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Generalized Linear Quadratic Gaussian and Loop Transfer Recovery Design of F-16 Aircraft Lateral Control System

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***Abstract*—This paper presents a generalized linear quadratic Gaussian and loop transfer recovery method to design an optimal lateral control system for an F-16 aircraft. The traditional linear quadratic Gaussian and loop transfer recovery method has been quite popular for aircraft control system design. In the proposed method we derive an optimal control law to minimize a generalized linear quadratic performance index to achieve better recovery quality in the LTR process and better some performance in time-domain responses and frequency-domain responses as well. The resulting controller can achieve a prescribed degree of stability even in the face of a non-minimum phase problem. Finally, the numerical simulation of an optimal lateral control system for an F-16 aircraft demonstrates the proposed method can provide good robustness and performance properties in both time- and frequency-domain responses.**

***Index Terms*—Generalized linear quadratic Gaussian, Loop transfer recovery, Optimal control**

I. Introduction

A powerful Linear Quadratic Gaussian and Loop Transfer Recovery (LQG/LTR) approach, originally proposed by Doyle *et al.* [1], is an optimal control design for many multivariable systems. In some practical circumstances the dynamics of controlled plant may not be exactly modeled, and there may be system disturbances and measurement noises in the plant. The traditional LQG/LTR method can provide good performance and guaranteed stability in the face of such noises. It allows such good robustness and performance properties with a prominent "loop shaping" concept for the principal gains of the return ratio in a two-step design procedure. The first step is to design an optimal state-feedback controller subject to a Linear Quadratic (LQ) performance index and also "loop shaping" a target feedback loop evaluated at the input of plant to meet the satisfactory specifications. The resulting static state-feedback controller has certain guaranteed robustness properties with infinite gain margin and at least 60° phase margin in each input channel. The second step is to design a Kalman filter to make the return ratio at the input of the LQG-compensated plant sufficiently close to the above target feedback loop as possible. The adoption of Kalman filter can provide some built-in robustness in the presence of system modeling uncertainty and noises [3]. The literatures [4]-[10] have adopted above design procedure and made much contribution to the improvement in the recovery quality of target feedback loop function for a general system that includes both minimum- phase and non-minimum phase systems. In addition, they have paid much attention to the Kalman filter design for the recoverable quality of the return ratio in frequency-domain requirements. On the other hand, Anderson *et al.* [11] have used the Hamilton-Jacobi equation to design a state-feedback controller with a prescribed degree of stability to minimize a modified LQ performance index. This motivates us to develop a Generalized LQG/LTR (GLQG/LTR) method with a Generalized LQ (GLQ) performance index to better the recovery quality and some performance.

From practical point of view, it is appropriate to design the return ratio at the output of the plant rather than the input point. The design procedure is dual to that described above. According to the separation principle, a Kalman filter is first designed to provide an optimal estimate of the state vector and to shape the principal gains of the target feedback loop at the output of the plant to meet the requirements. Then, an optimal state-feedback controller is designed to make the return ratio at the output of LQG-compensated plant sufficiently converge toward the resulting target feedback loop in the LTR process as close as possible. The proposed GLQG/LTR method also adopts above design procedure for practical consideration. In addition, we design a state-feedback controller subject to a tunable GLQ performance index in the LTR process. The resulting controller can provide a prescribed degree of stability.

For easy of presentation, the problem formulation of an optimal lateral control system for F-16 aircraft is first briefed in Section II. And then we design the Kalman filter for target feedback loop, as well as state-feedback controller with a prescribed degree of stability in the LTR procedure. In Section III, we demonstrate an optimal lateral control system design for an F-16 aircraft with the proposed method, and comparisons with that obtained by the traditional LQG/LTR method are also given. Finally, brief conclusions are drawn in Section IV.

II. Problem and Methodology Formulation

A. F-16 Aircraft Lateral Model

A nonlinear F-16 model has been linearized at some

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nominal flight condition by Stevens and Lewis [11]. The state variables of lateral dynamic and are sideslip β, bank angle ϕ, roll rate p, and yaw rate r. The state variables δ_r and δ_a are also introduced by the deflections of aileron and rudder actuators with the approximating transfer function 20.2/(s+20.2). The aircraft turn coordinator control system in Fig. 1 is to provide coordinated turns by making the bank angle follow a desired command and simultaneously keep the sideslip angle at zero as possible. Therefore, it is a diagonalizable plant with a two-channel input vector $u = \begin{bmatrix} u_\phi & u_\beta \end{bmatrix}^T$ and a two-variable output vector $y = \begin{bmatrix} \phi & \beta \end{bmatrix}^T$. The principal gains of the linearized aircraft dynamics at a nominal flight condition show constant values and wide separation of principal gains at lower frequencies. The plant is a type-0 system with constant steady state error in the performance of reference command tracking. To eliminate the steady-state error and make the speed of the system response in all input channels be nearly the same, two integrators are inserted in each input channel along with a pre-compensator to ensure the balance of the principal gains at lower frequency. Lewis [12] has developed the method to design the pre-compensator for the low-frequency balancing. The overall state vector including aircraft state variables, actuators, and integrators is defined as

$$x = \begin{bmatrix} \beta & \phi & p & r & \delta_a & \delta_r & \varepsilon_\phi & \varepsilon_\beta \end{bmatrix}^T \tag{1}$$

And the resulting augmented system is expressed in the form of state-variable model. The state equation is

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2}$$

with

$$A = \begin{bmatrix} -0.3220 & 0.0640 & 0.0364 & -0.9917 & 0.0003 & 0.0008 & 0 & 0 \\ 0 & 0 & 1 & 0.0037 & 0 & 0 & 0 & 0 \\ -30.6492 & 0 & -3.6784 & 0.6646 & -0.7333 & 0.1315 & 0 & 0 \\ 8.5395 & 0 & -0.0254 & -0.4764 & -0.0319 & -0.0620 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20.2 & 0 & -0.01 & -5.47 \\ 0 & 0 & 0 & 0 & 0 & -20.2 & -0.168 & 51.71 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{4}$$

And the output equation is

$$y = Cx(t) \tag{5}$$

with

$$C = \begin{bmatrix} 0 & 57.2958 & 0 & 0 & 0 & 0 & 0 & 0 \\ 57.2958 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

The lateral control system of F-16 aircraft is aimed to provide coordinated turns by making the bank angle swiftly track a desired command and simultaneously keep the sideslip angle at zero as possible. Therefore the tracking error is

$$e(t) = \begin{bmatrix} e_\phi(t) \\ e_\beta(t) \end{bmatrix} = \begin{bmatrix} r_\phi(t) - \phi(t) \\ r_\beta(t) - \beta(t) \end{bmatrix} \tag{7}$$

B. *Problem Definition*

Suppose the plant is generally described by the dynamic equations in the form of state-space representation shown in Fig. 2 as follows

$$\dot{x}(t) = Ax(t) + Bu(t) + \Gamma w(t) \tag{8}$$

and

$$y(t) = Cx(t) + v(t) \tag{9}$$

where $x(t) \in \Re^{\mathrm{n}}$, $u(t) \in \Re^{\mathrm{m}}$, and $y(t) \in \Re^{\mathrm{q}}$ are the state, input, and output vectors, respectively, $A \in \Re^{\mathrm{n\times n}}$, $B \in \Re^{\mathrm{n\times m}}$, $\Gamma \in \Re^{\mathrm{n\times p}}$, and $C \in \Re^{\mathrm{q\times n}}$ are the state, input of plant, input of disturbance, and output matrices, respectively. The system disturbance $w(t)$ and the measurement noise $v(t)$ are p- and q- dimensional uncorrelated Gaussian white noise processes with zero-mean, and the associated covariance matrices are defined as:

$$E\{w(t)w^T(\tau)\} = W(t)\delta(t-\tau) \tag{10}$$

$$E\{v(t)v^T(\tau)\} = V(t)\delta(t-\tau) \tag{11}$$

and

$$E\{v(t)w^T(\tau)\} = 0 \tag{12}$$

where $E\{\cdot\}$ is an expectation function operator, $W(t)$ and $V(t)$ are the system disturbance and measurement noise covariance matrices, respectively. The performance index can be modified in the form of generalized GLQ

$$J = E\{0.5\int_0^{t_f} \exp(2\alpha t)[e^T(t)Qe(t) + u^T(t)Ru(t)]dt\} \tag{13}$$

where Q and R are $\mathrm{m}\times\mathrm{m}$ positive semi-definite and positive-definite weighting matrices, respectively, and α is a nonnegative constant which can provide with the compensated plant with a prescribed degree of stability in the LQ regulation problem. If we choose $\alpha = 0$, then an optimal state-feedback controller is designed by the traditional LQG/LTR method. To have a prescribed relative degree of stability and better good recovery quality as well as performance properties, we adopt the proposed GLQG/LTR approach to have another degree of freedom to manipulate the nonnegative parameter α.

C. *Methodology Formulation*

According to the separation principle, we firstly design a Kalman filter to provide an optimal estimated state vector and shape the principal gains of the return ratio at the output of the plant to meet the system specifications, and then design an optimal state-feedback controller subject to the GLQ performance index in the LTR process.

1) Kalman filter design

The Kalman filter would be designed by substituting $x(t)$ with $\hat{x}(t)$, which is defined by the following state estimation equation

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f(y - C\hat{x}) \tag{14}$$

where K_f is a Kalman-filter gain matrix defined as

$$K_f = P_f C^T V^{-1} \tag{15}$$

and P_f is the covariance of $x - \hat{x}$, defined as

$$P_f = E[(x-\hat{x})(x-\hat{x})^T] \tag{16}$$

which can be determined by the following Filter Algebraic Riccati Equation (FARE)

$$AP_f + P_f A^T + \Gamma W \Gamma^T - P_f C^T V^{-1} C P_f = 0 \tag{17}$$

There are two assumptions for the existence of the Kalman filter. To assume that $(A, \Gamma\sqrt{W})$ is reachable means the system disturbance on the plant couples with the plant directly on the states, thus the input matrix of disturbance Γ is chosen as the identity matrix. Another assumption that $V > 0$ means the measurement noise corrupts the output vector. The Kalman-filter gain matrix for a general system can be determined by manipulating the covariance matrices W and V. The Kalman filter is used to shape the open-loop principal gains of the return ratio $G_t(s) = -C(sI - A)^{-1} K_f$ to meet the required specifications. $G_t(s)$ is called a target feedback loop. The associated sensitivity function and complementary sensitivity function are then defined as

$$S_f(s) = [I + C(sI - A)^{-1} K_f]^{-1} \tag{18}$$

and

$$T_f(s) = I - S_f(s) \tag{19}$$

The key points of a Kalman filter design are to meet the crossover frequencies of the principal gains of return ratio for the open-loop transfer function, to balance the principal gains as possible, and to adjust the low-frequency behavior. In addition, the suitable step responses of the target feedback loop are considered.

2) *Recovery target loop transfer function at plant output*

An optimal control law for the GLQ problem is derived as follows. According to the associated performance index defined as (13), the corresponding Hamilton function can be defined as

$$H = 0.5\exp(-2\alpha t)(\hat{x}^T Q x + u^T R u) + P^T(A\hat{x} + Bu) \tag{20}$$

The optimal control law can be derived by satisfying the Euler-Lagrange equations, which can be obtained

$$u = -\exp(-2\alpha t) R^{-1} B^T P \tag{21}$$

Assume that $P = P_c \hat{x}$. After some manipulations, we have

$$[\dot{P}_c + P_c A + A^T P_c - \exp(-2\alpha t) P_c B R^{-1} B^T P_c + \exp(-2\alpha t) Q]\hat{x} = 0 \tag{22}$$

Since (22) is always true for any $\hat{x}$, we have

$$\dot{P}_c + P_c A + A^T P_c - \exp(-2\alpha t) P_c B R^{-1} B^T P_c + \exp(-2\alpha t) Q = 0 \tag{23}$$

According to a sub-optimal control law, the control input can be defined

$$u(t) = -K_c \hat{x}(t) \tag{24}$$

where K_c is the gain matrix of state-feedback controller defined as

$$K_c = R^{-1} B^T P_c \tag{25}$$

where P_c is a positive definite symmetric matrix, which satisfies the following Controller Algebraic Riccati equation (CARE).

$$P_c(A + \alpha I) + (A^T + \alpha I)P_c - P_c B R^{-1} B^T P_c + Q = 0 \tag{26}$$

Therefore, the closed-loop dynamic equation of compensated system can be arranged as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} A & -BK_c \\ K_f C & A - BK_c - K_f C \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} B & \Gamma & 0 \\ B & 0 & K_f \end{bmatrix} [r_c(t) \quad w(t) \quad v(t)]^T \tag{27}$$

and

$$y(t) = [C \quad 0][x(t) \quad \hat{x}(t)]^T + [0 \quad 0 \quad I][r_c(t) \quad w(t) \quad v(t)]^T \tag{28}$$

Since the transfer function of observer-based state- feedback controller is

$$K(s) = -K_c(sI - A + BK_c + K_f C)^{-1} K_f \tag{29}$$

The return ratio at the output of compensated plant is

$$G(s)K(s) = -C(sI - A)^{-1} BK_c (sI - A + BK_c + K_f C)^{-1} K_f \tag{30}$$

The associated sensitivity function and complementary sensitivity function for the compensated plant are respectively defined as

$$S_{GK}(s) = [I + G(s)K(s)]^{-1} \tag{31}$$

and

$$T_{GK}(s) = S(s)G(s)K(s) \tag{32}$$

In the GLQG/LTR methodology, the closed-loop eigenvalues of compensated plant are just the union of the eigenvalues of a Kalman filter and those of an optimal state-feedback controller. The weighting matrices Q and R in the GLQ performance index are tunable parameters. We manipulate these parameters to recover the principal gains of the return ratio $G(s)K(s)$ at the output of compensated plant to the target feedback loop $G_t(s)$ as close as possible. In synthesis of a state-feedback controller, we can solve the CARE defined in (26) with $Q = I$ and $R = \rho I$. As $\rho \to 0$, it is obvious that $\lim_{\rho \to 0} G(s)K(s,\rho) = G_t(s)$. The higher gain matrix of state-feedback controller, the better recoverable quality in the LTR process. Besides, there is an additional parameter α in the proposed method, which can be tuned to get better performance.

III. Numerical Example and Simulation Results

The principal gains of the augmented F-16 aircraft with integrators and pre-compensator are shown in Fig. 3. At lower frequencies the principal gains is -20 dB/decade, and the augmented is a type-1 system with zero steady-state error in tracking the step response. In addition, the principal gains are well-balanced to have the approximately same speed of time-domain responses in the two input channels.

A. Kalman Filter Design for Target Feedback Loop

We design the Kalman filter by solving the FARE (9). There are three matrices in (9). Lewis et al [13] have chosen $\Gamma = I$, $W = diag\left(0.01 \quad 0.01 \quad 0.01 \quad 0.01 \quad 0 \quad 0 \quad 1 \quad 1\right)$, and $V = 1 \times I$. The Kalman-filter gain matrix is obtained

$$K_f = \begin{bmatrix} -0.0066 & 0.1300 & 0.1993 & -0.0932 & -0.1981 & 1.8662 & 0.6846 & 0.7290 \\ 0.0967 & -0.0066 & -0.1978 & -0.0201 & -0.1855 & 1.7653 & -0.7290 & 0.6846 \end{bmatrix}^T \tag{33}$$

The principal gains of target feedback loop $G_t(s)$, sensitivity function $S_f(s)$, and the complementary sensitivity function $T_f(s)$ at the plant output are shown in Figs. 4-5. The associated unit-step responses of bank angle are shown in Fig. 6.

B. Optimal Controller Design in LTR Process

The tunable parameters Q, R, and α are manipulated to shape the principal gains of return ratio, sensitivity function, and complementary sensitivity function for the compensated plant to get better recoverable quality. After some iterations, the parameters ρ and α are selected as

$$\rho = 1 \times 10^{-11} \tag{33}$$

and

$$\alpha = 12 \tag{34}$$

The gain matrix of state-feedback controller is obtained as

$$K_c = \begin{bmatrix} -8.5944 & 3.6563 & 0.3211 & 1.0384 & -0.0088 & -0.0009 & 0.0000 & -0.0000 \\ 1.7032 & 1.8251 & .0.0178 & -0.1012 & 0.0004 & 0.0001 & -0.0000 & 0.0000 \end{bmatrix} \times 10^7$$

(35)

In general, the smaller R, the better the loop transfer recovery. It should be noted that there is a tradeoff between the loop transfer recovery and the time-domain response.

The principal gains of return ratio, sensitivity function, and complementary sensitivity function of compensated plant are shown in Figs. 7-8. The unit-step responses of bank angle $\phi(t)$, and the associated actuator inputs are also shown in Figs. 9-10. For comparison purpose, the results obtained by the traditional LQG/LTR method with $\alpha = 0$ are also shown in Figs. 7-10. As shown in Fig. 7, the proposed method has better recovery quality at lower frequencies and higher frequencies as well. In Fig. 8 the largest principal gains of sensitivity function $S_{GK}(s)$ of the proposed method at low frequencies are decreased by 1 dB, and the rejection capability of system disturbance is increased. In addition, the proposed method has smaller condition number $\overline{\sigma}(T_{GK}) / \underline{\sigma}(T_{GK})$ at high frequencies for the complementary sensitivity function $T_{GK}(s)$, and this result makes the proposed method have better robustness in the face of high-frequency measurement noise. In the unit-step responses shown in Fig. 9, the maximum overshoot of bank angle $\phi(t)$ and cross coupling of sideslip angle $\beta(t)$ for the unit step responses are also reduced by the proposed method. In addition, the maximal transient commands of aileron and rudder actuators are decreased as shown in Fig. 10. Table I shows the root mean square (r.m.s.) of aileron and rudder actuators for the unit-step responses of bank angle, and the proposed GLQG/LTR method can also lessen the energy consumption of actuators. In face of system and measurement noises with $W = 1$ and $V = 1$, the white noise responses for both GLQG/LTR and LQG/LTR methods are all listed in Table II. These results show that the proposed GLQG/LTR method has better noise rejection and less energy consumption. Therefore, the GLQG/LTR method can make the compensated plant have good robustness and performance properties in the noise rejection.

IV. Conclusions

From the previous derivation, we have applied the GLQG/LTR method for not only minimum-phase system but non-minimum phase one as well. This makes the proposed technique more generalized in some practical applications. By numerical simulations and comparing these results obtained by the traditional LQG/LTR technique, the proposed method can achieve better robustness and performance properties in both frequency- and time-domain responses.

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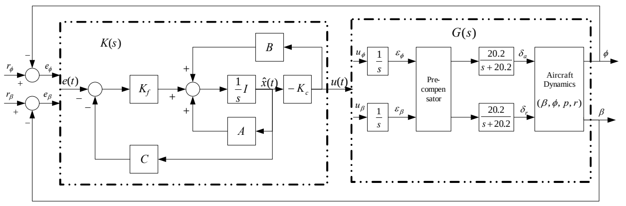

Fig. 1. Model of F-16 aircraft lateral control system

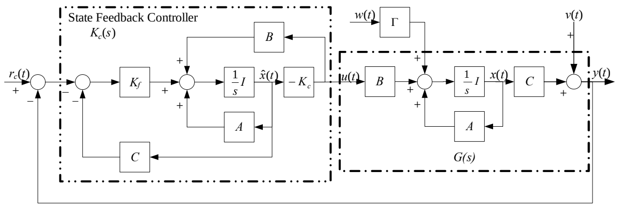

Fig. 2. GLQG/LTR control structure

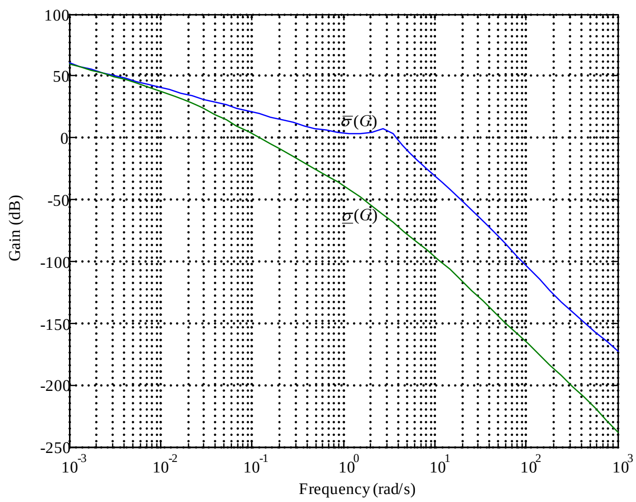

Fig. 3. Principal gain of $C(sI - A)^{-1}B$ for augmented plant

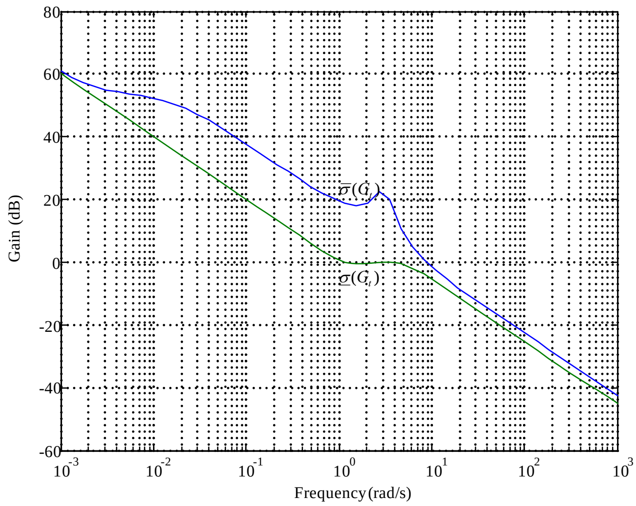

Fig. 4. Open-loop principal gain of return ratio $G_t(s)$.

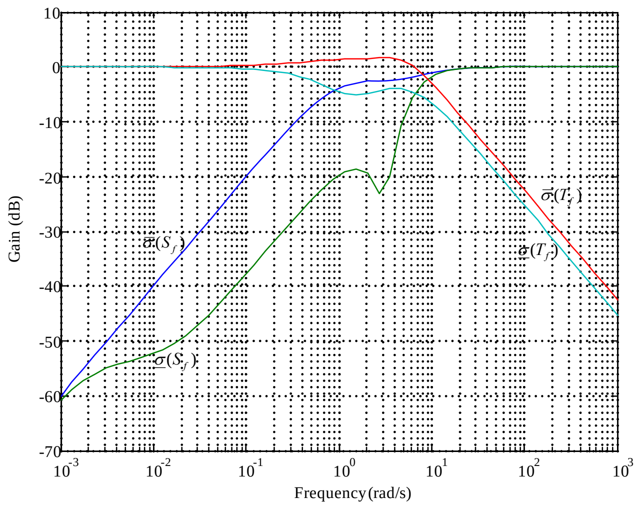

Fig. 5. Principal gain plots of $S_f(s)$ and $T_f(s)$ for $G_t(s)$.

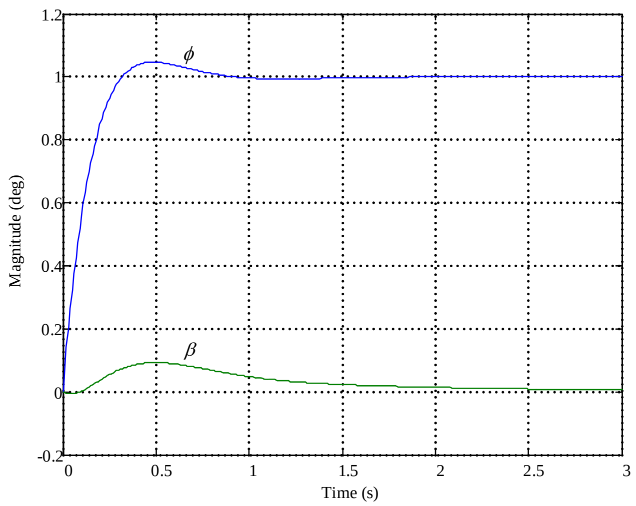

Fig. 6. Unit-step response of target feedback loop $G_t(s)$.

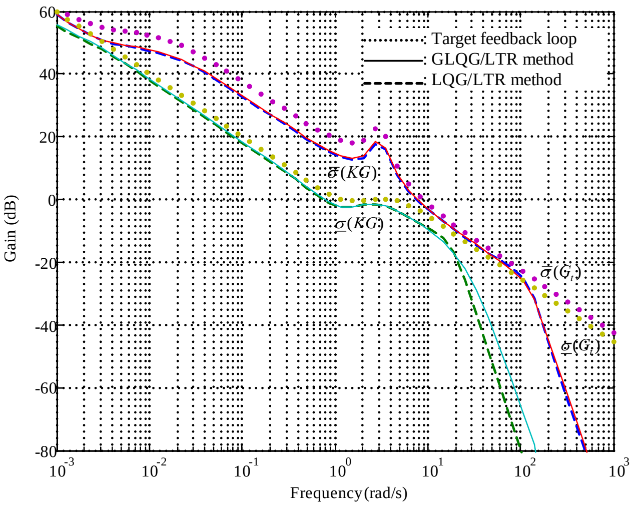

Fig. 7. Principal gain plots of return ratio $G(s)K(s)$.

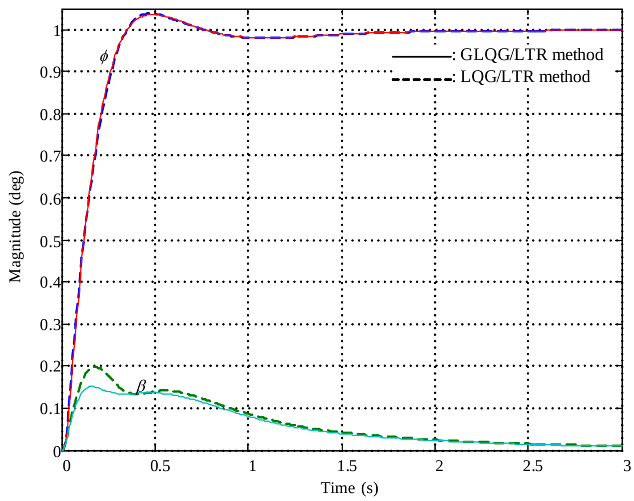

Fig. 9. Unit-step responses of φ .

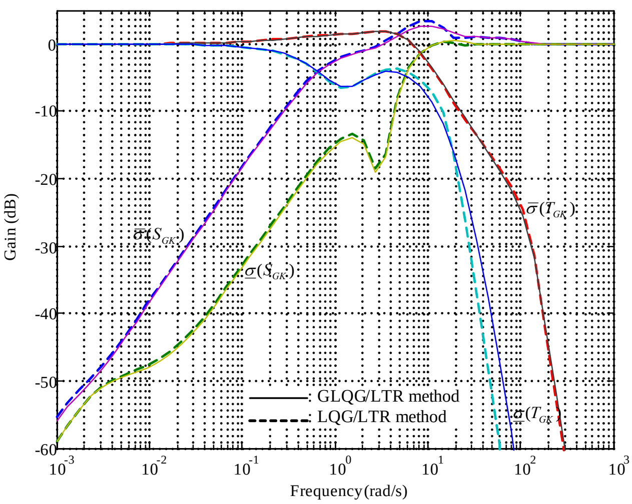

Fig. 8. Principal gains of $S_{GK}(s)$ and $T_{GK}(s)$.

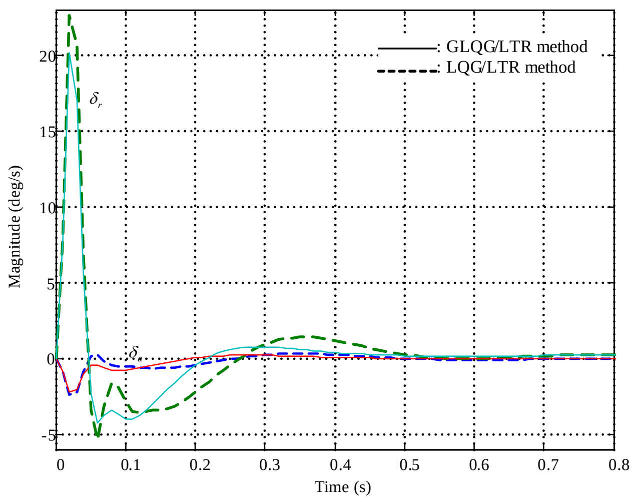

Fig. 10. Actuator inputs for unit-step responses of φ .

Table I Root mean square of actuators for unit-step responses of bank angle $\phi(t)$.

Item		GLQG/LTR	LQG/LTR
Unit step responses of φ	$\mathrm{rms}(\delta_a)$	0.2319	0.2455
	$\mathrm{rms}(\delta_r)$	1.7220	2.0380

Table II Covariances of output responses in face of white noises.

Item		GLQG/LTR	LQG/LTR
White noise response with $W=1$ and $V=1$	$E(\phi\phi^T)$	3.3780×10^3	3.4048×10^3
	$E(\beta\beta^T)$	3.5779×10^3	4.3484×10^3
	$E(\delta_a\delta_a^T)$	55.6590	57.2271
	$E(\delta_r\delta_r^T)$	191.6590	200.0845