15-745 Lecture 2

Dataflow Analysis
Basic Blocks
Related Optimizations

Dataflow Analysis

- Last time we looked at code transformations
  - Constant propagation
  - Copy propagation
  - Common sub-expression elimination
  - …
- Today, dataflow analysis:
  - How to determine if it is legal to perform such an optimization
  - (Not doing analysis to determine if it is beneficial)

A sample program

```
int fib10(void) {
  int n = 10;
  int older = 0;
  int old = 1;
  ir
  int i;

  if (n <= 1) return n;
  for (i = 2; i<n; i++) {
     result = old + older;
     older = old;
     old = result;
  }
  return result;
}
```

What are those numbers?

A Comment about the IR

```
1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13

8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
```


Simple Constant Propagation

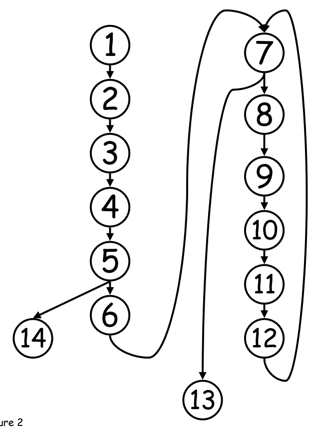
- Can we do SCP?
- How do we recognize it?
- What aren't we doing?
- Metanote:
  - keep opts simple!
  - Use combined power

```
1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13

8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
```

Reaching Definitions

- A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.




```
1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13
8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
```


Reaching Definitions (ex)

- 1 reaches 5, 7, and 14

14, Really?

Meta-notes:
- (almost) always conservative
- only know what we know
- Keep it simple:
  - What opt(s), if run before this would help
  - What about:

```
1: x <- 0
2: if (false) x<-1
3: … x …
```

  - Does 1 reach 3?
  - What opt changes this?

```
1:   n <- 10
2:   older <- 0
3:   old <- 1
4:   result <- 0
5:   if n <= 1 goto 14
6:   i <- 2
7:   if i > n goto 13
8:   result <- old + older
9:   older <- old
10:  old <- result
11:  i <- i + 1
12:  JUMP 7
13:  return result
14:  return n
```


Calculating Reaching Definitions

- A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.

- Build up RD stmt by stmt
- Stmt s, "d: v <- x op y", generates d
- Stmt s, "d: v <- x op y", kills all other defs(v)

Or,
- Gen[s] = { d }
- Kill[s] = defs(v) – { d }

Gen and kill for each stmt

	Gen	kill
1: n <- 10	1	
2: older <- 0	2	9
3: old <- 1	3	10
4: result <- 0	4	8
5: if n <= 1 goto 14		
6: i <- 2	6	11
7: if i > n goto 13		
8: result <- old + older	8	4
9: older <- old	9	2
10: old <- result	10	3
11: i <- i + 1	11	6
12: JUMP 7		
13: return result		
14: return n		

How can we determine the defs that reach a node?

We can use:
- control flow information
- gen and kill info

Computing in[n] and out[n]

- In[n]: the set of defs that reach the beginning of node n
- Out[n]: the set of defs that reach the end of node n

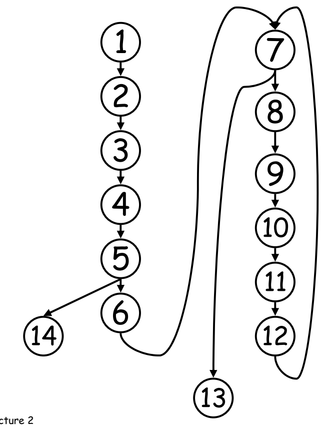
$$\text{in}[n] = \bigcup_{p \in pred[n]} out[p]$$

$$\text{out}[n] = gen[n] \cup (in[n] - kill[n])$$

- Initialize in[n]=out[n]={} for all n
- Solve iteratively

What is pred[n]?

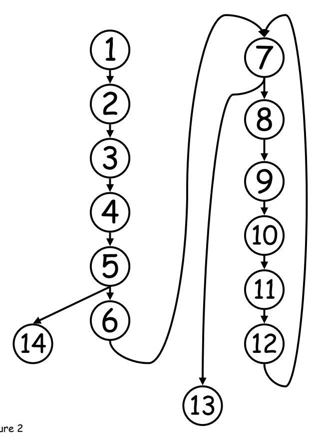
- Pred[n] are all nodes that can reach n in the control flow graph.
- E.g., pred[7] = { 6, 12 }




```
1:    n <- 10
2:    older <- 0
3:    old <- 1
4:    result <- 0
5:    if n <= 1 goto 14
6:    i <- 2
7:    if i > n goto 13
8:    result <- old + older
9:    older <- old
10:   old <- result
11:   i <- i + 1
12:   JUMP 7
13:   return result
14:   return n
```


What order to eval nodes?

- Does it matter?
- Lets do: 1,2,3,4,5,14,6,7,13,8,9,10,11,12




```
1:    n <- 10
2:    older <- 0
3:    old <- 1
4:    result <- 0
5:    if n <= 1 goto 14
6:    i <- 2
7:    if i > n goto 13
8:    result <- old + older
9:    older <- old
10:   old <- result
11:   i <- i + 1
12:   JUMP 7
13:   return result
14:   return n
```


Example:

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2,3	1-4
4: result <- 0	4	8		
5: if n <= 1 goto 14				
6: i <- 2	6	11		
7: if i > n goto 13				
8: result <- old + older	8	4		
9: older <- old	9	2		
10: old <- result	10	3		
11: i <- i + 1	11	6		
12: JUMP 7				
13: return result				
14: return n				

Example (pass 1)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2	1,2,3
4: result <- 0	4	8	1-3	1-4
5: if n <= 1 goto 14			1-4	1-4
6: i <- 2	6	11	1-4	1-4,6
7: if i > n goto 13			1-4,6	1-4,6
8: result <- old + older	8	4	1-4,6	1-3,6,8
9: older <- old	9	2	1-3,6,8	1,3,6,8,9
10: old <- result	10	3	1,3,6,8,9	1,6,8-10
11: i <- i + 1	11	6	1,6,8-10	1,8-11
12: JUMP 7			1,8-11	1,8-11
13: return result			1-4,6	1-4,6
14: return n			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
1: n <- 10	1			1
2: older <- 0	2	9	1	1,2
3: old <- 1	3	10	1,2	1,2,3
4: result <- 0	4	8	1-3	1-4
5: if n <= 1 goto 14			1-4	1-4
6: i <- 2	6	11	1-4	1-4,6
7: if i > n goto 13			1-4,6,8-11	1-4,6,8-11
8: result <- old + older	8	4	1-4,6,8-11	1-3,6,8-11
9: older <- old	9	2	1-3,6,8-11	1,3,6,8-11
10: old <- result	10	3	1,3,6,8-11	1,6,8-11
11: i <- i + 1	11	6	1,6,8-11	1,8-11
12: JUMP 7			1,8-11	1,8-11
13: return result			1-4,6	1-4,6
14: return n			1-4	1-4

An Improvement: Basic Blocks

- No need to compute this one stmt at a time
- For straight line code:
  - In[s1; s2] = in[s1]
  - Out[s1; s2] = out[s2]
- Can we combine the gen and kill sets into one set per BB?

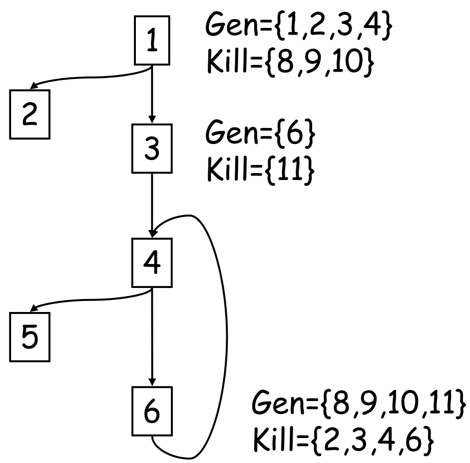
	Gen	kill
1: i <- 1	1	8,4
2: j <- 2	2	
3: k <- 3 + i	3	11
4: i <- j	4	1,8
5: m <- i + k	5	

- Gen[BB]={2,3,4,5}
- Kill[BB]={1,8,11}
- Gen[s1;s2]=
- Kill[s1;s2]=

BB sets

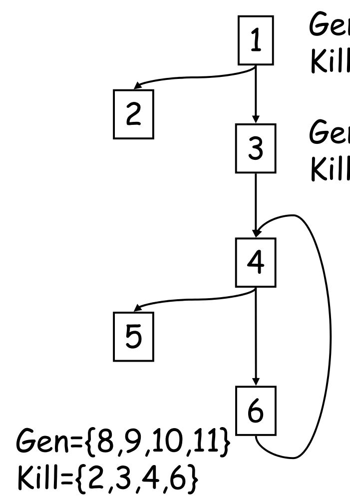
BB		Gen	kill		
	1: n <- 10	1			
	2: older <- 0	2	9		
1	3: old <- 1	3	10		
	4: result <- 0	4	8		
	5: if n <= 1 goto 14			1,2,3,4	8,9,10
3	6: i <- 2	6	11	6	11
4	7: if i > n goto 13				
	8: result <- old + older	8	4		
	9: older <- old	9	2		
6	10: old <- result	10	3		
	11: i <- i + 1	11	6		
	12: JUMP 7			8-11	2-4,6
5	13: return result				
2	14: return n				

BB sets



Node	In	out
1		1,2,3,4

BB sets



Node	In	out
1		1,2,3,4
3	1,2,3,4	1,2,3,4,6
4	1-4,6,8-11	1-4,6,8-11
6	1-4,6,8-11	1,8-11

Forward Dataflow

- Reaching definitions is a forward dataflow problem:
  It propgates information from preds of a node to the node
- Defined by:
  - Basic attributes: (gen and kill)
  - Transfer function: $out[b]=F_{bb}(in[b])$
  - Meet operator: $in[b]=M(out[p])$ for all $p \in pred(b)$
  - Set of values (a lattice, in this case powerset of program points)
  - Initial values for each node b
- Solve for fixed point solution

How to implement?

- Values?
- Gen?
- Kill?
- F_{bb}?
- Order to visit nodes?
- When are we done?
  - In fact, do we know we terminate?

Implementing RD

- Values: bits in a bit vector
- Gen: 1 in each position generated, otherwise 0
- Kill: 0 in each position killed, otherwise 1
- F_{bb}: out[b] = (in[b] | gen[b]) & kill[b]
- Init in[b]=out[b]=0

- When are we done?
- What order to visit nodes? Does it matter?

RD Worklist algorithm

Initialize: in[B] = out[b] = $\emptyset$
Initialize: in[entry] = $\emptyset$
Work queue, W = all Blocks in topological order
while (|W| != 0) {
  remove b from W
  old = out[b]
  in[b] = {over all pred(p) $\in$ b} $\cup$ out[p]
  out[b] = gen[b] $\cup$ (in[b] - kill[b])
  if (old != out[b]) W = W $\cup$ succ(b)
}

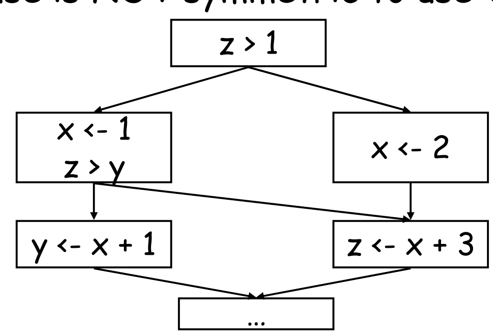
Storing Rd information

- Use-def chains: for each use of var x in s, a list of definitions of x that reach s

```
1:  n <- 10                1
2:  older <- 0             1            1,2
3:  old <- 1               1,2          1,2,3
4:  result <- 0            1-3          1-4
5:  if n <= 1 goto 14      1-4          1-4
6:  i <- 2                 1-4          1-4,6
7:  if i > n goto 13       1-4,6,8-11   1-4,6,8-11
8:  result <- old + older  1-4,6,8-11   1-3,6,8-11
9:  older <- old           1-3,6,8-11   1,3,6,8-11
10: old <- result          1,3,6,8-11   1,6,8-11
11: i <- i + 1             1,6,8-11     1,8-11
12: JUMP 7                 1,8-11       1,8-11
13: return result          1-4,6        1-4,6
14: return n               1-4          1-4
```


Def-use chains are valuable too

- Def-use chain: for each definition of var x, a list of all uses of that definition
- Computed from liveness analysis, a backward dataflow problem
- Def-use is NOT symmetric to use-def



Using RD for Simple Const. Prop.

```
1:  n <- 10                 1
2:  older <- 0              1            1,2
3:  old <- 1                1,2          1,2,3
4:  result <- 0             1-3          1-4
5:  if n <= 1 goto 14       1-4          1-4
6:  i <- 2                  1-4          1-4,6
7:  if i > n goto 13        1-4,6,8-11   1-4,6,8-11
8:  result <- old + older   1-4,6,8-11   1-3,6,8-11
9:  older <- old            1-3,6,8-11   1,3,6,8-11
10: old <- result           1,3,6,8-11   1,6,8-11
11: i <- i + 1              1,6,8-11     1,8-11
12: JUMP 7                  1,8-11       1,8-11
13: return result           1-4,6        1-4,6
14: return n                1-4          1-4
```

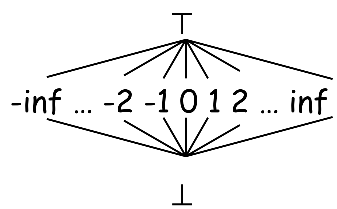

Better Constant Propagation

- What about:

  x <- 1
  if (y > z)
      x <- 1
  a <- x

Better Constant Propagation

- What about: x <- 1
  if (y > z)
      x <- 1
  a <- x
- Use a better lattice



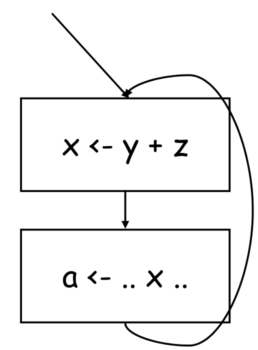
- Meet:

a	$\leftarrow a \wedge top$
bot	$\leftarrow a \wedge bot$
c	$\leftarrow c \wedge c$
bot	$\leftarrow c \wedge d$ (if $c \neq d$)

- Init all vars to: bot or top?

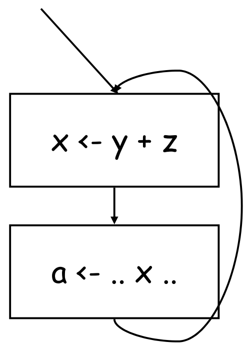
Loop Invariant Code Motion

- When can expression be moved out of a loop?



Loop Invariant Code Motion

- When can expression be moved out of a loop?
- When all reaching definitions of operands are outside of loop, expression is loop invariant
- Use ud-chains to detect
- Can du-chains be helpful?



Liveness (def-use chains)

- A variable x is live-out of a stmt s if x can be used along some path starting a s, otherwise x is dead.
- Why is this important?
- How can we frame this as a dataflow problem?

Liveness as a dataflow problem

- This is a backwards analysis
  - A variable is live out if used by a successor
  - Gen: For a use: indicate it is live coming into s
  - Kill: Defining a variable v in s makes it dead before s (unless s uses v to define v)
  - Lattice is just live (top) and dead (bottom)
- Values are variables
- In[n] = variables live before n
  $= out[n] - kill[n] \cup gen[n]$
- Out[n] = variables live after n
  $= \bigcup_{s \in succ(n)} In[s]$

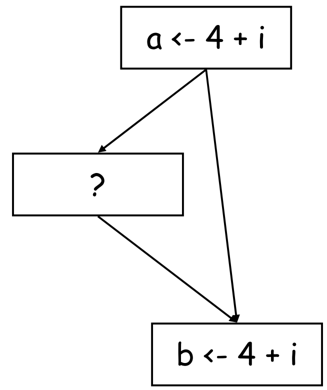
Dead Code Elimination

- Code is dead if it has no effect on the outcome of the program.
- When is code dead?

Dead Code Elimination

- Code is dead if it has no effect on the outcome of the program.
- When is code dead?
  - When the definition is dead, and
  - When the instruction has no side effects
- So:
  - run liveness
  - Construct def-use chains
  - Any instruction which has no users and has no side effects can be eliminated

When can we do CSE?



Available Expressions

- X+Y is “available” at statement S if
  - x+y is computed along every path from the start to S AND
  - neither x nor y is modified after the last evaluation of x+y

a <- b+c

b <- a-d

c <- b+c

d <- a-d

Computing Available Expressions

- Forward or backward?
- Values?
- Lattice?
- gen[b] =
- kill[b] =
- in[b] =
- out[b] =
- initialization?

Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- gen[b] = if b evals expr e and doesn't
  define variables used in e
- kill[b] = if b assigns to x,
  then all exprs using x are killed.
- out[b] = in[b] – kill[b] $\cup$ gen[b]
- in[b] = what to do at a join point?
- initialization?

Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- gen[b] = if b evals expr e and doesn't
  define variables used in e
- kill[b] = if b assigns to x, exprs(x) are killed
  out[b] = in[b] – kill[b] $\cup$ gen[b]
- in[b] = An expr is avail only if avail on ALL edges, so: in[b] = $\cap$ over all p$\in$ pred(b), out[p]
- Initialization
  - All nodes, but entry are set to ALL avail
  - Entry is set to NONE avail

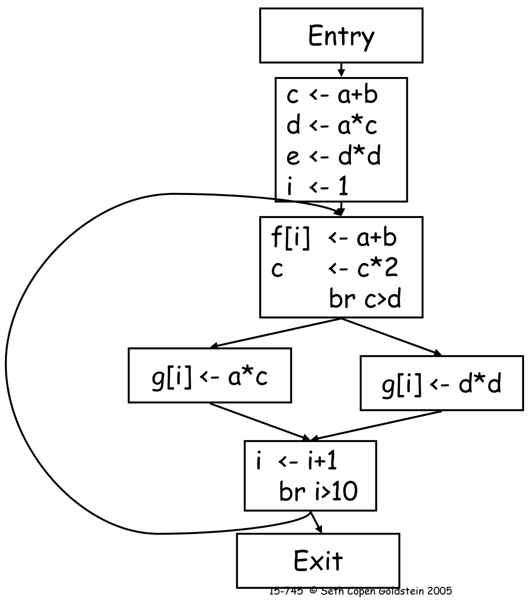
Constructing Gen & Kill

Stmt	Gen	Kill
t <- x op y	{x op y}-kill[s]	{exprs containing t}
t <- M[a]	{M[a]}-kill[s]	
M[a] <- b		
f(a, …)		{M[x] for all x}
t <- f(a,…)		

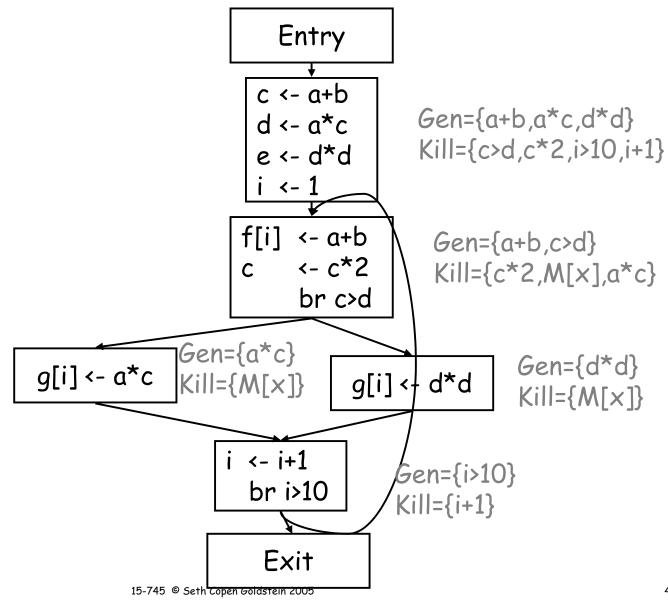
Constructing Gen & Kill

Stmt	Gen	Kill
t <- x op y	{x op y}-kill[s]	{exprs containing t}
t <- M[a]	{M[a]}-kill[s]	{exprs containing t}
M[a] <- b	{}	{for all x, M[x]}
f(a, …)	{}	{for all x, M[x]}
t <- f(a,…)	{}	{exprs containing t for all x, M[x]}

Example

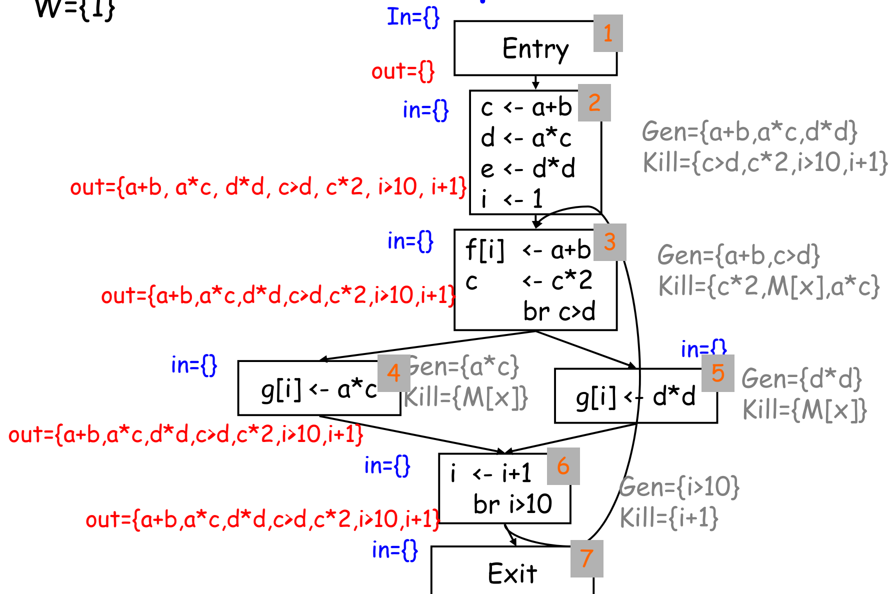


Example



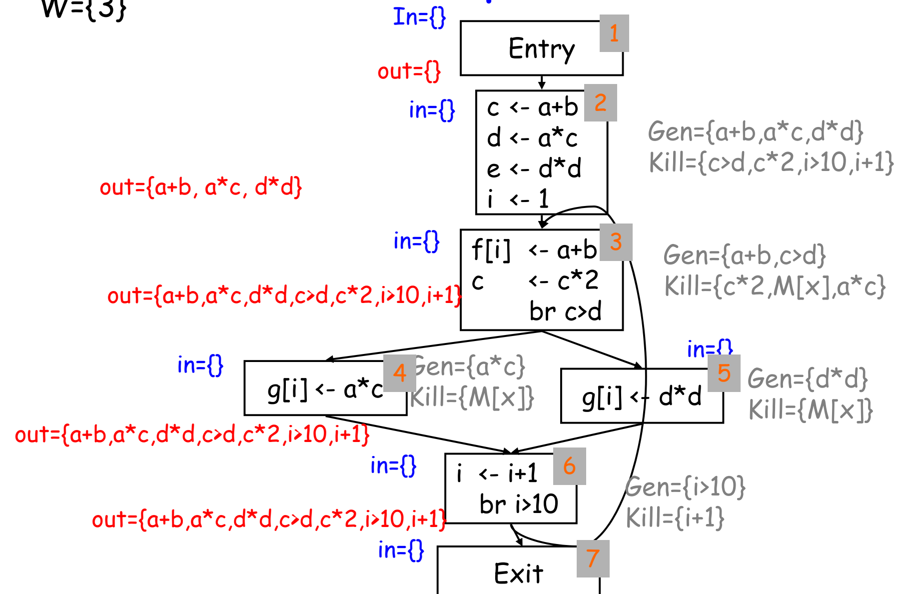
W={1}

Example



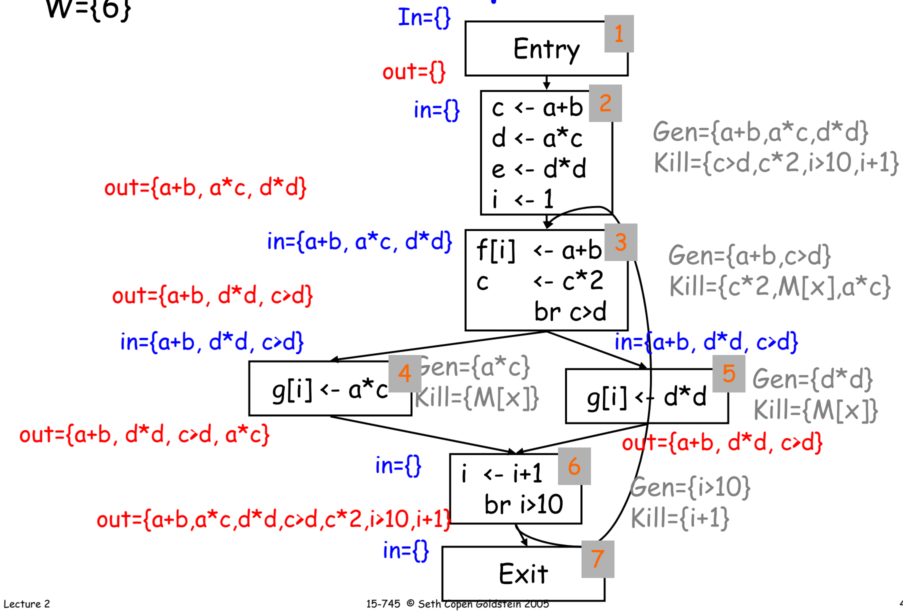
W={3}

Example



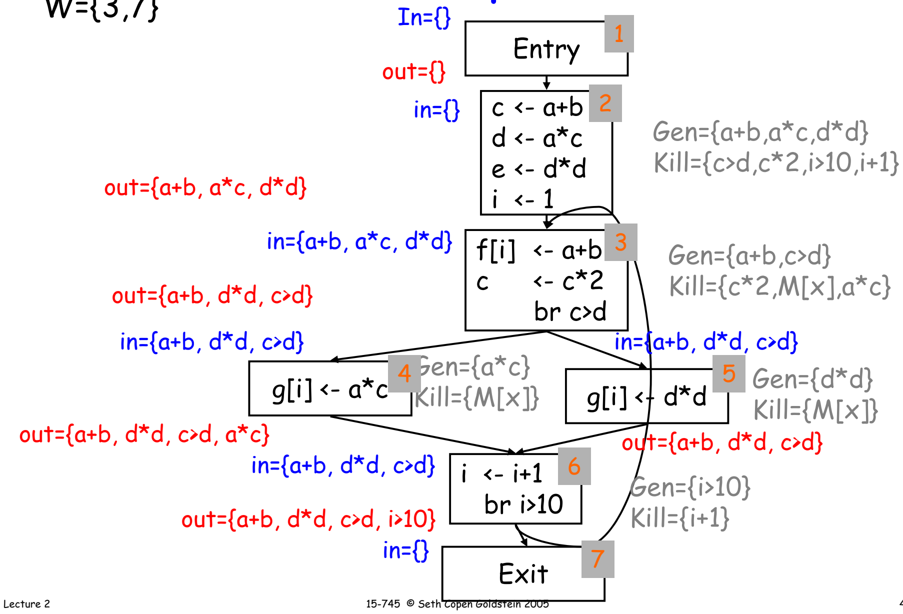
W={6}

Example



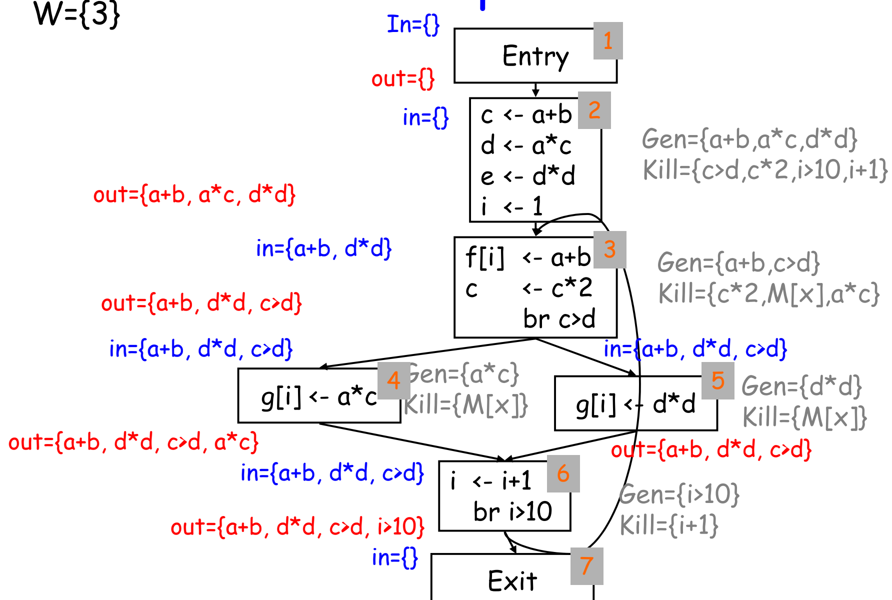
W={3,7}

Example



W={3}

Example



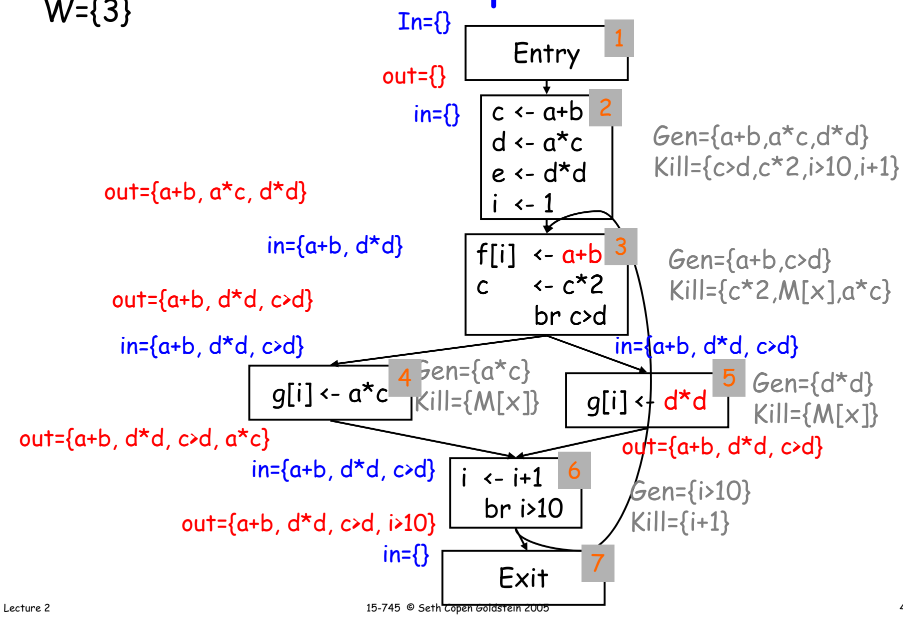
CSE

- Calculate Available expressions
- For every stmt in program

```
    If expression, x op y, is available {
        Compute reaching expressions for x op y
            at this stmt
        foreach stmt in RE of the form t <- x op y
                rewrite at: t' <- x op y
                            t <- t'
        }
        replace x op y in stmt with t'
    }
```

Example

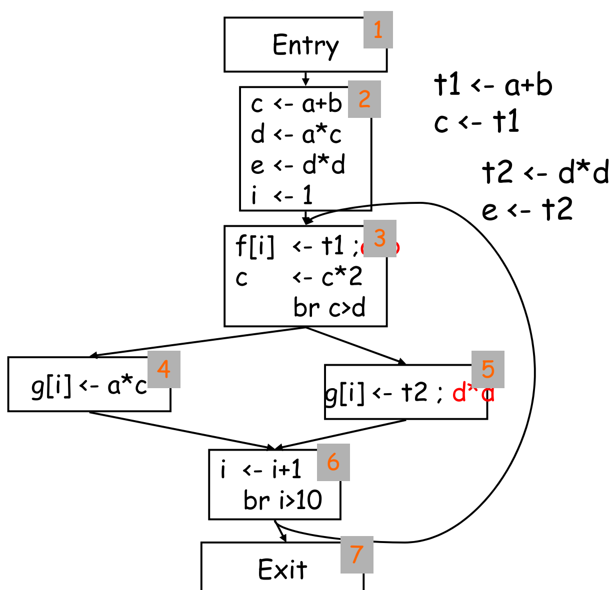
W={3}



Calculating RE

- Could be dataflow problem, but not needed enough, so …
- To find RE for x op y at stmt S
  - traverse cfg backward from S until
    - reach t <- x + y (& put into RE)
    - reach definition of x or y

Example



Dataflow Summary

	Union	intersection
Forward	Reaching defs	Available exprs
Backward	Live variables	

Later in course we look at bidirectional dataflow

Dataflow Framework

- Lattice
- Universe of values
- Meet operator
- Basic attributes (e.g., gen, kill)
- Traversal order
- Transfer function