
Contents

1	Introduction to Lévy Processes	1
1.1	Notation	1
1.2	Poisson Point Processes	3
1.3	The Lévy–Itô Decomposition	5
1.4	Lévy Processes as Markov Processes	7
2	Subordinators	9
2.1	Introduction	9
2.2	Basics	9
2.3	The Renewal Measure	10
2.4	Passage Across a Level	13
2.5	Arc-Sine Laws for Subordinators	15
2.6	Rates of Growth	16
2.7	Killed Subordinators	17
3	Local Times and Excursions	19
3.1	Introduction	19
3.2	Local Time of a Markov Process	19
3.3	The Regular, Instantaneous Case	20
3.4	The Excursion Process	22
3.5	The Case of Holding and Irregular Points	23
4	Ladder Processes and the Wiener–Hopf Factorisation	25
4.1	Introduction	25
4.2	The Random Walk Case	25
4.3	The Reflected and Ladder Processes	27
4.4	Applications	30
4.5	A Stochastic Bound	35

5	Further Wiener–Hopf Developments	41
5.1	Introduction	41
5.2	Extensions of a Result due to Baxter	41
5.3	Les Équations Amicales of Vigon	43
5.4	A First Passage Quintuple Identity	49
6	Creeping and Related Questions	51
6.1	Introduction	51
6.2	Notation and Preliminary Results	52
6.3	The Mean Ladder Height Problem	53
6.4	Creeping	56
6.5	Limit Points of the Supremum Process	59
6.6	Regularity of the Half-Line	61
6.7	Summary: Four Integral Tests	64
7	Spitzer’s Condition	65
7.1	Introduction	65
7.2	Proofs	65
7.2.1	The Case $\rho = 0, 1$	66
7.2.2	A First Proof for the Case $0 < \rho < 1$	66
7.2.3	A Second Proof for the Case $0 < \rho < 1$	68
7.3	Further Results	69
7.4	Tailpiece	80
8	Lévy Processes Conditioned to Stay Positive	81
8.1	Introduction	81
8.2	Notation and Preliminaries	81
8.3	Definition and Path Decomposition	83
8.4	The Convergence Result	86
8.5	Pathwise Constructions of (X, \mathbb{P}^\dagger)	89
8.5.1	Tanaka’s Construction	89
8.5.2	Bertoin’s Construction	91
9	Spectrally Negative Lévy Processes	95
9.1	Introduction	95
9.2	Basics	95
9.3	The Random Walk Case	99
9.4	The Scale Function	100
9.5	Further Developments	104
9.6	Exit Problems for the Reflected Process	109
9.7	Addendum	112

10 Small-Time Behaviour	115
10.1 Introduction	115
10.2 Notation and Preliminary Results	115
10.3 Convergence in Probability	117
10.4 Almost Sure Results	126
10.5 Summary of Asymptotic Results	131
10.5.1 Laws of Large Numbers	131
10.5.2 Central Limit Theorems	131
10.5.3 Exit from a Symmetric Interval	132
References	133
Index	139
List of Participants	141
List of Short Lectures	145