15-453: Formal Languages, Automata, and Computability **L. Blum**
Homework #1 **Due Date: January 21, 2014**

Rules: Collaborate as you like, but everyone must write up their own solutions. Cite all references that you use (books, people, yourself) at the end of your writeup. All proofs should contain: (1) a one-line phrase with your proof strategy, (2) an informal paragraph outlining your idea, and (3) the gritty details. Be as precise and rigorous as you can! Please print out the solutions to each problem on a separate sheet of paper.

1

Draw DFAs for the following languages. Briefy justify why your DFAs recognize the correct language.

(a) The language of strings in $\{a, b, c\}$ containing the string "*cab*" at least twice.

(b) The language of strings in $\{a, b, c\}$ containing the string "*cab*" at most twice.

(c) The language of strings in $\{a, b, c\}$ whose length is either even or divisible by 5 (or both!)

(d) The language of strings in $\{a, b, c\}$ containing at least one a and an even number of b's.

2

For any string $s \in \{1, 2, 3\}^*$, let $\text{Sum}(s)$ denote the sum of its digits. For example, $\text{Sum}(1213) = 7$, $\text{Sum}(22) = 4$, and $\text{Sum}(212) = 5$. Let L be the language

$$L = \{x \in \{1, 2, 3\}^* \mid \text{Sum}(s) \equiv 0 \mod 5\}.$$

Prove that L is regular by providing a DFA that recognizes it.

3

For any language A, define the operation MAX by

$$\text{MAX}(A) = \{w \in A \mid \text{no string containing } w \text{ as a proper prefix is in } A\}.$$

Prove that regular languages are closed under MAX.

Hint: Assume you have a DFA that recognizes A. How would you modify it to recognize $\text{MAX}(A)$?

4

Let L_1 and L_2 be languages over some common alphabet Σ. Define a new language

$$L_1/L_2 = \{w \in \Sigma^* \mid \exists\ x \in L_2 \text{ such that } wx \in L_1\}.$$

Prove that if L_1 is regular, then L_1/L_2 is regular.

5

Include a References section. Cite all sources that you used and people, including yourself, that you collaborated with on this homework.