OBJECTS RECOGNITION BY MEANS OF PROJECTIVE INVARIANTS CONSIDERING CORNER-POINTS.

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ABSTRACT

This paper presents an object recognition technique based on projective geometry for industrial pieces that satisfy geometric properties. First at all, we consider some methods of corner detection which are useful for the extraction of interest points in digital images. For object recognition by means of projective invariants, an excessive number of points to be processed supposes a greater complexity of the algorithm We present a method that allows to reduce the points extracted by different corner detection techniques, based on the elimination of non-significant points, using the estimation of the straight lines that contain those points. Secondly, these groups of points are then used to build projective invariants which allow us to distinguish one object from another. Experiments with different pieces and real images in grey-scale show the validity of this approach.

Keywords: object recognition, corner detector, projective invariants, projective geometry.

1. INTRODUCTION

Object recognition is an essential part of any high-level robotic system. In the last years, there has been variety of approaches to tackle the problem of object recognition. However, there is not a general technique that allows to recognize any type of objects, independently of its intrinsic properties (color, shape, texture, size, ...). Existing recognition techniques can be classified by the way objects are represented in the model data base or by the type of features use. On the first kind we can find geometric representations (silhouettes, superquadrics, algebraic surfaces, complex representation,..) and appearance-base representations, where wide variety of techniques exist, differing in which image information is used and the how the data is stored, but in all, the representations are learned from the images. By the type of features used, we can find techniques based on global features (such as area, compactness,..) or on local features (line segments,…). These approaches are clearly limited to a certain applications, and perhaps the synthesis of sets of these techniques might resolve the problem of object recognition. On the other hand, the projective geometric provides new tools that allow a generic recognition of the object, through the employment of geometric invariants of points, straight lines or conics existing in the object. [Bandlow98], [Tarel00].

The remainder of the paper is organized as follows. In section 2 we review the corner detection techniques employed to detect significant points on the image. In section 3 we present a method that allows to eliminate those points previously detected that correspond with false corner points. In following sections we present some concepts of the projective invariant theory. Based on these concepts in section 5 we show the procedure to construct the projective invariants with the selected points and the algorithm proposed to recognized the objects in the image. Finally in section 6 we show some experiments carried out with the proposed procedure.

2. CORNER DETECTION

Vision-based object recognition begins with an extraction of global or local features from the object image. In our recognition system the features extracted are the corners from the image object. We can define a corner as the image points belonging to a contour where the contour presents a local maximum curvature or as the intersection of two o more contours. [Deriche93].

There are a lot of corner detection techniques. Most of them can be classified into two groups: the first group are based in operations over an object image with a pre-extraction of edges, and the other one consists of approaches that work directly at the gray scale level.

In the experiments presented in following sections, we used classical techniques as the Beaudet, Kitchen, Noble and Harris detectors, and one more recent, the SUSAN detector which provided better results for the pieces we have considered.

In [Beaudet78] was proposed a rotationally invariant operator called DET. This operator is obtained using a second order Taylor's expansion of the intensity surface I(x,y):

$$DET = I_{xx} I_{yy} - I_{xy}^{2} \tag{1}$$

The corner detection is based on the thresholding of the absolute value of the extrema of this operator. DET can be estimated as the Hessian determinant, H, which is related to the product of the principal curvatures $k_{min} \cdot k_{máx}$, called the Gaussian Curvature:

$$H = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix} \tag{2}$$

$$k_{min} \cdot k_{max} = \frac{DET}{1 + I_x^{2} + I_y^{2}} \tag{3}$$

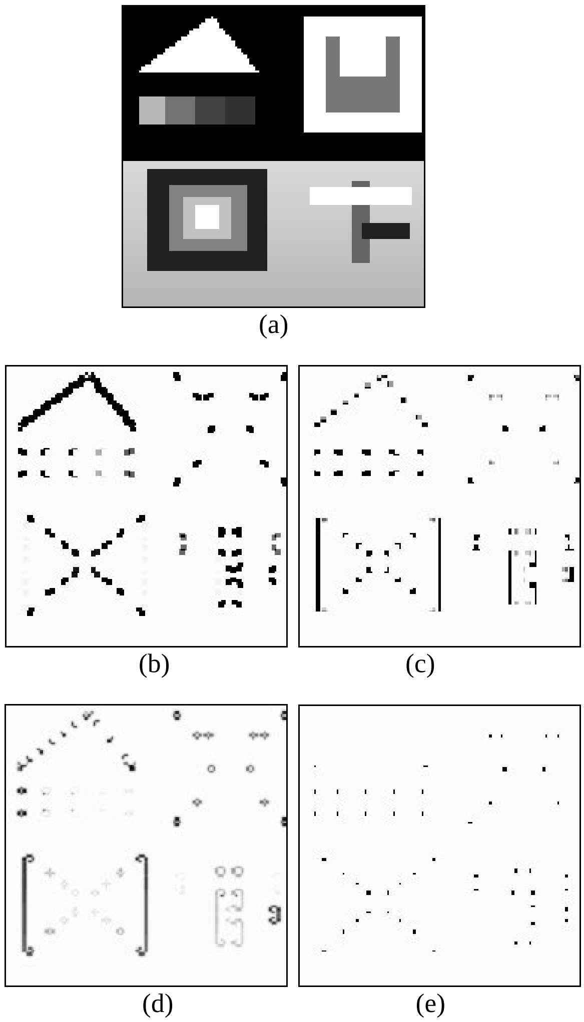
For a pixel I(x,y): if $k_{min} \cdot k_{máx} = 0$, the pixel is a parabolic point; if $k_{min} \cdot k_{máx} > 0$, the pixel is an elliptic point and if $k_{min} \cdot k_{máx} < 0$, the pixel is a hyperbolic point.

DET and the Gaussian Curvature have the same sign because the denominator of Eq. 3 is always positive. Near a corner, DET gives a positive and negative response on both sides of the edge.

Kitchen proposed a measure of cornerness based on the changed of gradient direction along an edge contour multiplied by the local gradient magnitude (Eq.4), the maximum values of k show the possible corners [Kitchen82].

$$k = \frac{I_{xx} I_y^{2} + I_{yy} I_x^{2} - 2 I_{xy} I_x I_y}{I_x^{2} + I_y^{2}} \tag{4}$$

Noble given a theoretical formulation for the corner detection problem using differential geometry [Noble88].



(a) Original. (b) Beaudet. (c) Kitchen.
(d) Noble. (e) SUSAN.
Figure 1

The corner detector SUSAN (Smallest Univalue Segment Assimilating Nucleus) [Smith97] measures the certainty that a pixel of the image be a corner as of its area USAN, that is defined like the total number of pixels in a neighbourhood that they have similar values of intensity, in certain degree, to the value of intensity of the pixel that is considered. The detector SUSAN situates a circular mask around the pixel respected as the nucleus, calculates the area USAN, decides if the pixel is a corner reducing the size of the area USAN of a specific threshold (normally the half or less than the total area USAN), and eliminates the false corners by means of a suppression of not most maximum.

To compare these corner detectors techniques in Fig.1 we show the results obtained using a synthetic image. As we can see, SUSAN corner detector offers best results over objects with very defined boundaries. The objects we are used in our experiments have this property.

3. ELIMINATION OF NON-SIGNIFICANT POINTS.

Often, the corner detectors analysed recognize certain points as corners which do not have the characteristic to be corner-points (false corner points). We consider a corner-point any point at which two or more edges converge. Similarly, an edge is considered as any of the borders of object's contour which separates two planes of different surfaces or which indicates an abrupt change of luminance.

In this section a method is proposed which allows to eliminate false corner-points. This method is based on the knowledge of the possible straight lines present in the input image. There are a lot of procedures that allow us to determine the straight line that best approximates a set of points (border detection, Hough transform).

For each one of the possible corner-points detected $P_j(x,y)$, a set of point lists lr_i is constructed. This set indicates whether the points belong to each one of the different straight lines detected r_i . In order to determine whether a point belongs to a straight line, we verify if the point satisfies the straight-line equations allowing a threshold, ε. The selection of the threshold is important, so if ε is too great, we eliminate all the points in the image.

According to the type of straight lines, we have differents sets:

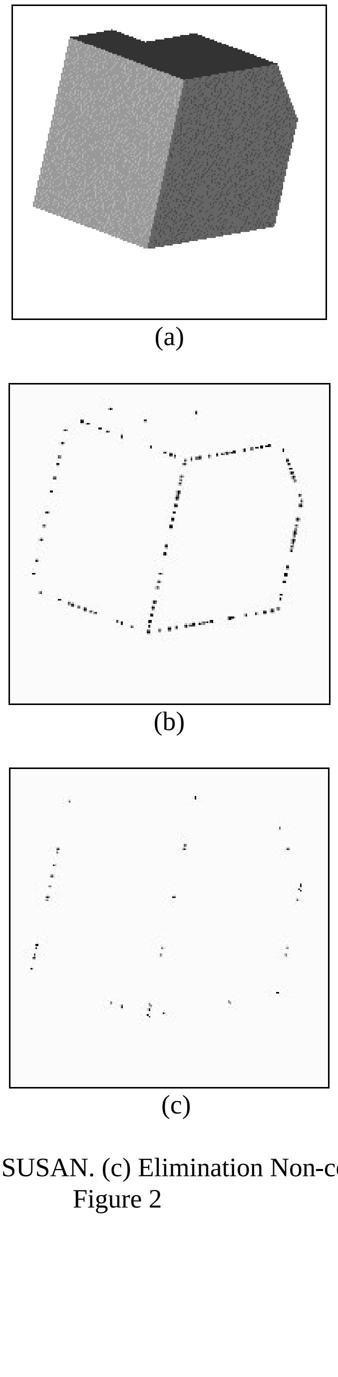
$$lr_i = \begin{cases} \bigcup P_j \,/\, y_j = a_i \cdot x_j + b_i \pm \varepsilon \\ \bigcup P_j \,/\, y_i = y_j \pm \varepsilon \\ \bigcup P_j \,/\, x_i = x_j \pm \varepsilon \end{cases} \tag{5}$$

where lr_i is the i-set formed by all the corner points that belong to a straight line r_i, and a_i and b_i are the straight line coefficients obtained by the Hough transform.

Afterwards, the points that belong to each of the straight lines (horizontal, vertical or inclined) are ordered based on their values as coordinates x and/or coordinates y. Once these points have been ordered, we eliminate all the points that belong to the straight line but are not the ends. Thus, we eliminate all the inner points.

During the process of eliminating the false corner points, we may find points that satisfy more than one equation of the straight line. Such points are internal points of the straight line r_i which can be the ends of other straight lines r_j. For such cases, a previous pre-processing is required before its elimination as inner points.

An example of the elimination of points is shown in Fig. 2.



(a) Original. (b) SUSAN. (c) Elimination Non-corners.
Figure 2

4. BASIC THEORY OF PROJECTIVE INVARIANT.

Projective geometry is based on the geometry inherent inside a central transformation (camera model). In transformations more usual, as Euclidean or Similarity transformations exist invariants properties well known, as the length and the area in the Euclidean or the ratio of lengths and the angles in the Similarity transformation. In Projective transformation there also are invariants, the most known is the ratio of ratio of lengths (or cross-ratio). In this section, we review some aspects of projective invariants built with points from the images. [Hartley00].

Given five general 3D points[1] P_i , $i=1,..,5$ of an object, and p_i , $i=1,..,5$ which correspond to points of the image plane, respectively, two projective invariants on a plane are defined as follows, [Song00]:

$$I_1 = \frac{|M_{431}|\cdot|M_{521}|}{|M_{421}|\cdot|M_{531}|} \qquad I_2 = \frac{|M_{421}|\cdot|M_{532}|}{|M_{432}|\cdot|M_{521}|} \tag{6}$$

where M_{ijk} represents the determinant of a 3x3 matrix, which is composed of the coordinates of three points p_i,p_j,p_k of an image plane and the corresponding points of an object plane (coordinates of the world), respectively.

We rewrite Eq.2 as follows:

$$I_1 = \frac{|p_4 p_3 p_1|\cdot|p_5 p_2 p_1|}{|p_4 p_2 p_1|\cdot|p_5 p_3 p_1|}$$
$$I_2 = \frac{|p_4 p_2 p_1|\cdot|p_5 p_3 p_2|}{|p_4 p_3 p_2|\cdot|p_5 p_2 p_1|} \tag{7}$$

Nevertheless, the representations of these invariants I_1, I_2 are sensitive to changes. Therefore, the value of an invariant associated to a set of points depends frequently on the order in which these points are considered during their computation. This is why $\bar{J}$ is introduced as a more stable invariant when there is a permutation of points:

$$\bar{J} = (J_1, J_2, J_3, J_4, J_5) = \left(J(l_1), J(l_2), J\left(\frac{l_1}{l_2}\right), J\left(\frac{l_2-1}{l_1-1}\right), J\left(\frac{l_1}{l_2}\cdot\frac{l_2-1}{l_1-1}\right) \right) \tag{8}$$

with

$$J[l] = \frac{2l^6 - 6l^5 + 9l^4 - 8l^3 + 9l^2 - 6l + 2}{l^6 - 3l^5 + 3l^4 - l^3 + 3l^2 - 3l + 1} \tag{9}$$

where l can fulfil any of the following relations, for l_1 and l_2 .

$$l_1 = I_1(p_1, p_2, p_3, p_4, p_5)$$
$$l_2 = I_1(p_2, p_1, p_3, p_4, p_5) \tag{10}$$

In this way, the $\bar{J}$ invariant (Eq.9) obtained by means of the procedure is independent of the order of the points chosen to calculate it.

Projective invariants play an important role in the method proposed in following section to recognize the objects in the image. Due to five points in a plane of an object $(P_1, P_2, P_3, P_4, P_5)$ determine the invariant I_1, and the projection of these points of the object over the image plane $(p_1, p_2, p_3, p_4, p_5)$ determine the same invariant, if we study the invariants generated through the corner points of the image we will know the object with those invariants.

5. METHODOLOGY.

In this section, we explain how to recognize pieces using projective invariants of these objects. The main idea consist of to calculate off line some projective invariants of the model of the objects that we have to recognize on line. These projective invariants constitute the data base of the reference invariants. Then, we calculate all the possible projective invariants with the corner detected in the image and matching them with the data base of the reference invariants previously calculated.

From the input images, corner points are extracted (Fig.3). The corners can be obtained using different corner operators or corner detectors (see section 2).

We should mention that some of these corner-detectors detect small pixel-regions as corners. That is to say, the detected corners are formed by several pixels and it is necessary the compute the centres of gravity to reduce such regions to one single pixel.

Afterwards, we construct candidate sets of five points and for each combination (set of five points) the invariant J_i is computated. As the number of combinations C_5^n (Eq.11) is very high, the computational cost is increased exponentially. This factor forces the reduction of the number of possible combinations to avoid an overload of the system. One

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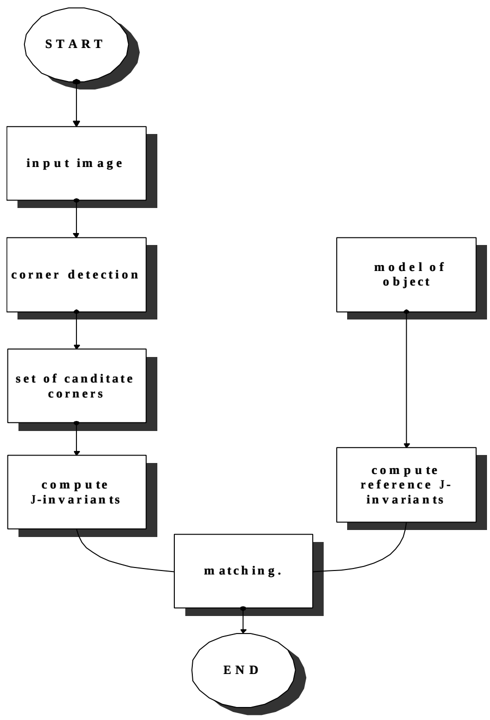
[1] For an object 3D, a point is represented in homogeneous coordinates as *P= (P1,P2,P3,1)*, and for an image, *p=(p1,p2,1)*.

constraint that must be added is not to calculate the invariants of combinations that have three or more collinear points.

$$C_5^n = \frac{n!}{(n-5)!\cdot 5!} \tag{11}$$

The J_i calculated are compared with the models of the pieces, defined as $Jref_i$. (See Fig. 3).

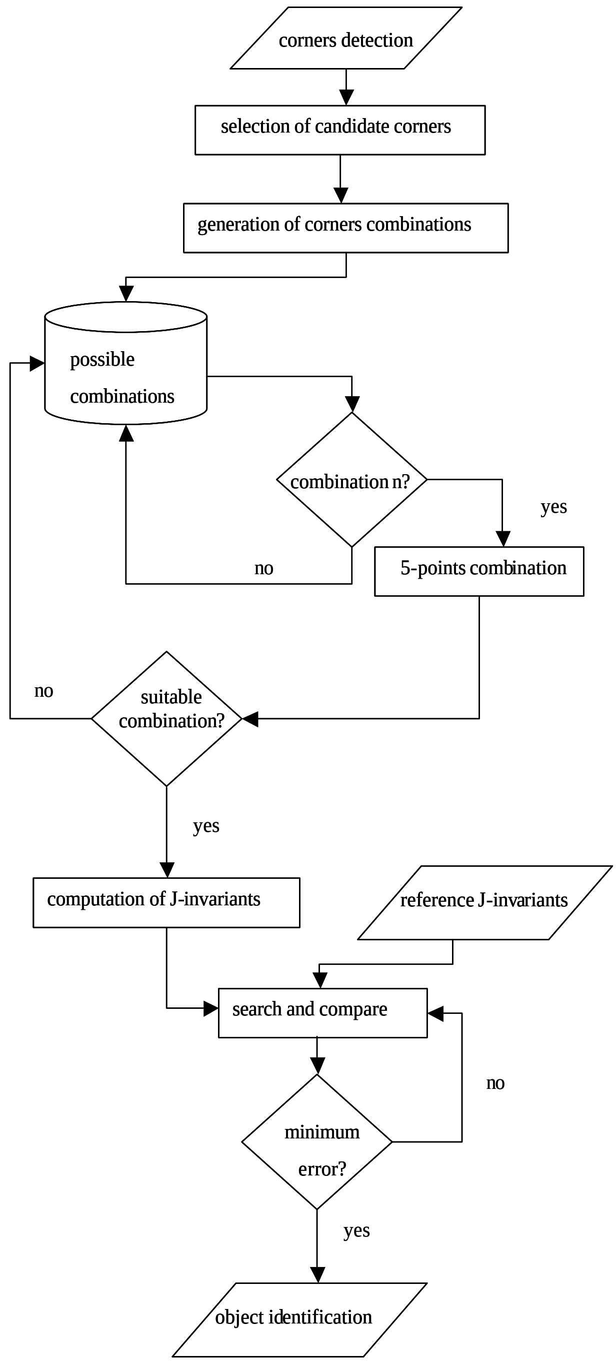
Each of these vectors of J-invariants (calculated as combinations of points) is compared to a vector or several vectors of reference J-invariants. The vectors of reference J-invariants identify each one of the models stored in the database. As such, for each piece stored in the database we have a set of reference J-invariants.



Object Recognition System.
Figure 3

The degree of similarity is measured by the mean squared error. The minimum error will provide the correct matching, and it allows us to state which piece-model of the database is represented in the image captured. Due to in an image several corner pixels would be detected, a great number of invariants would be generated. Some of these invariants would be very similar to other of the different models. So, to determine the model of the piece with the better match with the object in the image it is necessary taking into account not only the best match (minimum error between all reference and the image invariants), but the average of the four minimum errors

In Fig. 4, we illustrate the algorithm developed to recognize the objects in the image through its projective invariants.



Recognition process detailed.
Figure 4

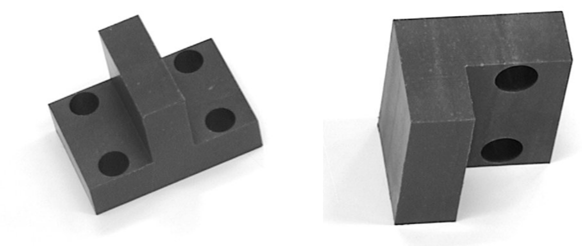
6. EXPERIMENTS ON REAL IMAGES.

The proposed recognition technique has been evaluated on real images from different pieces. The system has eight different models or pieces. Some of the pieces are shown on Fig.5, all of them have any side with 5 or more corners.

Different experiments have been carried out, by changing the size of the set of the model J-invariants (*Jref$_i$*.). On Fig. 6 we can see the error obtained during the matching with the J_i and *Jref$_i$* using just a set of 4 *Jref$_i$*. Fig.6 shows the results achieved after comparing 12 images with the reference invariants of 4 models. To compare these invariants we have chosen the average of the minimum 4 errors between every image invariant and the reference invariants of each model. We have represented the inverse of this average error. Fig. 7 show the same experiment but we have taking into account the average of the minimum 8 errors (8 *Jref$_i$*), comparing again the 12 images with the reference invariants of 2 models.

Adding a noise equivalent to the displacement of the corner detected approximately 2 pixels, does not introduce error in the recognition. In Fig. 8 we illustrate the results achieved considering an error in the corner detection method. As we can see the 12 images evaluated allow perfectly to separate the model A and B and recognize model A as the object in the image.

Finally in Fig.9 we show the results achieved when we compare four images of two different models (image 1 and 2 from object B, and image 3 and 4 from object D) with the reference invariants of all possible models.



(a) (b)

Two objects from the set.
Figure 5

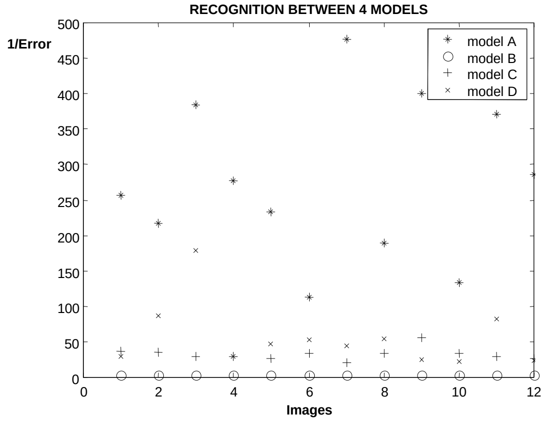


Figure 6

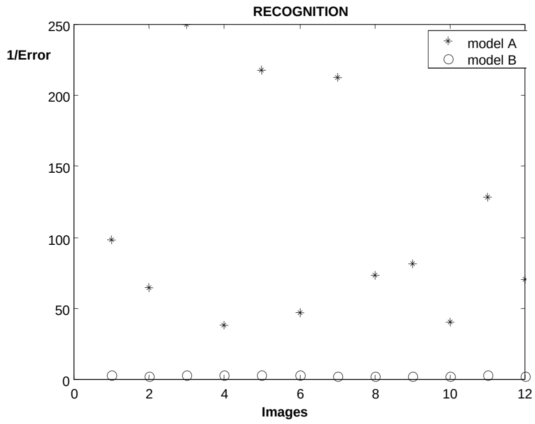


Figure 7

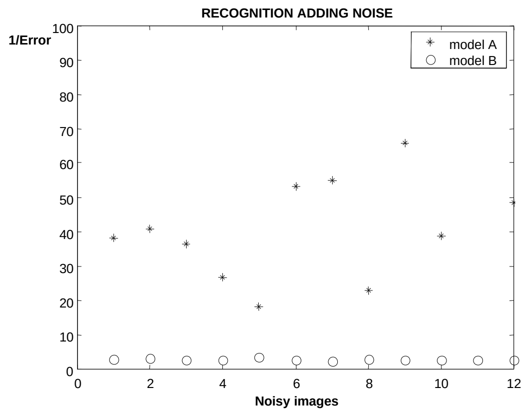


Figure 8

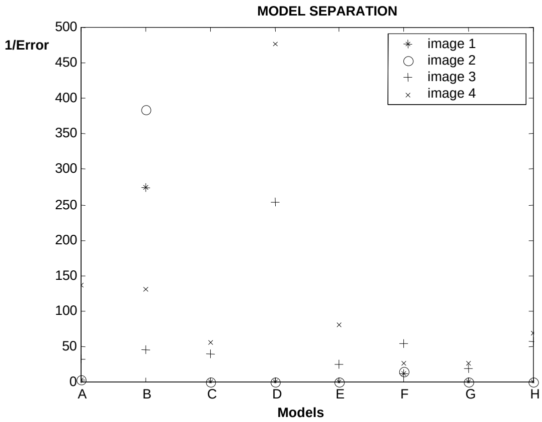


Figure 9

7. CONCLUSIONS.

In this paper, an object recognition system through projective invariants has been presented. The algorithm proposed allows to recognised the object in the image comparing the J-invariant of several points in the image with the J-invariants of every model previously calculated. Furthermore, the method employed allows us to partially reduce the false points detected, and so reducing the search space for the calculated J-invariants. Experiments show that just a small set of J-invariants can identify the object, reducing the computational cost.

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