} + 2x_{21}$$
$$+3x_{22} + 5x_{23} + 4x_{31} + 6x_{32} + 10x_{33} < 16\big]$$
$$+\big[\Pr\big[2x_{11} + 4x_{12} + 5x_{13} + 2x_{21} + 3x_{22}$$
$$+5x_{23} + 4x_{31} + 6x_{32} + 10x_{33} > 16$$

subject to

$$x_{11} + x_{12} + x_{13} = 1 \quad (13)$$
$$x_{21} + x_{22} + x_{23} = 1 \quad (14)$$
$$x_{31} + x_{32} + x_{33} = 1 \quad (15)$$
$$x_{ij} = 0,1 \quad i = 1,2,3;\ j = 1,2,3$$

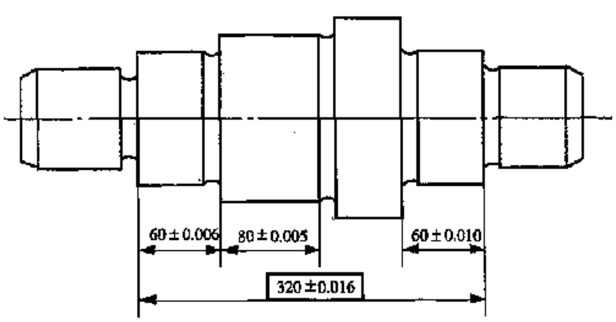

Figure 1 Tolerance of shaft
Source: Kusiak and Feng (1995)

Table I standard deviation and variance of components

σ_{11}	0.67	σ_{11}^2	0.44
σ_{12}	1.33	σ_{12}^2	1.78
σ_{13}	1.67	σ_{13}^2	2.78
σ_{21}	0.67	σ_{21}^2	0.44
σ_{22}	1.00	σ_{22}^2	1.00
σ_{23}	1.67	σ_{23}^2	2.78
σ_{31}	1.33	σ_{31}^2	1.78
σ_{32}	2.00	σ_{32}^2	4.00
σ_{33}	3.33	σ_{33}^2	11.11

Using Excel Solver to find the optimality of the model, the solution is $x_{13} = x_{23} = x_{33} = 1$ and the objective value is 134. Substituting the values obtained from the worst-case model, the costs show as 156.

V. CONCLUSION AND DISCUSSION

This paper gives a comparative study of worst-case and probabilistic tolerance syntheses. Comparing the optimization models, the model of worst-case considers only manufacturing component costs while assuming that all assemblies are in the tolerance limits. Actually, using worst-case model does not guarantee that all assemblies fall in the tolerance limit. There is a probability that an assembly is out of the tolerance limit. If that is the case, the loss of rejecting the assembly is incurred. Worst-case model does not take this loss in the account. Hence, the total cost of worst-case model seems to be lower than that of the probabilistic model. In the probabilistic model, both types of costs including manufacturing component costs and loss due to rejected assemblies are considered. The objective is trying to minimize the total costs while the probability of rejected assemblies is not limited. Based on the example, the probabilistic tolerance synthesis gives lower overall costs but if considering deeply, it also gives higher probability of rejected assemblies. In the case that assembly performance is required, high value of unit loss should be set. The result then tends to be the same as that of worst-case analysis.

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