Inductance Calculation Techniques --- Approximations and Handbook Methods

Marc T. Thompson, Ph.D.
marctt@aol.com

President, Thompson Consulting	**Adjunct Professor of Electrical Engineering**
25 Commonwealth Road	**Worcester Polytechnic Institute**
Watertown, MA 02472	**Worcester, MA 01609**

Abstract

A variety of practical techniques are shown in this tutorial on inductance calculation methods. Although numerical methods can give very accurate answers they do not in general give good insight into design and scaling laws. A set of back-of-the-envelope estimates is given which helps develop intuition into the design and scaling of inductors and transformers. Results of some practical examples are compared to calculations from finite-element analysis, with good comparison between answers.

1   Introduction

The calculation of inductance of wire loops is of practical importance in power electronics, motor design and other applications. Techniques for approximating the inductance of structures often rely on simulations of complicated formulae (using, for instance MATLAB or other tools) or through finite-element analyses (FEA). These numerical techniques are very useful for analysis, but give little <u>design</u> insight. In this paper, approximation techniques and handbook methods are shown for air-core structures that do not easily lend themselves to closed-form solutions. A set of references is also given which is useful for finding the inductance of many different loop shapes.

2   Review of closed-form methods

Following is a brief review of several analytic techniques for inductance calculation. Methods covered are use of Maxwell's equations, energy methods, use of the speed of light, and magnetic circuit analogies.

2.1   Review of Maxwell's equations

Maxwell's equations [1] couple electric fields to magnetic fields, and explain how electromagnetic waves are created. There are four Maxwell's equations, but in magnetic design we only need three: Ampere's Law, Gauss' Magnetic Law and Faraday's Law.

By Ampere's Law, a flowing current creates a magnetic field (Figure 1a), or:

$$\oint_C \vec{H} \cdot d\vec{l} = \oint_S \vec{J} \cdot d\vec{A}$$

where $\vec{H}$ is the magnetic field (Amps/meter) and $\vec{J}$ is current density (Amps/meter^2).

Gauss' Magnetic Law says that the flux density integrated over any closed surface equals zero, or:

$$\oint_S \vec{B} \cdot d\vec{A} = 0$$

This can be illustrated by considering a "Y" shape pipe of magnetic material (Figure 1b). If we assume that the fields are uniform and confined to the pipe, Gauss' law shows that:

$$B_1 A_1 + B_2 A_2 = B_3 A_3$$

Faraday's law shows the mechanisms by which a changing magnetic flux generates eddy currents in a conducting material. The relationship in a conducting (or "Ohmic") material relating the current density and electric field is $\vec{J} = \sigma \vec{E}$ and we can derive (Figure 1c):

$$\frac{1}{\sigma} \oint_C \vec{J} \cdot d\vec{l} = -\frac{d}{dt} \oint_S \vec{B} \cdot d\vec{A}$$

The term on the right of this equation is the negative of the time rate of change of the magnetic flux passing through the surface. This is how induced currents are created in conductors.
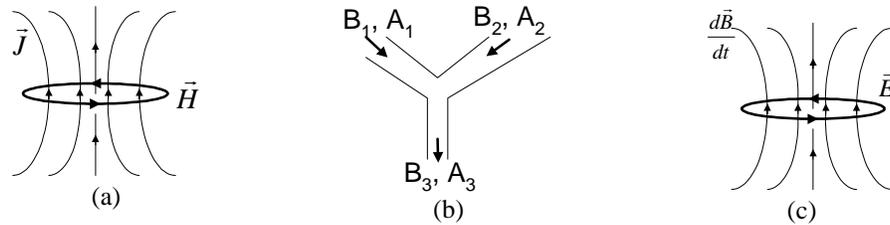
Figure 1. Figures illustrating use of Maxwell's equations. (a) Ampere's law. (b) Gauss' magnetic law. (c) Faraday's law.

Use of Maxwell's equations, or "the brute force method" is the easiest to understand, and often the most difficult to implement. The procedure is as follows:

- Calculate the magnetic flux density B everywhere
- Use this value to calculate the flux Φ linked by the windings
- Once the flux is known, multiply by N to get flux linkage $\lambda = N\Phi$.
- The inductance is the flux linkage divided by the coil current, or $L = \lambda/I$.

2.2 Energy methods

It is commonly known that energy stored in an inductor is:

$$E_m = \frac{1}{2}LI^2$$

In some magnetic structures, the field over all space is easily found and the energy stored in the inductance can be directly calculated. The energy is found by integrating the magnetic flux density over all volume, as:

$$E_m = \frac{1}{2\mu_o}\iiint B^2 dV$$

2.3 Use of the speed of light

Another technique is using the speed of light and transmission line theory. If the capacitance per unit length (C_o) of your structure is known, the inductance per unit length (L_o) is found by:

$$c^2 = \frac{1}{L_o C_o}$$

where c is the speed of light. Furthermore, the speed of light in a medium is given by:

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

where μ and ε are the magnetic permeability and dielectric permittivity of the material, respectively. Since the capacitance of many common structures is known, by using this method you can easily find the inductance. Inductance of the microstrip line (Figure 2) is found by:

$$L_o = \frac{1}{c^2 C_o} = \mu\varepsilon\left(\frac{d}{\varepsilon w}\right) = \mu\frac{d}{w} \Rightarrow L = \mu\frac{dl}{w}$$

Note that the inductance of the microstrip line can be made arbitrarily small by adjusting the geometry.
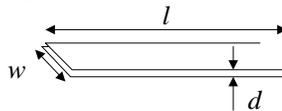
Figure 2. Microstrip line

2.4 Magnetic circuit analogies

The structure of the magnetic laws suggests a possible electrical circuit analogy. In a magnetic circuit, flux (Φ) is forced to flow by the total Ampere-turns (NI) driving the circuit. Using this analogy between electrical and magnetic circuits, we can write:

$$V \Leftrightarrow NI, \quad I \Leftrightarrow \Phi, \quad R \Leftrightarrow \Re$$

In this case, current is analogous to flux, with the proportionality constant being the

reluctance of the magnetic path. The analogy between resistance and reluctance is:

$$R = \frac{l}{\sigma A} \Leftrightarrow \Re = \frac{l}{\mu A}$$

The magnetic circuit method is particularly useful for gapped magnetic circuits, and for circuits with multiple paths (where the simple analogy to resistances in parallel is easily seen).

3   Review of handbook methods

3.1   Straight conductors, wire loops and use of filaments

The self-inductance of a straight conductor of length l and radius R, neglecting the effects of nearby conductors (i.e. assuming that the return current is far away) is given by [2, pp. 35]:

$$L \approx \frac{\mu_o l}{2\pi}\left[\ln\left(\frac{2l}{R}\right) - \frac{3}{4}\right]$$

Many other magnetic structures can be modeled as simpler structure, since inductance is in general a weak function of loop shape for loops with loop radius much larger than wire cross section. For the shapes shown in Figure 3, inductances as shown in the Table 1 are weak functions of wire radius R but a strong function of total wire length.

Figure 3. Some common structures. (a) Circular wire loop. (b) Parallel wire line.

Table 1. Results for some common structures

Circular wire loop	$L = \mu_o a\left[\ln\left(\frac{8a}{R}\right) - 1.75\right]$
Parallel wire line of length l	$L \approx \frac{\mu_o l}{\pi}\left[\ln\left(\frac{d}{R}\right) + \frac{1}{4} - \frac{d}{l}\right]$

These results suggest an interesting result for a polygon of wire [2]. The inductance of a generic polygon of wire with perimeter p and area A may be approximated by:

$$L \approx \frac{\mu_o p}{2\pi}\left[\ln\left(\frac{2p}{R}\right) + 0.25 - \ln\left(\frac{p^2}{A}\right)\right]$$

Note that this function is strongly dependent on the perimeter and weakly dependent on loop area and wire radius. For this reason, the inductance of complicated shapes can often be well approximated by a simpler shape with the same perimeter and/or area.

Conductors can be replaced by filaments in order to calculate inductances, often with very accurate results. For straight filaments made of parallel conductors, with length l and filament-filament spacing d (*Figure 4*a), the mutual inductance is [2, pp. 31]:

$$M_{12} = \frac{\mu_o l}{2\pi}\left[\ln\left(\frac{l}{d} + \sqrt{1 + \frac{l^2}{d^2}}\right) - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l}\right]$$
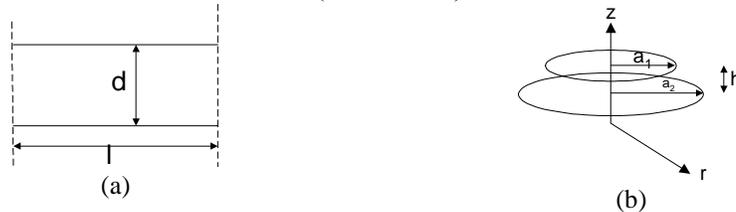

Figure 4. Mutual inductance between filaments. (a) Straight conductors (b) Loops

For the coaxial loops, the mutual inductance between loops as found by Maxwell is:

$$M_{12} = \mu_o \sqrt{a_1 a_2} \, \frac{2}{k} \left[\left(1 - \frac{k^2}{2} \right) K(k) - E(k) \right], \; k = \sqrt{\frac{4 a_1 a_2}{(a_1 + a_2)^2 + h^2}}$$

where *K(k)* and *E(k)* are elliptic integrals. Mutual inductance is a very important parameter to calculate, as if the mutual inductance M_{12} is found the force between loops can be found as

$$f_{12} = -I_1 I_2 \nabla M_{12}$$

3.2 Disk coils

Grover [2, pp. 94] tabulates results for the inductance of a generic round loop with rectangular cross section, with mean radius *a*, axial thickness *b*, and radial width *c* (Figure 5a). This shape is useful to use in motor and Maglev modeling. The inductance is:

$$L = \frac{\mu_o}{4\pi} a P F$$

P and *F* are unitless constants. *P* is a function of the coil normalized radial thickness *c/2a* (Figure 5b) and applies to a coil of zero axial thickness (*b* = 0), and *F* accounts for the finite axial length of the coil. For *b* << *c* and *c* <<*a* (coils resembling thin disks) the factor *F* ≈ 1, an important limiting case. Therefore, for a thin disk coil with double the mean radius *a*, there is a corresponding doubling of the inductance. If the coil is made of *N* turns of wire, and if *c* << *a* the inductance can be approximated by multiplying the above expression by N^2.
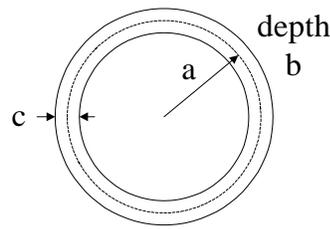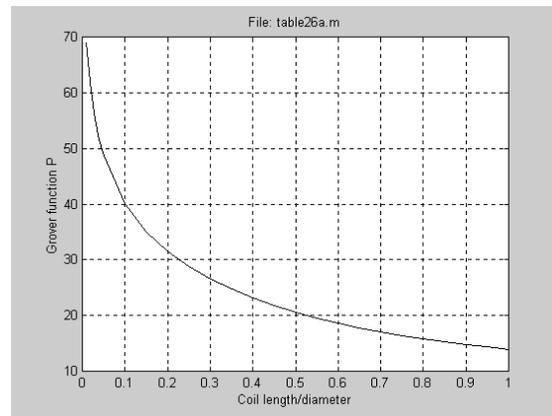
Figure 5. Disk coil. (a) Geometry. (b) Function P

4 Conclusions

This work shows a variety of handbook and approximate inductance calculation results for different shaped conductors. The results are useful as they show scaling relationships that depend on geometry and number of turns. In many cases, simple back-of-the-envelope calculations suffice to calculate the inductance very accurately. A more comprehensive bibliography covering inductance calculation techniques is found on the website listed in the REFERENCES section below [3].

References

[1] James C. Maxwell, *Electricity and Magnetism*, vols. 1 and 2, Dover Publications, 1954
[2] Frederick W. Grover, *Inductance Calculations: Working Formulas and Tables*, Dover Publications, Inc., New York, 1946
[3] Marc T. Thompson, website:
   http://members.aol.com/marctt/Technical/Inductance_References.htm