Fuzzy Logic Tracking Control for Unicycle Mobile Robots

Oscar Castillo, Luis T. Aguilar, and Sélene Cárdenas

Abstract— **This paper addresses the problem of trajectory tracking control in an autonomous, wheeled, mobile robot of unicycle type using Fuzzy Logic. The Fuzzy Logic Control (FLC) is based on a backstepping approach to ensure asymptotic stabilization of the robot's position and orientation around the desired trajectory, taking into account the kinematics and dynamics of the vehicle. We use the Mamdani inference system to construct a controller, with nine IF-THEN rules and the centroid of area method as our deffuzification strategy where the input torques and velocities are considered as linguistic variables. The performance of this FLC are illustrated in a simulation study.**

Index Terms— **Fuzzy logic, tracking control, unicycle mobile robots, backstepping.**

I. INTRODUCTION

A. Overview

A UNICYCLE mobile robot is an autonomous, wheeled vehicle capable of performing missions in fixed or uncertain environments. Mobile robots have attracted considerable interest in the robotics and control research community, because they posses nonholonomic properties caused by noninte-grable differential constraints. The motion of a nonholonomic mechanical systems [4] is constrained by its own kinematics, so its control laws are not derivable in a straightforward manner (Brockett condition [6]). Backstepping methodology is a suitable choice in solving the tracking control problem for mobile robots, in both kinematic and Euler-Lagrange models. Recent developments include the introduction of an adaptive tracking controller based on a backstepping approach [12], and a dynamical extension that enables the integration of kinematic and torque controllers into a nonholonomic mobile robot [11]. The backstepping approach consists of two steps: 1) finding the velocities that stabilize the kinematic model and 2) finding a control law that will ensure the converge of real velocities to those values.

Fuzzy logic control (FLC) ([31]) has been recognized for its effectiveness in the control of industrial processes [3], mechanical systems [27], chemical processes [9], medicine [5], pattern recognition [2], and others (see [17] and [23]). In essence, FLC provides an algorithm which can convert a linguistic control strategy based on expert knowledge to an automatic control strategy. The conceptual advantage of FLC is very useful when the processes are too complex for analysis, or when the available resource of information are inexact or uncertain. There exist three different fuzzy inference systems, known as the Mamdani ([21]), Tagaki-Sugeno ([25]), and Tsukamoto [28] fuzzy models. These are frameworks built on the concepts of fuzzy sets, fuzzy IF-THEN rules, and fuzzy reasoning. Recent representative papers on the use of FLC in mobile robots include [1], [7], [8], [13], [16], [18], [22], and [29]. The Tagaki-Sugeno approach is the most commonly used fuzzy logic model in the tracking control problem of autonomous vehicles, because it provides remarkable experimental results and a more systematic approach to FLC design (see [26] and references therein).

B. Contribution

In this paper we introduce a Fuzzy Logic Controller based on the Mamdani fuzzy model and the Centroid of Area defuzzification method to force a unicycle mobile robot to follow a desired trajectory. Currently, many research papers consider only the kinematic model (steering system) (see *e.g.*, [18], [20], [24], [30], and references therein) to solve the tracking control problem, where the velocity, used as input control, is assumed to be supplied by the mobile robot whose dynamic of the actuators is neglected. However, real prototype mobile robots have actuated wheels whose slip rate, rolling, inertia moments, and mass distribution contribute to the forces exerted on the structure of the vehicle thus affecting the accuracy and full manoeuvrability of the robot. Motivated by this, the vehicle dynamics, represented by the Euler-Lagrange equations, is considered to convert a steering system into control inputs for the actual vehicle ([10], [11]). We use triangle- and trapezoid-shaped membership functions in our control design, with three fuzzy partitions and nine fuzzy rules. It is worth noting that FLC does not require information of the parameters of the Euler-Lagrange equation (masses, inertias, damping, etc.), thus avoiding an extra work on its identification.

C. Organization

This paper is organized as follows: Section II presents the problem statement and the kinematic and dynamic model of the unicycle mobile robot. Section III introduces the fuzzy logic control system using a Mamdani-type model where the wheel input torques, linear velocity, and angular velocity will be considered as linguistic variables. Section IV provides a simulation study of the unicycle mobile robot using the
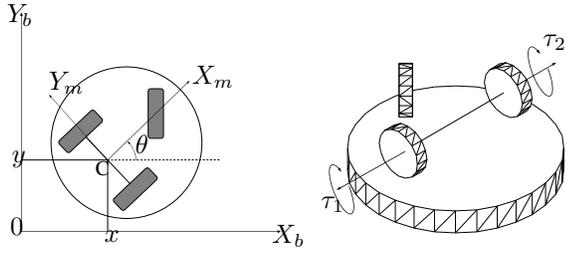
Fig. 1. Schematic diagram (*left*) of Unicycle mobile robot with actuated wheels (*right*).

controller described in Section III. Finally, Section V presents the conclusions.

II. PROBLEM STATEMENT

In this paper, we consider a unicycle mobile robot as a case study. The robot body is symmetrical around the perpendicular axis and the center of mass is at the geometric center of the body. It has two driving wheels fixed to the axis that passes through C and one passive orientable wheel that is placed in front of the axis and normal to it. The two fixed wheels are controlled independently by motors, and the passive wheel prevents the robot from tipping over as it moves on a plane. In what follows, we assume that the motion of passive wheel can be ignored in the dynamics of the mobile robot.

We propose a fuzzy logic controller for the unicycle mobile robot governed by the following equation of motion [15]:

$$\dot{q} = \underbrace{\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}}_{S(q)} \underbrace{\begin{bmatrix} \nu \\ \omega \end{bmatrix}}_{v} \qquad (1)$$

$$M(q)\dot{v} + C(q,\dot{q})v + Dv = \tau + \kappa(t) \qquad (2)$$

where $q = (x, y, \theta)^T$ is the vector of generalized coordinates; $v = (\nu, \omega)^T$ is the vector of velocities; $\tau = (\tau_1, \tau_2)$ is the vector of torques applied to the wheels of the robot; $\kappa(t)$ is the 2×1 uniformly bounded disturbance vector; $M(q)$ is a 2×2 positive-definite inertia matrix; $C(q,\dot{q})v$ is the vector of centripetal and Coriolis forces; and D is a 2×2 diagonal positive-definite matrix. Equation (1) represents the kinematics of the system, where (x, y) is the position in the $X-Y$ (world) reference frame; θ is the angle between the heading direction and the x-axis; ν and ω are the linear and angular velocities respectively; and τ_1 and τ_2 denote the torques of the right and left wheel, respectively (see Fig. 1). Furthermore, the system (1)-(2) has the following nonholonomic constraint:

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0, \qquad (3)$$

which corresponds to a no-slip wheel condition preventing the robot from moving sideways [19]. The system (1) fails to meet Brockett's necessary condition for feedback stabilization [6]. This implies that no continuous static state-feedback controller exists that stabilizes the closed-loop system around the equilibrium point.

The control objective is established formally as follows: Given a continuously differentiable, bounded trajectory and orientation q_d, we must design a fuzzy logic controller τ such that the positions $q(t)$ reach the desired posture $q_d(t)$, that is

$$\lim_{t\to\infty} \|q_d(t) - q(t)\| = 0. \qquad (4)$$

III. FUZZY CONTROL DESIGN

We now present the synthesis of the FLC to achieve stabilization of a unicycle mobile robot around a desired path. The backstepping approach plays an important role.

The system (1)-(2) is in cascade interconnection; that is, the kinematic subsystem (1) is controlled only indirectly through the velocity vector v. Stabilizing control laws for systems in such a hierarchical form can be designed using the method of backstepping [14], which consists of the following two steps:

i) We must find a velocity vector $v_r = v$ such that the kinematic system (1) be asymptotically stable.

ii) A control input τ must be found, such that

$$\lim_{t\to t_s} \|v_r(t) - v(t)\| = 0 \qquad (5)$$

where $t_s < \infty$ is the reachability time.

In (5), we consider that real mobile robots have actuated wheels, so the control input is τ; that is, the actual control input τ is designed so as to stabilize the system (2) without destabilizing the system (1) by forcing $v \to v_r$. Moreover, if (5) is satisfied in an infinite time ($t_s = \infty$) then v_r will be different from v during $t < \infty$, with the result that the mobile robot will be neither positioned nor oriented at the desired point. Figure 2 illustrates the feedback connection which involves the fuzzy controller.

A. Control of the kinematic model

To solve the tracking control problem for the kinematic model, we followed the procedure proposed by Fierro and Lewis [11] and Lee *et al*. [15]. Suppose the reference trajectory $q_d(t)$ satisfies

$$\dot{q}_d(t) = \begin{bmatrix} \cos\theta_d(t) & 0 \\ \sin\theta_d(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{1d}(t) \\ v_{2d}(t) \end{bmatrix} \qquad (6)$$

where $\theta_d(t)$ is the desired orientation, and $v_{1d}(t)$ and $v_{2d}(t)$ denote the linear and angular desired velocities, respectively. In the robot's local frame, the error coordinates can be defined as

$$\begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} \cos\theta(t) & \sin\theta(t) & 0 \\ -\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_e(q_d(t)-q(t))} \begin{bmatrix} x_d(t) - x(t) \\ y_d(t) - y(t) \\ \theta_d(t) - \theta(t) \end{bmatrix}, \qquad (7)$$
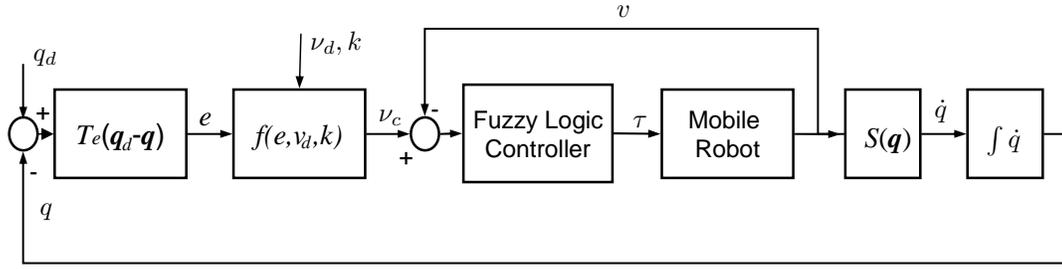
Fig. 2. Feedback connection.

where (x_d, y_d) is the desired position in the world $X - Y$ coordinate system. The tracking error model is given by

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} v_2 e_2 - v_1 + v_{d1} \cos e_3 \\ -v_2 e_1 + v_{d1} \sin e_3 \\ v_{d2} - v_2 \end{bmatrix}}_{f(e, v_d, k)}. \tag{8}$$

We are now in position to state the main result of this subsection in the form of theorem (the proof can be found in [11]).

Theorem 1 ([11]): Let the tracking error system (8) be driven by the control law

$$v_{r1} = v_{d1} \cos e_3 + k_1 e_1 \tag{9}$$

$$v_{r2} = v_{d2} + v_{d1} k_2 e_2 + k_3 \sin e_3 \tag{10}$$

where k_1, k_2, and k_3 are strictly positive constants. If $v_1 = v_{r1}$ and $v_2 = v_{r2}$, then the origin of the closed-loop system [(8)-(10)] is asymptotically stable.

B. Fuzzy logic control synthesis

We proceed to derive a Fuzzy Logic Controller that can force the real velocities of the mobile robot (1)-(2) to match those required in equations (9)-(10) of theorem 1 and thus satisfy the control objective given in (4). In this paper we adopt the *Mamdani Fuzzy model*.

The *fuzzy rules* are presented as a mapping from the linear and angular velocity errors (the linguistic input variables):

$$e_\nu = v_{r1} - v_1 \tag{11}$$

$$e_\omega = v_{r2} - v_2 \tag{12}$$

to the required torque (τ_1, τ_2) (the linguistic output variables). We point out that equations (7)-(10) allows us to express velocity information in terms of the position measurements only. The membership functions, depicted in Figure 3, are shaped like triangles and trapezoids with three fuzzy partitions denoted as *negative* (N), *zero* (Z), and *positive* (P). The universe of discourse is normalized into the range $[-1, 1]$. The
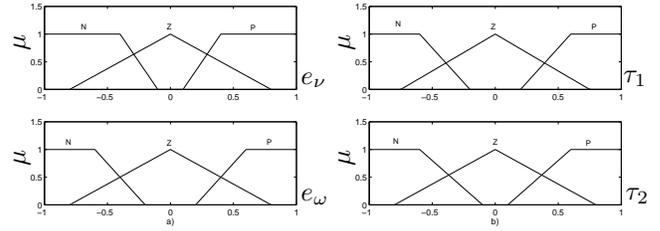

Fig. 3. *a)* Membership functions of the input variables (e_ν, e_ω), and *b)* Membership functions of output variables (τ_1, τ_2).

TABLE I

FUZZY RULES

e_ν	e_ω		
	N	**Z**	**P**
N	N/N	N/Z	N/P
Z	Z/N	Z/Z	Z/P
P	P/N	P/Z	P/P

nine *fuzzy rules* are:

R_1 : IF e_ν is Z and e_ω is Z THEN τ_1 is Z and τ_2 is Z,
R_2 : IF e_ν is Z and e_ω is P THEN τ_1 is Z and τ_2 is P,
R_3 : IF e_ν is Z and e_ω is N THEN τ_1 is Z and τ_2 is N,
R_4 : IF e_ν is P and e_ω is P THEN τ_1 is P and τ_2 is P,
R_5 : IF e_ν is P and e_ω is N THEN τ_1 is P and τ_2 is N,
R_6 : IF e_ν is P and e_ω is Z THEN τ_1 is P and τ_2 is Z,
R_7 : IF e_ν is N and e_ω is N THEN τ_1 is N and τ_2 is N,
R_8 : IF e_ν is N and e_ω is P THEN τ_1 is N and τ_2 is P,
R_9 : IF e_ν is N and e_ω is Z THEN τ_1 is N and τ_2 is Z.

The fuzzy rules can be summarized in Table 1. The elements in the first row and column of Table 1 are the input linguistic variables, while the others correspond to the output linguistic variables.

We use the Centroid of Area as our defuzzification strategy, *i.e.*

$$z_{COA} = \frac{\int_Z \mu_A(z) z \, dz}{\int_Z \mu_A(z) \, dz} \tag{13}$$

where $\mu_A(z)$ is the aggregated output of membership function, and $\int$ denotes the union of $(z, \mu(z))$ pairs. Figure 4 shows the input-output curves.
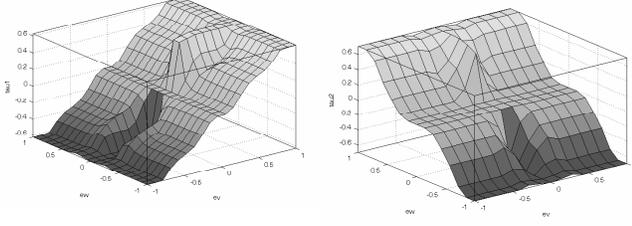
Fig. 4.   Overall input-output curves.

IV. SIMULATION RESULTS

In this section, we evaluate, through computer simulation performed in MATLAB® and SIMULINK®, the ability of the proposed controller to stabilize the unicycle mobile robot, defined by (1), (2). The following matrix values

$$M(q) = \begin{bmatrix} 0.3749 & -0.0202 \\ -0.0202 & 0.3739 \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0.1350\dot{\theta} \\ -0.1350\dot{\theta} & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix},$$

were taken from [10].

In the simulations, we choose

$$v_d(t) = \begin{cases} v_{d1}(t) = 0.25 - 0.25\cos(\frac{2\pi t}{5}) \\ v_{d2}(t) = 0 \end{cases} \quad (14)$$

as desired linear v_{d1} and angular velocities v_{d2}, subject to the initial conditions

$$q(0) = (0.1, 0.1, 0)^T \text{ and } v(0) = 0 \in \mathbb{R}^2.$$

The gains k_i of equations (9)-(10) were set to the values of 5.

Figure 5 shows the posture errors, $X - Y$ position, velocity errors, and input torques for the closed-loop system for the FLC presented in Section 3. It should be noted that the horizontal and vertical displacements are brought to zero in 0.5 [sec] and 1.0 [sec], respectively, while the orientation converge at $t = 1.2$ [sec]. The controller brings the velocity errors to zero at $t_s = 0.25$ [sec]. Figure 5 also demonstrates the fast switching of the input control. Clearly, the proposed controller achieves regulation of the velocity errors in a finite time thus satisfies the control objective.

We demonstrate the robustness properties of the closed-loop system by injecting impulse disturbances every two seconds

$$\kappa(t) = \delta(t - t_k) \quad t_k = 2, 4, \ldots$$

where $\delta(t)$ is the Delta-Dirac function. Simulation results are depicted in Figure 6, which shows the closed-loop responses for the displacement errors, posture, velocity error, and input torques. Note that the position and orientation errors remain stable, in spite of the external disturbances.
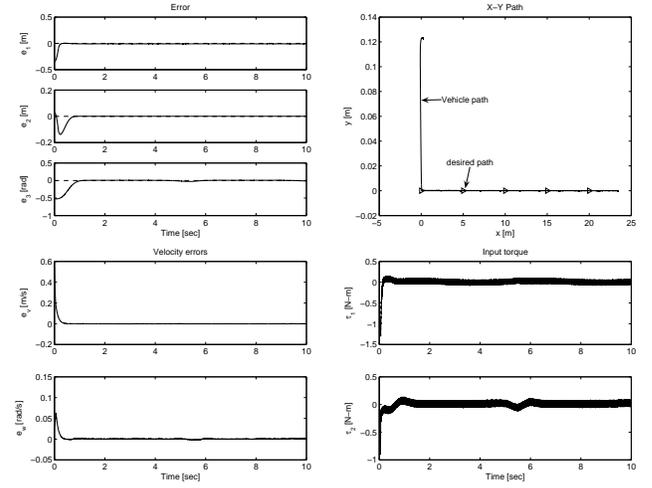

Fig. 5.   Simulation results for the closed-loop system: the unperturbed case.
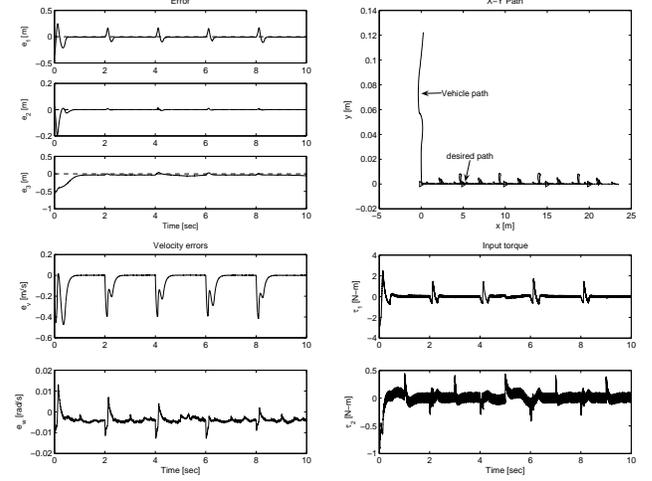

Fig. 6.   Simulation results for the closed-loop system: the perturbed case.

V. CONCLUSIONS

This paper addressed the problem of tracking control around a desired position and orientation, taking into account the dynamics of the vehicle, for unicycle mobile robots. The proposed solution is based on the backstepping approach, with an internal loop governed by a Mamdani-based fuzzy logic controller (FLC). The FLC forces the real velocities towards the values required to achieve the control objective. Two inputs (the linear and angular velocity errors) and two outputs (the torques) were used to create nine IF-THEN fuzzy rules, resulting in minimal software complexity. Future research topics of interest include the problem of tracking control when only displacement measurements are available, and adapting this work to more complex systems such as four-wheeled robots.

REFERENCES

[1] Abdessemed, F., Benmahammed, K., Monacelli, E., "A fuzzy-based re-active controller for a non-holonomic mobile robot," *Journal of Robotics and Autonomous Systems*, V47, pp. 31–46, 2004.

[2] Ajiboye A.B., Weir, R.F., "A heuristic fuzzy logic approach to EMG pattern recognition for multifunctional prosthesis control," *IEEE Trans. on Neural Systems and Rehabilitation Engineering*, V13, N3, pp. 280–291, 9/05.

[3] Awais, M.M., "Application of internal model control methods to industrial combustion," *Applied Soft Computing* V5, pp. 223–233, 2005.

[4] Bloch, A.M. *Nonholonomic mechanics and control*, New-York: Springer, 2003.

[5] Boegl, K., Adlassniga, K-P., Hayashic, Y. Rothenfluhd, T.E., Leitich, H., "Knowledge acquisition in the fuzzy knowledge representation framework of a medical consultation system," *Artificial Intelligence in Medicine*, V30, pp. 1–26, 2004.

[6] Brockett, R.W. "Asymptotic stability and feedback stabilization," in: R.S. Millman and H.J. Sussman, (Eds.), *Differential Geometric Control Theory*, Boston: Birkhauser, pp. 181–191, 1983.

[7] Chiu, C-S., Lian, K-W., Liu, P., "Fuzzy gain scheduling for paralell parking a car-like robot," *IEEE Trans. Contr. Syst. Technol.*, V13, N6, pp. 1084–1092, 11/05.

[8] Das, T., Kar, I.N., "Design and implementation of an adaptive fuzzy logic-based controller for wheeled mobile robots," *IEEE Trans. Contr. Syst. Technol.*, V14, N3, pp. 501–510, 05/06.

[9] Dash, S., Rengaswamy, R., Venkatasubramanian, V. "Fuzzy-logic based trend classification for fault diagnosis of chemical processes," *Computers & Chemical Engineering*, V27, pp. 347–362, 2003.

[10] Duc Do, K., Zhong-Ping J., Pan, J. "A global output-feedback controller for simultaneous tracking and stabilization of unicycle-type mobile robots," *IEEE Trans. Automat. Contr.*, V30, N3, pp. 589–594, 06/04.

[11] Fierro R., Lewis, F.L., "Control of a nonholonomic mobile robot: backstepping into dynamics," *Proc. 34th IEEE Conf. on Decision and Control*, New Orleans USA, pp. 3805–3810, 1995.

[12] Fukao, T., Nakagawa, H., Adachi, N., "Adaptive tracking control of a nonholonomic mobile robot," *IEEE Trans. Robot. Automat.*, V16, N5, pp. 609–615, 09/00.

[13] Hagras, H.A. "A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots," *IEEE Trans. Fuzzy Syst.*, V12, N4, pp. 524–539, 08/04.

[14] Kristic, M., Kanellakopoulos, I., Kokotovic, P. *Nonlinear and adaptive control design*, Wiley-Interscience, 1995.

[15] Lee, T-C., Song, K-T., Lee, C-H., Teng, C-C., "Tracking control of unicycle-modeled mobile robot using a saturation feedback controller," *IEEE Trans. Contr. Syst. Technol.*, V9, N2, pp. 305–318, 03/01.

[16] Lee, T.H., Lam, H.K., Leung, F.H.F, Tam, P.K.S. "A practical fuzzy logic controller for the path tracking of wheeled mobile robots," *IEEE Control Systems Magazine*, V23, N2, pp. 60–65, 04/03.

[17] Lee, C.C., "Fuzzy logic in control systems: Fuzzy logic controller–Part I," *IEEE Trans. Syst., Man., Cybern.*, V20, N2, pp. 404–418, 03/90.

[18] Li, T.H., Chang, S.J., Tong, W., "Fuzzy target tracking control of autonomous mobile robots by using infrared sensors," *IEEE Trans. Fuzzy Syst.*, V12, N4, pp. 491–501, 08/04.

[19] Liberzon, D. *Switching in Systems and Control*. Birhauser, 2003.

[20] Lo Bianco, C.G., Piazzi, A., Romano, M., "Smooth motion generation for unicycle mobile robots via dynamic path inversion," *IEEE Trans. Robot.*, V20, N5, pp. 884–898, 10/04.

[21] Mamdani, E.H., "Applications of fuzzy algorithms for simple dynamic plant," *Proc. of IEE*, V121, pp. 1585–1588, 1974.

[22] Maeda, M., Maeda Y., and Murakami, S., "Fuzzy drive control of an autonomous mobile robot," *Fuzzy Sets and Systems*, V39, pp. 195–204, 1991.

[23] Mendel, J.M., "Fuzzy logic systems for engineering: A tutorial," *Proc. of the IEEE*, pp. 345–377, 1995.

[24] Samson, C., "Control of chained systems applications to path following and time-varying point stabilization of mobile robots," *IEEE Trans. Automat. Contr.*, V40, N1, pp. 64–77, 01/95.

[25] Sugeno M., Kang, G.T., "Structure identification of fuzzy models," *Fuzzy Sets and Systems*, V28, pp. 15–33, 1988.

[26] Tagaki, T., Sugeno, M., "Fuzzy identification of systems and its application to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, V15, pp. 116–132, 1985.

[27] Tang, Y., Vélez-Díaz, D., "Robust fuzzy control of mechanical systems," *IEEE Trans. Fuzzy Syst.*, V11, N3, pp. 411–418, 06/03.

[28] Tsukamoto, Y., "An approach to fuzzy reasoning method," in: M.M. Gupta, R.K. Ragade, and R.R. Yager, (Eds.), Advance in Fuzzy Set Theory and Applications North-Holland, Amsterdam, pp. 137–149, 1979.

[29] Vadakkepat, P., Miin, O.C., Peng, X., Lee, T.H., "Fuzzy bevavior-based control of mobile robots," *IEEE Trans. Fuzzy Syst.*, V12, N4, pp. 559–564, 08/04.

[30] Yim, H., Butler, A.C., "Motion planning using fuzzy logic control with minimum sensors," in *Proc. of the 1995 IEEE International Symposium on Intelligent Control* pp. 558–564, 1995.

[31] Zadeh, L.A., "Fuzzy sets," *Information and control*, v8, pp. 338–353, 1965.

Oscar Castillo is Full time professor of the Computer Science Department of Tijuana Institute of Technology since 1990. He holds a Ph.D. in Computer Science from Kensington University, USA (1996-1998). Master Degree in Applied Mathematics from National Polytechnic Institute of Mexico (1983-1985), and a Bachelor in Science Degree in Electrical Engineering from Tijuana Institute of Technology (1978-1983). He is a member of IEEE Neural Networks Society, IFSA, NAFIPS and New York Academy of Sciences. His main research areas are Fuzzy Logic, Neural Networks, Genetic Algorithms, Robotics and Control of Dynamical Systems. He is currently member of the Technical Committee on Fuzzy Systems of the IEEE. He has received several awards for his achievements in research and teaching during his work in Tijuana Institute of Technology. He has received many research grants from government and industry to direct his many research projects. He has also collaborated with researchers from other countries in several projects of great importance. He has over 180 technical papers in Journals, Proceedings, and Edited Books. Because of his many contributions he has been elected to the Editorial Board of several Journals in Computational Intelligence. He is author of the book entitled "Soft Computing for Control of Non-Linear Dynamical Systems" published by Springer-Verlag, Germany in January 2001, co-author of the book "Modelling, Simulation and Control of Non-Linear Dynamical Systems" published by Taylor and Francis, Great Britain in January 2002, and author of the book "Soft Computing and Fractal Theory for Intelligent Manufacturing", published by Springer-Verlag, Germany in January 2003.

Luis T. Aguilar received his Bachelor's degree in Industrial Electronics Engineering from Tijuana Institute of Technology in 1994; the M. Sc. degree in Digital Systems from Centro de Investigación y Desarrollo de Tecnología Digital (CITEDI-IPN); and the Ph. D. degree in Electronics and Telecommunications, specialized in automatic control, from CICESE, in Ensenada, in 1998 and 2003, respectively. From 1994 to 1996, he was a project engineer at Sanyo Corporation, and from 1996 to 2004, he was a part-time professor at Electronics Engineering Department of Tijuana Institute of Technology. He is currently a Researcher at CITEDI-IPN since 2004. His research interest include Control of Nonsmooth Electromechanical Systems, Non-Holonomic Systems, Output Feedback, Sliding Mode Control, and Nonlinear H-infinity Control.

Sélene Cárdenas received the M. Sc. degree in Computer Science in 1995 from Tijuana Institute of Technology. Currently, she is a part-time professor at Computer Science Department of Tijuana Institute of Technology.