Proof sketch that Manson/Pugh allows reordering

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Consider a program P and the program P' that is obtained from P by reordering two adjacent statements x and y. Let x be the statement that comes before y in P, and after y in P'. The statements x and y may be any two statements such that

- reordering x and y doesn't eliminate any happens-before edges, except for reversing the one between x and y,
- x and y are not conflicting accesses to the same variable,
- x and y are not both synchronization actions, and
- the intra-thread semantics of x and y allow reordering (e.g., x doesn't store into a register that is read by y).

Assume that we have a valid execution E' of program P'. To show that the transformation of P into P' is legal, we need to show that there is a valid execution E of P that has the same observable behavior as E'.

Assume $E' = \langle S, so, hb', co \rangle$. We are going to show that $E = \langle S, so, hb, co \rangle$ is also a valid execution of P. Let a_x and a_y denote the actions corresponding to the statements x and y. Because of the reordering the happens-before ordering may be different but we know that $hb - \{a_x \rightarrow a_y\} \subseteq hb' - \{a_y \rightarrow a_x\}$. Clearly, if E' is consistent then E is consistent, so we only need to worry about showing that co is justified as the causal order of E.

- Assume that $co = \alpha a_y \beta a_x \gamma$.

  We don't need to worry about any actions that were prescient in E'. The justification of those prescient actions in E' will also justify them in E.

  The only action that could be prescient in E but not E' is a_y. If a_y is not prescient in E', we know a_x is the only action that comes after a_y in the causal order such that $a_x \stackrel{hb}{\rightarrow} a_y$, and that if a_y is a read, it sees a value in α. In order to justify the prescient action a_y in E, we need to show that for each execution of P whose causal order starts with α, the action a_y is allowed to occur and if a_y is a read that a_y can see a write of the value seen by a_y in E..

  We know intra-thread semantics will cause a_y to occur, since all actions other than a_x that occur before a_y in program order are in α, and we have assumed as a condition for the reordering that a_x does not effect the intra-thread semantics of a_y. If a_y is a read that sees a write w in α, we know it can see w in any execution with a causal order that starts with α.

- Alternatively, assume that $co = \alpha a_x \beta a_y \gamma$. Then any action in E that is prescient is also prescient in E', and the justifications used to show that those actions are justified in E' will also show that those actions are justified in E.